

ADVANCES IN  
FRACTURE RESEARCH

Editors

K. SALAMA, K. RAVI-CHANDAR,  
D. M. R. TAPLIN, P. RAMA RAO

Volume 5



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# **Advances in Fracture Research**

PROCEEDINGS OF THE 7th INTERNATIONAL  
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HOUSTON, TEXAS, 20-24 MARCH 1989

*Editors*

**K. SALAMA, K. RAVI-CHANDAR  
D. M. R. TAPLIN, P. RAMA RAO**

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## **XII. NDE AND EXPERIMENTAL TECHNIQUES**



# Strength Evaluation by Ultrasonic Methods

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## ABSTRACT

Ultrasonic methods to detect and characterize cracks are discussed with a view towards the non-destructive determination of the reduction of strength due to the presence of flaws. The relation between quantitative ultrasonics and strength considerations is considered in some detail. Mathematical modelling for the direct problem of scattering of ultrasonic waves by cracks, as well as for the inverse problem of crack characterization from scattering data, is reviewed. For a problem of quasi-static loading, the Mode-I stress intensity factor induced by the presence of a crack, has been computed directly from the results of a related inverse scattering problem. The analytical work is based as the assumption of a perfect crack geometry. The effects of deviations from a perfect geometry due to near-tip zones of different mechanical properties, partial crack closure and the presence near the crack tips of discrete secondary scatterers such as microcracks, voids and inclusions, are also discussed.

## KEYWORDS

Quantitative non-destructive evaluation, ultrasonics, crack characterization, stress intensity factors.

### 1. Introduction

Cracks in structural components are undesirable, and one can take the point of view that any component which contains a detectable crack should be rejected. Within the theoretical framework of the mechanics of fracture, this would, however, not be a productive point of view, because not all cracks are necessarily harmful over the service life of the component. A more effective approach is to detect and then characterize a crack by a non-destructive method, i.e., determine its location, size, shape and orientation. When the geometrical configuration of a crack is known, the maximum value of the stress intensity factor can be calculated, and the criticality of the crack can be assessed. An even more desirable result would be if a non-destructive test would yield direct information on the maximum value of the stress intensity factor for specified service loads, without the intermediate step of characterization of the crack geometry.

Both approaches, with the second being as yet unproven in practical applications, require sophisticated methods of quantitative non-destructive evaluation.

A single crack is only one kind of material discontinuity. Cracks are particularly objectionable since they are very obvious causes of catastrophic failure, but voids, cavities, inclusions, interfaces, distribution of cracks, or in general terms damaged regions of a material, may have equally deleterious effects on the strength of components. By the use of appropriate non-destructive evaluation methods it should be possible to discriminate between a broad spectrum of flaws and to determine the relevant characteristics of each kind. For purposes of specificity within the allotted length of this paper, the attention will, however, be restricted to components containing cracks.

Most methods of non-destructive evaluation provide only limited information. For strength evaluation it is, however, not good enough just to detect a flaw or the presence of inferior material properties. Quantitative information is required. This need has given rise to a more rigorous and fundamental approach to non-destructive evaluation which is called Quantitative NDE (QNDE).

Nondestructive evaluation (testing) methods include radiography, eddy current methods, dye penetrants, ultrasonic methods, optical methods, thermal wave imaging, x-ray and neutron scattering methods and methods based on nuclear magnetic resonance. Each method has its advantages and disadvantages for particular applications.

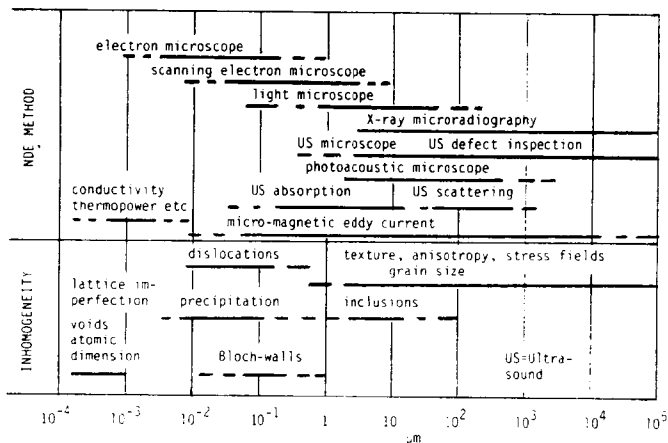


Fig. 1 Range of linear dimensions of material inhomogeneities, and examination methods, after Höller [1].

Some methods of quantitative non-destructive evaluation are laboratory methods, others are already being used extensively in the field. The distinction is primarily one of resolution of linear dimensions of micro-structural parameters. The smallest dimensions can be resolved only with sensitive laboratory equipment. On the other hand macrolevel cracks can be detected and characterized with robust equipment that can easily be transported and put in operation. Figure 1, after Höller [1], shows the range of linear dimensions of various material inhomogeneities in the lower part, while the upper part lists available examination methods. If we

restrict our attention to inhomogeneities with dimensions above one micron, then inclusions, grain sizes, texture, anisotropy and residual stresses cover more than four orders of magnitude of linear dimensions. It should be noted that ultrasonic resolutions has come down to microns by gigahertz ultrasonic microscopy. Ultrasonic scattering becomes effective slightly below 10 microns in materials with low absorption.

The great advantage of ultrasonic techniques is that they are relatively simple. Mechanical waves are used to penetrate a material and so mechanical properties and defects, that are most closely related to useful life and eventual failure, are measured directly. A considerable fraction of the research efforts in QNDE is, therefore, concerned with ultrasonic measurement techniques. This paper will consider only ultrasonic methods for quantitative non-destructive evaluation. The emphasis will be on bodies that contain one or more compact flaws, particularly cracks.

A compact flaw acts as a reflector of ultrasonic wave motion. A bigger flaw reflects more sound, and hence the amplitude of the voltage produced by a piezoelectric transducer that is exposed to the reflected sound is related to the dimensions of the crack. Unfortunately, quantitative characterization of a flaw based on amplitude measurements may be very inaccurate unless the measurement is carefully calibrated. The reason is that part of the incident energy that insonifies the flaw is scattered rather than specularly reflected. A calculation of the total fields which includes scattering effects is rather complicated. Another problem with amplitude considerations is that damping in the material and scattering by secondary sources, such as microstructure, microcracks, voids and inclusions, tend to reduce the measured signals, particularly as the frequency increases. Hence the productive use of amplitude considerations requires careful adjustment of the results for the effects mentioned above.

In an alternative approach it is attempted to characterize a flaw on the basis of measured travel times for signals which travel different paths, that are related to the geometrical configuration of the flaw. This time of flight approach, which is an application of elastodynamic ray theory, is less susceptible to damping effects since these affect wave speeds to a lesser extent than amplitudes. To be useful the pulses must be quite short, or the flaws should be relatively large. The pulses that are generated by mechanisms other than specular reflection may be very small, sometimes so small that they cannot be distinguished from noise that is produced in the system, unless sophisticated data processing techniques are applied.

In general terms, two approaches to ultrasonic flaw detection and characterization have been taken. The imaging approach seeks to process the scattered field in such a manner that a visual outline of the object is produced on a display. The inverse-scattering approach attempts to infer geometrical characteristics of a flaw from either the angular dependence of its far-field scattering amplitude at fixed frequency, or from the frequency dependence of its far-field scattering amplitude at fixed angles.

Imaging is conceptually simple, but difficult to implement with ultrasonic signals. The basic idea is to collect signals scattered by the flaw into an experimentally accessible aperture and then to recombine these signals such that the ones scattered from a particular point on the flaw add coherently at a unique point on the image. At other points in the image, these same signals should not add in phase. Not surprisingly, the main difficulty with imaging is to obtain an acceptable level of resolution. A number of innovative instruments and signal processing methods have been developed. An important example is the acoustic microscope. This instrument was initially developed in the gigahertz frequency regime, where

the wavelength is on the order of a micrometer and resolution approaches that of optical microscopes. The acoustic microscope can, however, also "see" under the surface of a solid, and it can image both microstructure and flaws. Imaging is an important method of detection and characterization, but it falls somewhat outside the scope of this paper. For an interesting discussion of its potential, but also of the current problems with imaging, we refer to the review paper by Thompson and Thompson [2].

In experimental work on quantitative flaw definition by the scattered-field method, either the pulse-echo method with one transducer or the pitch-catch method with two transducers is used. The transducer(s) may be either in direct contact with the specimen, or transducer(s) and specimen may be immersed in a water bath. Most experimental setups include instrumentation to gate out and spectrum analyze the signal diffracted by a flaw. The raw scattering data generally need to be corrected for transducer transfer functions and other characteristics of the system, which have been obtained on the basis of appropriate calibrations. After processing, amplitudes and phase functions, are available as functions of the frequency and the scattering angle. These experimental data can then be directly compared with theoretical results. For the inverse scattering method, the experimental data can be interpreted with the aid of analytical methods, to characterize the scatterer.

In recent years several methods have been developed to investigate scattering of elastic waves by interior cracks as well as by surface-breaking cracks, in both the high- and the low-frequency domains. The appeal of the high-frequency approach is that the probing wavelength is of the same order of magnitude as the length-dimensions of the crack. This gives rise to interference phenomena which can easily be detected. The advantage of the low frequency approach is that useful approximations can be based on static results. A large body of numerical results has been developed for the direct problem of elastodynamic scattering by an inhomogeneity. In particular, several numerical programs based on the use of the T-matrix method and the boundary element method have been developed.

The solution to the direct scattering problem, that is, the computation of the field generated when an ultrasonic wave is scattered by a known flaw, is a necessary preliminary to the solution of the inverse problem, which is the problem of inferring the geometrical characteristics of an unknown flaw from either the angular dependence of the amplitude of the scattered far-field at fixed frequency, or from the frequency dependence of the far-field amplitude at fixed angle. In recent years solutions to the inverse problem have been obtained by the use of nonlinear optimization methods.

Other applications of ultrasonic wave methods, e.g., to acoustic emission techniques and distributed property measurements are not discussed here, due to length limitations. More complete discussions of ultrasonic QNDE methods can be found in recent review papers by Thompson and Thompson [2], Fu [3], and Thompson [4]. The role of elastodynamic scattering problems in quantitative non-destructive evaluation has also been reviewed by, e.g., Gubernatis [5] and Bond et al. [6]. Interesting practical applications have been discussed by Coffey and Chapman [7].

Section 2 starts the main body of this paper with a discussion of the relation between quantitative ultrasonics and strength considerations. The principles and objectives of mathematical modelling both for ultrasonic wave scattering and for static analysis of the fields of stress and deformation near the edge of a crack are discussed in Section 3. Sections 4 and 5 are concerned with two methods for the direct scattering problem: the time domain finite difference method, and ray theory methods.

Corresponding methods for the inverse problem of crack characterization are also briefly discussed. Frequency domain techniques are discussed in Section 6. Most of the results of this Section are based on a representation integral for the scattered displacement field. In Section 7 we discuss an inverse method to characterize a crack of general shape in an elastic solid, by the use of ultrasonic crack-scattering data in conjunction with the integral representation for the scattered-field. For a given set of scattered field data the inverse problem is formulated as a nonlinear optimization problem. For the case of normal incidence discussed in this paper, the solution gives the location of the crack, and the crack-opening volume induced by the probing ultrasonic field. For a problem of quasi-static loading, the Mode-I stress intensity factor induced by the presence of the crack has been computed directly from the results of a related inverse scattering problem. Section 8 is concerned with the effect of near-tip zones of different mechanical properties on the scattered field. Some results for scattering by a partially closed crack are given in Section 9. Effects of the presence of discrete scatterers such as microcracks, voids and inclusions near a crack tip are discussed in Section 10.

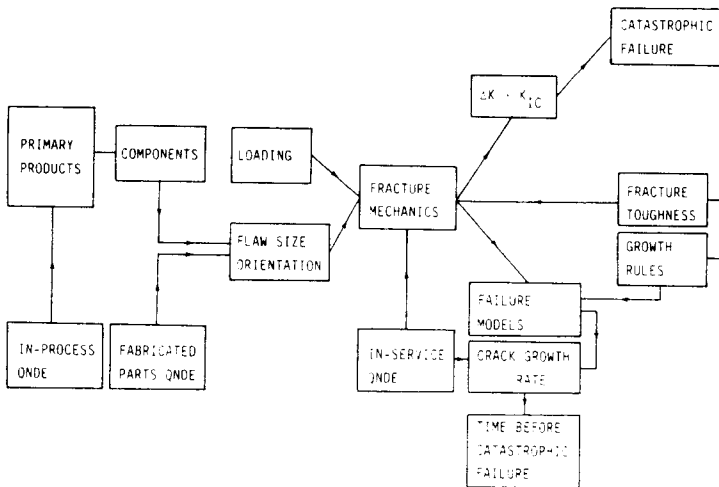


Fig. 2 Role of quantitative non-destructive evaluation in the life-cycle of a component.

## 2. Strength Considerations

The role of quantitative non-destructive evaluation during the various stages of the life cycle of a structural component is illustrated in Fig. 2. The schematic depiction is meant to apply to metal components which are subjected to cyclic loading, and hence may sustain metal fatigue. As indicated in Fig. 1, Quantitative NDE methods should enter in the material processing stage, to play a role in maintaining material quality of primary products. At this stage QNDE methods ensure that primary products do not contain cracks or other flaws whose dimensions exceed a certain specified level. Plates, sheets and strips are examples of primary metal products that should be inspected and subjected to quality control before being used for the fabrication of parts. In the next stage, QNDE methods should be applied to characterize flaws that have been induced in the process of

fabricating components. The maximum dimensions of such cracks, in conjunction with the magnitude of the cyclic load, can be used to calculate the maximum values of corresponding stress intensity factors. Naturally these maximum values,  $\Delta K$ , should be less than the fracture toughness. If this is indeed the case, then a crack may still propagate, but at a controlled rate which in principle is predictable. Hence within the framework of the "fail safe" or "damage tolerant" philosophy, a part containing a macroscopic flaw is acceptable if it can be shown that at the predicted stress levels, the flaw will not grow to critical size during the design lifetime. Reliable quantitative methods of non-destructive in-service inspection of parts are clearly essential for a successful implementation of the damage tolerant philosophy. Flaws which at the time of in-service inspection are greater in size than consistent with the design lifetime, must be detected and characterized. On the other hand the QNDE procedure should not reject components that contain only smaller size flaws. A part should be returned to service if no flaws are found, or if it can be shown that the size of a detected flaw is small enough that it will propagate to failure only over a period substantially larger than the next inspection interval. Very considerable life cycle cost savings can be achieved with this "retirement for cause" procedure. Retirement for cause procedures have been discussed in considerable detail in the literature, see e.g. Refs. [8]-[9].

Clearly, it is extremely important that QNDE procedures, (for the present discussion: ultrasonic techniques) can reliably determine the location, size, shape and orientation of cracks. In the sequel we will discuss the methods and their mathematical background. A more direct connection skips the geometrical crack-characterization stage and attempts to directly determine stress intensity factors under service loads from ultrasonic scattering data. In the remainder of this Section we will discuss this interesting connection between fracture mechanics and scattering of ultrasonic waves, which was noted by Budiansky and Rice [10].

The incidence of ultrasonic waves on a crack generates stress intensity factors along the edge of the crack. The calculation of dynamically induced stress intensity factors is a problem of long-standing interest in dynamic fracture mechanics, see, e.g., [11].

For the sake of simplicity of exposition, let us consider a flat crack of arbitrary shape in the  $x_1, x_2$ -plane, and let a plane longitudinal wave of the special form

$$u_3(x_3, t) = \delta(t - x_3/c_L) \quad (1)$$

be incident on the crack. Here  $\delta(\cdot)$  is the Dirac delta function and  $c_L$  is the longitudinal wave speed. Let the corresponding crack-opening volume be denoted by  $V^\delta(t)$ . Now, if the incident wave is of the more general form

$$u_3(x_3, t) = f(t - x_3/c_L) H(t - x_3/c_L) \quad (2)$$

it follows immediately by linear superposition that the corresponding crack-opening volume may be written as

$$V(t) = \int_0^t f(t-s) V^\delta(s) ds \quad (3)$$

The asymptotic form of  $V(t)$  as time increases, depends on the stress field corresponding to Eq.(2). If the stress component  $\sigma_{33}(x_3, t)$  which corresponds to Eq.(2) approaches a finite limit as  $t \rightarrow \infty$ , i.e., as



$$\lim_{t \rightarrow \infty} [-(\lambda+2\mu)/c_L] f'(\tau-x_3/c_L) = \sigma_{33}^{st} \quad (4)$$

then  $V(t)$  also approaches a finite limit, which just equals the static crack-opening volume induced by the static stress  $\sigma_{33}^{st}$ .

Equation (3) is a convolution integral. It is well known that the Fourier transform (over time) of (3), which is indicated by a bar, is of the form

$$\bar{V}(\omega) = \bar{f}(\omega) \bar{V}^\delta(\omega) \quad (5)$$

It is also known that the long-time value of a quantity is related to the value at small  $\omega$  of its Fourier transform. It can be shown that

$$\begin{aligned} v^{st} &= \lim_{\omega \rightarrow 0} i\omega \bar{V}(\omega) \\ &= \lim_{\omega \rightarrow 0} i\omega \bar{f}(\omega) \bar{V}^\delta(\omega) \\ &= -\frac{c_L}{\lambda+2\mu} \sigma_{33}^{st} \lim_{\omega \rightarrow 0} \frac{\bar{V}^\delta(\omega)}{i\omega} \end{aligned} \quad (6)$$

Now, if it would be possible to obtain  $\bar{V}^\delta(\omega)$  from an ultrasonic test, then  $v^{st}$  corresponding to the static stress  $\sigma_{33}^{st}$  could be obtained directly from Eq. (6).

Data obtained from ultrasonic tests generally yield displacements (or their Fourier transforms) at a limited number of points of observation. As shown in the sequel it is possible to extract crack-opening volumes as functions of the frequency from measured displacements, on the basis of elastodynamic modeling formulas. Once a crack-opening volume has been obtained, it is possible to estimate the maximum value of a related static stress intensity factor.

The approach is based on a formula stated by Budiansky and O'Connell [12] which relates the static crack opening volume to the static stress intensity factor by

$$v^{st} = \frac{1-\nu}{3\mu} \sigma_{33}^{st} \int_S \rho_c [k_I(l, \omega)]^2 dl \quad (7)$$

where  $S$  is the edge of the crack and  $\rho_c$  is a length parameter of the crack. Also,  $k_I = K_I/\sigma_{33}^{st}$  is the reduced mode I stress-intensity factor. According to Budiansky and Rice [10] the right hand side of (7) can be approximated by an expression in terms of the maximum value of  $k_I$ . This results in

$$v^{st} = \frac{(1-\nu)\pi^3}{24\mu} \sigma_{33}^{st} [(k_I)_{\max}]^6 \quad (8)$$

or

$$(k_I)_{\max} = \left[ \frac{24\mu}{(1-\nu)\pi^3} \frac{v^{st}}{\sigma_{33}^{st}} \right]^{1/6} \sigma_{33}^{st} \quad (9)$$

The Fourier transform of a plane longitudinal wave which is incident on the crack in an ultrasonic test, may be written as