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Circuits, Signals, and Systems

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Preface

The core curriculum taken by all undergraduates in the Department of Electrical Engineering and Computer Science at MIT consists of four courses. Until a few years ago, two of these were relatively traditional introductory electrical engineering courses in circuit theory and linear system theory, and two were introductory computer science courses in languages and architectures. By 1978, however, it had become clear that the needs and interests of the students in the department are diverse. For some, the core electrical engineering courses are the first step on a path leading to professional careers in electronic circuit and device design, control and communication system design, or engineering applications of electromagnetic fields and waves. For other students, these core courses are the last they will take with an engineering or physical science flavor. In 1979—facing up to this diversity—the department redesigned the core electrical engineering subjects in both content and style. Topics such as elementary electronic devices and circuits were added, the mathematical emphasis was broadened to include more applications, and modest laboratory exercises were incorporated to provide further experience with engineering reality. Complementary changes were introduced in the core computer science subjects, as described in other books in this series.

This book has evolved from a set of lecture notes for the second of the electrical engineering core subjects. The background assumed is an appreciation of the constitutive relations for common electrical circuit elements (including simple semiconductor devices and operational amplifiers), some skill at exploiting Kirchhoff's Laws to write dynamic equations for simple circuits in either node or state form, and an ability to solve such dynamic equations in simple cases when the drives are either zero or (possibly complex) exponentials. Chapter 1 reviews and somewhat extends this background material. A textbook for the prerequisite course is in preparation.

Chapters 2–4 introduce operational methods (the unilateral Laplace transform), system functions, and the complex frequency domain in a circuit context, leading to an input-output (functional or "black-box") characterization of linear time-invariant (LTI) circuit behavior. Interconnections of LTI systems are explored in Chapters 5 and 6, with particular emphasis on the practical and conceptual consequences of feedback. The next two chapters use the precise parallels between lumped continuous-time systems (described by differential equations and Laplace transforms) and discrete-time systems (described by difference equations and Z-transforms) both to review the mathematical structure of LTI circuits as presented in the earlier chapters and to introduce important applications of that structure in a broader context.

Discrete-time systems also provide a convenient vehicle for introducing the input-output characterization of LTI systems directly in the time domain through the unit sample response and convolution. This is done in Chapter 9. In Chapter 10, these ideas are extended to continuous-time systems. The mathematical subtleties of continuous-time impulses are carefully explored in Chapter 11 and resolved through an operational or generalized-function approach.

Examination of the general convolutional characterization of the black-box behavior of LTI systems reveals two overlapping categories of systems—those that are *causal* (although not necessarily stable) with inputs specified for $t > t_0$ and with the effects of past inputs implied by a state at $t = t_0$, and those that are *stable* (although not necessarily causal) with inputs typically specified for all time, $-\infty < t < \infty$. The first category—causal systems—may be loosely identified as systems of *control* type; appropriate analytical tools include the ones described in the first half of the book. The second category—stable systems—may be loosely described as systems of *communication* type; appropriate analytical tools are based on bilateral transforms—particularly (in an introductory treatment that does not require complex function theory as a prerequisite) the Fourier transform as described in the second half of the book.

The eigenfunction property of complex exponentials for LTI systems and the significant role thereby conferred on sums or integrals of exponentials as signal representations are explored in Chapters 12 and 13, and the fundamental properties of Fourier series and transforms are derived. The implications of these properties for such important applications as sampling, filtering, and frequency shifting (modulation) are discussed in the next four chapters. Throughout (but especially in Chapter 16, where duration-bandwidth relationships and the uncertainty principle are derived) emphasis is placed on the insights that can be gained from looking at system behavior simultaneously in the time domain and the frequency domain. To facilitate this process, time-frequency (duality) relationships are developed in as symmetric a way as possible. The power of this approach is particularly evident in the detailed examination of communication system engineering principles in Chapter 17.

Chapter 18 develops an application of a different kind—digital signal processing. The discrete-time Fourier transform (derived in Chapter 14 as a dual interpretation of the Fourier series formulas) is employed to probe various approaches to the use of digital hardware for carrying out operations such as low-pass filtering of analog waveforms.

In the next to last chapter, it is shown that knowledge of such averages of the input waveform as correlation functions and power density spectra is sufficient to determine corresponding averages of the output waveforms of LTI systems. The relationship of characterizing signals in terms of averages to the idea of a random process is discussed. Chapter 20 applies these results to explain the performance advantages of such wideband communication systems as PCM and FM.

Throughout, I have sought to balance two somewhat conflicting requirements. On the one hand, as befits an introductory course designed for sophomores and juniors, I have tried to hold the mathematical level to the absolute

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minimum necessary for an explication of the basic principles being discussed. On the other hand, as befits the breadth and maturity of our students' interests, I have chosen to talk about those topics (independent of difficulty or subtlety) that seem most exciting in terms of their mathematical, philosophical, or applicational interest. The first requirement, for example, has led me to resist the temptation to formulate the circuit dynamic equations in matrix form. The second has induced me to introduce certain aspects of generalized functions and random processes even though the discussion may be dangerously oversimplified. The balance is particularly tricky in connection with proofs of mathematical theorems; in general, I have ignored rigorous formalisms, trying to follow Heaviside's advice: "The best of all proofs is to set out a fact descriptively so that it can be seen to be a fact." I have also been sensitive to Bertrand Russell's aphorism: "A book should have either intelligibility or correctness; to combine the two is impossible." This is, perhaps, a bit bleak, but where I have had to choose, I have opted for intelligibility.

The problems at the end of each chapter (except the last) are an integral part of the text. Some are intended to provide practice in the topics of the chapter; the simplest of these are separately identified as exercises and usually include answers. Many of the problems, however, are designed to extend or amplify the text material. Reading through all the problems, to discover at least the kinds of topics discussed there, should be considered an essential part of studying each chapter.

At MIT, the material in this book is the basis for a 14-week course meeting about 5 hours a week in groups of various sizes (2 large lectures, 2 smaller recitations, and 1 very small tutorial). The lectures and accompanying demonstrations attempt to convey broad insights and perspectives that are difficult to communicate in other ways; they are also used to extend topics such as computational methods and the bilateral Laplace transform that are only briefly discussed in the text. Four laboratory assignments are part of the course; presently these cover the design of active filters, a comparison of numerical integration techniques for differential equations, a study of systems constructed from tapped delay lines, and the properties of FM modulators and phase-locked loops.

Not all topics in every chapter of the book are covered each term; not all topics that are covered are mastered by every student. Nevertheless, after five years of experience with the course in this form, we believe that most students come away with a broad understanding of complex system behavior as well as a set of elementary skills that can serve as adequate preparation for more advanced courses on more specialized topics. Although the book has been designed as an introductory text rather than a comprehensive treatise or a handbook of useful formulas and algorithms for solving problems, it could, I believe, be used in other ways. In particular, complemented with further notes and practice material of the instructor's choosing, the book would make an appropriate basis for a year-long senior-level preprofessional course in many curricula.

Despite what it may say on the title page, no book of this kind is the work of one person. Both the content and style have been enormously influenced by

my early associations with that great teacher Ernst Adolph Guillemin. Over the more than 30 years that I've been teaching some of this material, numerous colleagues and students have also had an enormous impact. I am particularly happy to acknowledge the significant contributions of Hal Abelson, Rob Buckley, Mike Dawson, Bob Gallager, Lennie Gould, Bart Johnson, Bob Kennedy, Marvin Keshner, Jae Lim, Peter Mathys, Bill Schreiber, Cam Searle, Steve Senturia, Gerry Sussman, Art Smith, Dick Thornton, George Verghese, Stuart Wagner, Alan Willsky, John Wyatt, Mark Zahn, and Victor Zue. I also deeply appreciate the many hours devoted to this book and the notes that preceded it by the MIT Press editor, Larry Cohen; by a number of secretaries over the years, including in particular Barbara Ricker and Sylvia Nelson; by Pat McDowell, who drew the figures; and by Amy Hendrickson, who assisted me in the final typesetting in TEX on computer facilities generously made available by Lou Braida. My wife Sandy and my children, who gave up numerous opportunities so that Daddy could work on "the book," deserve my most heartfelt thanks. Finally, I am grateful to a succession of Department Heads and Deans (and most recently to the Bernard M. Gordon Engineering Curriculum Development Fund) for the financial support that made this book possible, and for the faith—in the absence of any compelling evidence—that it might someday be completed.

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1

DYNAMIC EQUATIONS AND THEIR SOLUTIONS FOR SIMPLE CIRCUITS

1.0 Introduction

The goal of this first chapter is twofold: to remind the reader of the basic principles of electrical circuit analysis, and to formulate these principles in appropriate ways so that we can develop them further in the chapters to come. Circuits (or networks) are, of course, arrangements of *interconnected elements*. But the word "circuit" can refer either to a *real* reticulated structure that we build in the laboratory out of elements such as resistors, capacitors, and transistors, interconnected by wires or printed-circuit busses, or it can refer to a *model* that we develop abstractly. For the most part in this book, we shall be discussing circuits in this latter sense (although we should never forget for long that, as engineers, we are interested in circuit models primarily as aids to the design and understanding of real systems). Our first task is therefore to define what we choose the words "interconnected" and "elements" to imply as abstractions. The circuit model then becomes a graphic way of specifying a set of *dynamic equations* that describe the behavior of the circuit. But such a description is usually only implicit; in the latter part of this chapter, we shall explore how simple dynamic equations can be solved to yield an explicit specification of the circuit response to simple stimuli. A goal of later chapters will be to extend and refine the ideas of this chapter into a collection of powerful tools for the analysis and design of the complex systems that characterize modern engineering practice.

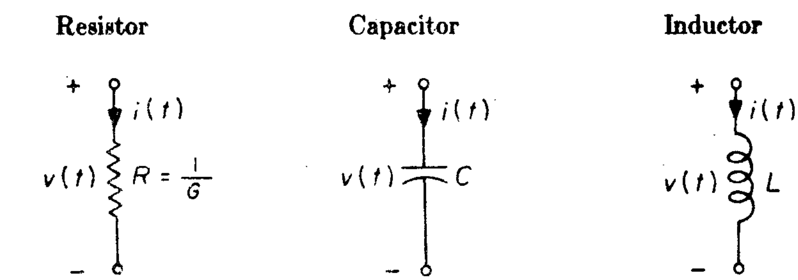
1.1 Constitutive Relations for Elements

In models of electrical circuits, the elements or branches are characterized by equations (called *constitutive relations*) relating branch voltages and currents.* The simplest abstract electrical elements are the linear resistors, capacitors,

*It is perhaps useful to point out that most of the ideas to be studied in this book also apply to a variety of other situations in which the important dynamic variables are *efforts* and *flows* (e.g., mechanical forces and velocities, temperatures and heat flows, chemical potentials and reaction rates). In addition, many models proposed in the social and biological sciences are described by equations similar to those we shall be investigating. Some texts go to elaborate lengths to formalize these *analogies*. We are not convinced such efforts are worthwhile, since most students make the necessary translations easily. Examples from non-electrical applications are scattered throughout the problems in this book.

inductors, and ideal sources described in Figure 1.1–1. Note that the reference directions for current, $i(t)$, and voltage, $v(t)$, in the constitutive relations are always *associated* as shown: that is, the positive direction for $i(t)$ is selected to be through the element from the positive reference terminal for $v(t)$ towards the negative terminal. The units of $i(t)$ and $v(t)$ are *amperes* and *volts* respectively.

Circuit elements may have more than two terminals. Perhaps the most important abstract multiterminal element is the *ideal controlled* (or *dependent*) *source,* of which there are four basic types as shown in Figure 1.1–2. Ideal controlled sources arise most commonly as idealizations for such active elements as transistors and op-amps in their linear regions. The *ideal op-amp,* for example, is an important special case of an ideal voltage-controlled voltage source obtained



Resistor	Capacitor	Inductor
$v(t) = Ri(t)$	$i(t) = C\dfrac{dv(t)}{dt}$	$v(t) = L\dfrac{di(t)}{dt}$
or	or	or
$i(t) = Gv(t)$	$v(t) - v(0) = \dfrac{1}{C}\int_0^t i(\tau)\,d\tau$	$i(t) - i(0) = \dfrac{1}{L}\int_0^t v(\tau)\,d\tau$
Units of R are ohms (Ω)	Units of C are farads (F)	Units of L are henries (H)

$v(t) = v_0(t)$ independent of $i(t)$ $\qquad$ $i(t) = i_0(t)$ independent of $v(t)$

Figure 1.1–1. Simple linear 2-terminal lumped electrical elements and their constitutive relations. Note how current sources are distinguished from voltage sources; the orientation of the arrow or of the + and – signs inside the source symbol identifies the positive reference direction for the source quantity.

in the limit as the gain, α, becomes very large. It has its own special symbol as shown in Figure 1.1–3. The ideal op-amp is always used in a feedback circuit that achieves a finite output voltage by driving the input voltage difference, $\Delta v(t)$, nearly to zero. Other examples of multiterminal elements, such as coupled coils and transformers, transducers, and gyrators, are discussed in the problems at the end of Chapter 3.

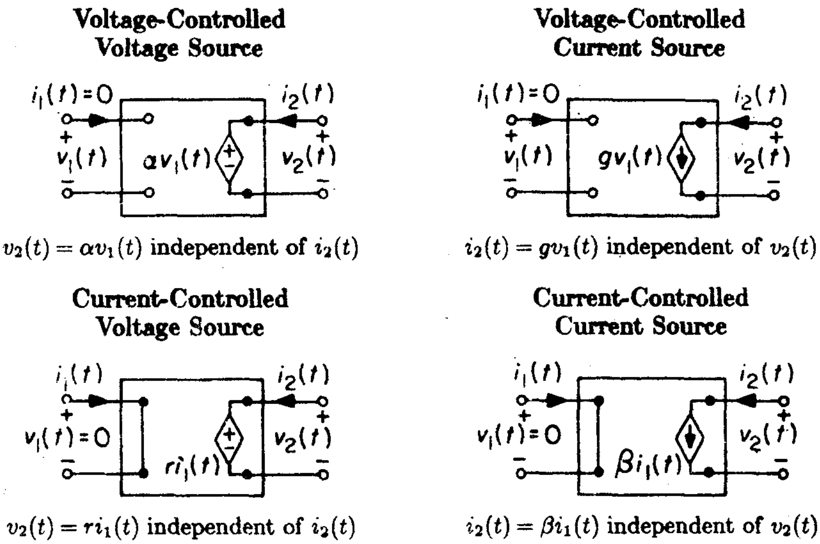


Figure 1.1–2. Ideal controlled (dependent) sources and their constitutive relations. Note that diamonds are used to identify dependent sources and circles to identify independent sources.

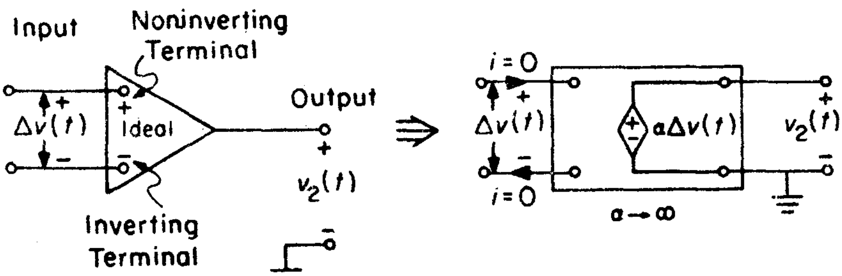


Figure 1.1–3. Ideal op-amp.

The ideal 2-terminal elements (excluding the independent sources) shown in Figures 1.1–1, 2, 3 are *linear*; that is, their dynamic variables satisfy the

> *SUPERPOSITION (LINEARITY) PRINCIPLE:**
>
> If $i'(t)$ and $v'(t)$ are *any* pair of functions that satisfy the constitutive relation of an element, and if $i''(t)$ and $v''(t)$ are any other pair satisfying the same constitutive relation, then the element is said to obey the *superposition principle* (or equivalently to be *linear*) if the pair of functions $i(t) = ai'(t) + bi''(t)$ and $v(t) = av'(t) + bv''(t)$ also satisfy the constitutive relation for any choices of the constants a and b.

The 2-terminal elements described in Figures 1.1–1, 2, 3 (again excluding independent sources) also satisfy the

> *TIME-INVARIANCE PRINCIPLE:**
>
> If $i(t)$ and $v(t)$ are *any* pair of functions that satisfy the constitutive relation of the element, then the element is *time-invariant* if $i(t-T)$ and $v(t-T)$ also satisfy the constitutive relation for *any* value of T.

Circuits composed entirely (except for independent sources) of linear time-invariant elements are examples of *linear time-invariant (LTI) systems*. The concept of an LTI system is more general, however, as we shall see in later chapters.

The most common *non*-linear element is probably the *diode*, whose idealized constitutive relation is shown in Figure 1.1–4. Also shown in this figure is the constitutive relation for what is surely the most important time-varying element—the *switch*. Circuits containing non-linear or time-varying elements are extremely useful. (See Problems 1.13–1.15 for some examples.) But the analysis of such circuits is often difficult. There are relatively few general principles or techniques for studying the behavior of non-linear circuits; each new circuit is likely to present a new analytical problem. In contrast, the theory of LTI systems consists of a rich collection of theorems, concepts, and methods providing powerful tools for understanding and design. As a result, the necessary non-linearities in practical electronic circuits are often restricted to isolated locations interconnected by LTI systems. Such an arrangement may vastly simplify the analysis while providing enough design freedom to achieve the desired dynamic effects. When such isolation and localization are impossible, as for example in some high-speed integrated circuits, the design process may reduce to employing numerical methods to study the performance of the device as various parameters are systematically varied. Computerized circuit simulation programs intended for this purpose have been developed, but the wide availability of such simulation

*The extension of these definitions to multiterminal elements is straightforward. For other examples of 2-terminal elements that are or are not linear and/or time-invariant, see Exercise 1.1.

programs has not eliminated the need to understand the mathematics of LTI systems, which remains a powerful *language* in terms of which complex system behavior can be discussed.

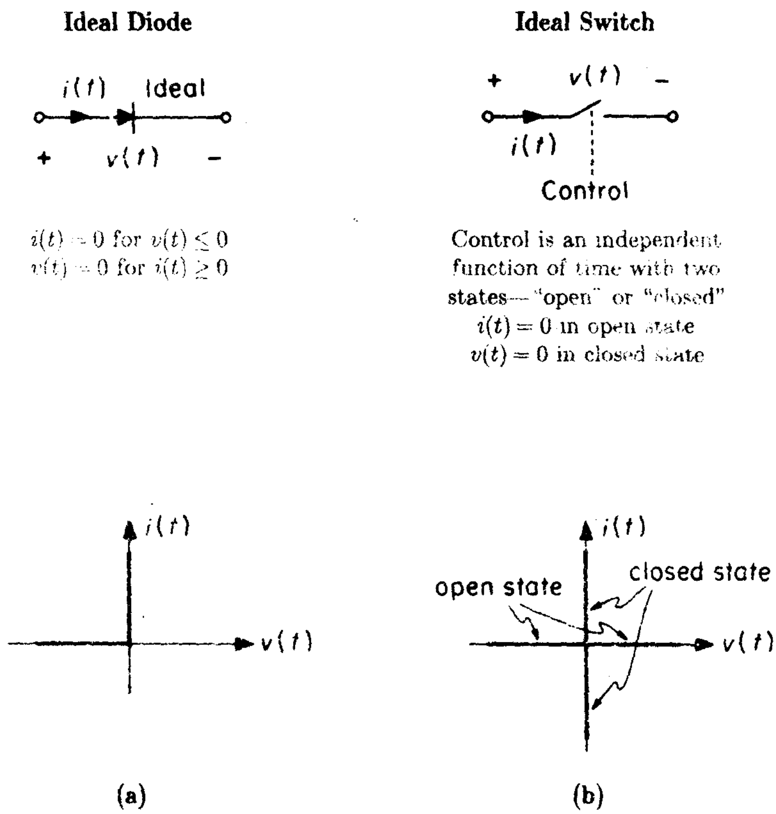


Figure 1.1–4. The ideal diode (a) and the ideal switch (b).

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