Volume Holography and Volume Gratings

L. SOLYMAR AND D. J. COOKE

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Department of Engineering Science
University of Oxford

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Preface

A glance through the contents of existing books on holography will show that scant attention has so far been given specifically to the volume properties of holograms. It is primarily in the belief that this apparent discrimination is not wholly justified that we put forward the present work.

Although it was appreciated quite early in the history of holography that the thickness of the hologram might be of some significance, it was for some time doubtful whether the additional insight obtained by taking into account this third dimension could possibly repay the mathematical labour required. Indeed, we must concede that for many purposes a plane holographic model is still perfectly adequate. The need to analyse holograms as volume elements could not be denied, however, as questions concerning efficiency, superposition and frequency selectivity acquired importance, and it was then realized that the behaviour of these "volume holograms" bore close resemblance to well known phenomena in quite different disciplines, such as the scattering of light by acoustic waves, the diffraction of X-rays and electron beams by crystals, and diffraction by surface gratings in integrated optics. One may thus identify a vast field of study which, in the absence of a better term, could be called that of "volume gratings".

In consequence, our approach to holography, and the tools of analysis which it is our purpose to present, have their origin in earlier work in related fields, and they continue to be adapted to find application in new ones, for example four-wave mixing and surface acoustic wave reflective-array devices. We wish to emphasize that it is not the aim of our book to review this whole subject, our primary interest lying in the theory and applications of volume holograms, and in the materials used for recording them, since the latter will determine their properties. We shall, nevertheless, while employing the terminology appropriate to holography, attempt to keep the treatment general, and to stress the similarity to other types of volume gratings when the opportunity arises. In particular, in the chapter devoted to applications, we shall, besides reviewing volume holographic displays and optical elements, include also applications of grating devices based on acousto-optic and electro-optic interaction, all kinds of integrated optics structures, and brief notes as well on purely acoustic Bragg devices, multilayers and four-wave mixing.

The audience we have in mind could be broadly divided into the following categories:

(1) final year undergraduates with some knowledge of optics and electromagnetic theory, for whom a gradual introduction is provided and who should be able to read the first five chapters without difficulty;
(2) graduate students who are working in the field;
(3) research workers in various branches of volume holography interested in the state of the art;
(4) research workers in the neighbouring fields who want to learn about the potential of volume gratings in general.

We wish here to thank our colleagues Cass Benlarbi, Paul Bloch, Mike Jordan, Rick Kerle, Martin Owen, Ted Paige, Philip Russell, Dilano Saldin, Colin Sheppard, Richard Syms, Andrew Ward and Tony Wilson who helped to shape the ideas presented in this book. Our thanks are also due to Eric Ash and Antonio Leite of University College, London, and Nick Phillips of Loughborough University for many helpful discussions. We are particularly indebted to Rudi Kompfner, one of the most imaginative engineers of this century, without whose initiative we would have never moved into this exciting field, and who gave constant encouragement to our various endeavours until his untimely death.

We should further like to acknowledge our debt to Professor A. Korpel of Iowa University and Dr. J.-P. Huignard of Thomson-CSF for sending us original photographs of their experimental results. The credit for the illustrations should go to Mrs. J. Takacs. Finally, one of us (D.J.C.) would like to thank the Warden and Fellows of Merton College for the support of a Junior Research Fellowship.

L. SOLYMAR
D. J. COOKE

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1 Derivation of the Basic Equations

1.1 Introduction

The study of volume gratings may be regarded as the study of waves in periodic and nearly periodic media. The waves may be of different kinds, such as electromagnetic, acoustic and particle waves, each one, strictly speaking, requiring a different mathematical description. Acoustic waves are mostly needed in *producing* the periodically varying permittivity; we shall have less occasion to talk about their properties in periodic media. Electron diffraction is, of course, very extensively used in the study of materials and the properties of electron waves are based on an equation (Schrödinger's) which looks quite different from the others. Nevertheless, in the case of stationary processes, there is hardly any difference between the mathematical description of the various waves. Considering further that our main interest is in the optical region concerning various manifestations of volume holography, and that X-rays also happen to be electromagnetic waves, we feel it will be sufficient to restrict our introduction to the relevant properties of electromagnetic waves.

Our aim is to derive the expressions needed later. The treatment will, of necessity, be brief. It cannot provide a complete introduction; it can serve only to refresh the memory of the reader, and to give ready access to formulae. The book likely to prove most useful in providing more details is that of Born and Wolf [1], though we shall often refer to a book [2] by one of the present authors on electromagnetic theory for the simple reason that we are more familiar with it.

We shall start the treatment with Maxwell's equations in Section 1.2; the wave equation and Poynting's theorem are derived in Sections 1.3 and 1.4 and boundary conditions are given in Section 1.5; the plane wave solution is discussed in Section 1.6 and the corresponding refraction problem in Section 1.7; finally, the geometrical optics approach for a simple two-dimensional case is presented in Section 1.8.

1.2 Maxwell's equations

We start with Maxwell's equations. These can be found in any textbook on electromagnetic theory [2], usually in the following form

$$\nabla \wedge \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \tag{1.1}$$

$$\nabla \wedge \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{1.2}$$

$$\nabla . \boldsymbol{D} = \rho \tag{1.3}$$

$$\nabla . \boldsymbol{B} = 0 \tag{1.4}$$

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} \tag{1.5}$$

$$\boldsymbol{B} = \mu \boldsymbol{H} \tag{1.6}$$

where $\boldsymbol{H}$ = magnetic field strength, $\boldsymbol{E}$ = electric field strength, $\boldsymbol{B}$ = magnetic flux density, $\boldsymbol{D}$ = electric flux density, $\boldsymbol{J}$ = current density, ρ = charge density, μ = permeability, ε = permittivity and ∇ is the differential vector operator.

We shall now introduce ε_r, the relative permittivity (also called the relative dielectric constant) and μ_r, the relative permeability, with the relations

$$\varepsilon = \varepsilon_0 \varepsilon_r \quad \text{and} \quad \mu = \mu_0 \mu_r \tag{1.7}$$

In SI units, the values of ε_0 and μ_0 are given as

$$\varepsilon_0 = 8{\cdot}85 \times 10^{-12}\,\mathrm{A\,s\,V^{-1}m^{-1}}, \qquad \mu_0 = 4\pi \times 10^{-7}\,\mathrm{V\,s\,A^{-1}m^{-1}} \tag{1.8}$$

The relative permittivity and permeability are characteristic of a given material. In free space, $\varepsilon_r = \mu_r = 1$.

1.3 Derivation of the wave equation

We shall now eliminate the variables $\boldsymbol{H}$, $\boldsymbol{B}$ and $\boldsymbol{D}$ from Equations (1.1)–(1.6). Taking the curl of Equation (1.2) and using Equations (1.1), (1.5) and (1.6), we get

$$\nabla\wedge(\nabla\wedge \boldsymbol{E}) = -\mu\frac{\partial}{\partial t}(\nabla\wedge \boldsymbol{H}) = -\mu\left(\frac{\partial \boldsymbol{J}}{\partial t} + \varepsilon\frac{\partial^2 \boldsymbol{E}}{\partial t^2}\right) \tag{1.9}$$

leading to

$$\nabla\wedge(\nabla\wedge \boldsymbol{E}) + \varepsilon\mu\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = -\mu\frac{\partial \boldsymbol{J}}{\partial t} \tag{1.10}$$

In all cases of interest we shall be concerned with harmonic time variation at an angular frequency ω. Hence the form $\exp(j\omega t)$ is assumed, leading to the modified form of the above equation,

$$\nabla\wedge(\nabla\wedge \boldsymbol{E}) - \omega^2\mu\varepsilon \boldsymbol{E} = -j\omega\mu \boldsymbol{J} \tag{1.11}$$

We shall further assume that at the frequencies of interest, and for the materials under consideration, the current density at any given point will be proportional to the electric field,

$$\boldsymbol{J} = \sigma \boldsymbol{E} \tag{1.12}$$

where σ is another material constant known as the conductivity. Substituting Equation (1.12) into (1.11), the latter may be written in the form

$$\nabla\wedge(\nabla\wedge \boldsymbol{E}) + \gamma^2 \boldsymbol{E} = 0 \tag{1.13}$$

where

$$\gamma^2 = j\omega\mu\sigma - \omega^2\mu\varepsilon \tag{1.14}$$

Equation (1.13) is one of the convenient forms from which the study of volume gratings could start: we shall use it in Chapter 9, where the full vectorial problem will be treated. For most other problems, it is more convenient to use the vector relation

$$\nabla\wedge(\nabla\wedge \boldsymbol{E}) = \nabla(\nabla . \boldsymbol{E}) - \nabla^2 \boldsymbol{E} \tag{1.15}$$

What can we say about $\nabla . \boldsymbol{E}$? In materials where no free charge is permitted one might be tempted to take it equal to zero. However, some care must be exercised, because in our problems ε will be a function of the spatial coordinates. Hence the mathematical relationship to consider in the absence of free charge is

$$\nabla . \boldsymbol{D} = 0 = \varepsilon\nabla . \boldsymbol{E} + \boldsymbol{E} . \nabla\varepsilon \tag{1.16}$$

Thus $\nabla . \boldsymbol{E}$ will not be zero provided $\boldsymbol{E} . \nabla\varepsilon$ is finite. There is however a wide range of problems (actually including most of those treated in the present book) where ε varies only in the plane perpendicular to the direction of $\boldsymbol{E}$, leading to $\boldsymbol{E} . \nabla\varepsilon = 0$ and consequently $\nabla . \boldsymbol{E} = 0$. Equation (1.13) then takes the form

$$\nabla^2 \boldsymbol{E} - \gamma^2 \boldsymbol{E} = 0 \tag{1.17}$$

Each of Equations (1.10), (1.13) and (1.17) is usually referred to as the wave equation.

We shall say more about the physical significance of γ in Section 1.6. Here we shall be concerned with some alternative forms. One way of expressing γ is in terms of a complex dielectric permittivity, which may be deduced from Equation (1.14) as follows:

$$\gamma = [-\omega^2\mu\varepsilon + j\omega\mu\sigma]^{1/2} = j\omega(\mu\varepsilon_0)^{1/2}\left[\varepsilon_r - j\frac{\sigma}{\omega\varepsilon_0}\right]^{1/2} \qquad (1.18)$$

It is now customary to introduce the following notations,

$$\varepsilon' = \varepsilon_r \qquad \text{and} \qquad \varepsilon'' = \frac{\sigma}{\omega\varepsilon_0} \qquad (1.19)$$

where ε' and ε'' are referred to as the real and imaginary parts of the complex permittivity.

In wave propagation problems, γ is usually divided into real and imaginary parts as

$$\gamma = \alpha + j\beta \qquad (1.20)$$

where α and β may be determined by comparing the above equation with equation (1.14). We shall perform this calculation only for the case where the conductivity is small, leading to

$$\gamma = j\omega(\mu\varepsilon)^{1/2}\left[1 - j\frac{\sigma}{\omega\varepsilon}\right]^{1/2} \approx j\omega(\mu\varepsilon)^{1/2}\left[1 - j\frac{\sigma}{2\omega\varepsilon}\right] \qquad (1.21)$$

whence

$$\beta = \omega(\mu\varepsilon)^{1/2} \qquad \text{and} \qquad \alpha = \frac{\sigma}{2}\left(\frac{\mu}{\varepsilon}\right)^{1/2} \qquad (1.22)$$

We may also relate α to ε''; to the same accuracy, this comes to

$$\alpha = \tfrac{1}{2}\frac{\varepsilon''}{\varepsilon_r}\beta \qquad (1.23)$$

1.4 Poynting's theorem

The aim is now to derive an expression from Maxwell's equations which may serve as the power conservation theorem of electromagnetic theory.

Let us take the scalar product of Equation (1.2) with $\boldsymbol{H}$, and subtract from it the scalar product of (1.1) with $\boldsymbol{E}$. We obtain

$$\boldsymbol{H}.(\nabla\wedge\boldsymbol{E}) - \boldsymbol{E}.(\nabla\wedge\boldsymbol{H}) = -\boldsymbol{H}.\frac{\partial\boldsymbol{B}}{\partial t} - \boldsymbol{E}.\frac{\partial\boldsymbol{D}}{\partial t} - \boldsymbol{E}.\,\boldsymbol{J} \qquad (1.24)$$

Using a well known vector identity (see, for example, ref. 2) and the relationship between current density and electric field as given by Equation (1.12), we may rewrite (1.24) in the form

$$\nabla.(\boldsymbol{E}\wedge\boldsymbol{H}) = -\frac{\partial}{\partial t}[\tfrac{1}{2}\varepsilon E^2 + \tfrac{1}{2}\mu H^2] - \sigma E^2 \tag{1.25}$$

We shall now define

$$\boldsymbol{P}_d = \boldsymbol{E}\wedge\boldsymbol{H} \tag{1.26}$$

as the Poynting vector representing power density, and

$$W = \tfrac{1}{2}\varepsilon \boldsymbol{E}^2 + \tfrac{1}{2}\mu \boldsymbol{H}^2 \tag{1.27}$$

as the energy contained in the electric and magnetic fields.

Integrating Equation (1.25) for a volume τ we get

$$\int_\tau \nabla.\boldsymbol{P}_d d\tau + \frac{\partial}{\partial t}\int_\tau W d\tau + \sigma\int_\tau E^2 d\tau = 0 \tag{1.28}$$

Using Gauss' theorem [2], the first term in the above equation becomes

$$\oint_A \boldsymbol{P}_d.d\boldsymbol{A} \tag{1.29}$$

which is equal to the net power flowing through the closed surface A of volume τ. Noting further that the third term in Equation (1.28) represents the energy dissipated, we may now state the power conservation theorem, known as Poynting's theorem, as follows: the net power crossing a closed surface, plus the rate of change of stored energy, plus the power dissipated, must be equal to zero. In other words, the power leaving the closed surface may be different from that entering only if the stored energy changes, or if some of the power is dissipated.

Assuming the $\exp(j\omega t)$ time dependence, we need to rewrite Equation (1.28) in its complex form. We are then interested only in the real part, which takes the form

$$\int_\tau \nabla.\boldsymbol{P}_d d\tau + \tfrac{1}{2}\sigma\int_\tau |\boldsymbol{E}|^2 d\tau = 0 \tag{1.30}$$

where $\boldsymbol{P}_d$ is now redefined as

$$\boldsymbol{P}_d = \tfrac{1}{2}\,\mathrm{Re}(\boldsymbol{E}\wedge\boldsymbol{H}^*) \tag{1.31}$$

and where * denotes the complex conjugate.

1.5 Boundary conditions

At the boundary of two different media, all our field quantities need to satisfy certain conditions, a consequence of the differential equations they must obey. For our purpose it is sufficient to consider the case when there are neither surface charges nor surface currents present. The boundary conditions take then the form [2],

$$\boldsymbol{n} \wedge (\boldsymbol{E}_2 - \boldsymbol{E}_1) = 0, \qquad \boldsymbol{n}.(\boldsymbol{D}_2 - \boldsymbol{D}_1) = 0 \tag{1.32}$$

and

$$\boldsymbol{n} \wedge (\boldsymbol{H}_2 - \boldsymbol{H}_1) = 0, \qquad \boldsymbol{n}.(\boldsymbol{B}_2 - \boldsymbol{B}_1) = 0 \tag{1.33}$$

where $\boldsymbol{n}$ is the vector normal to the boundary surface and the subscripts 1 and 2 refer to the boundary values of the field quantities in media 1 and 2 respectively. In words, the tangential components of $\boldsymbol{E}$ and $\boldsymbol{H}$, and the normal components of $\boldsymbol{D}$ and $\boldsymbol{B}$, must be conserved across a boundary.

1.6 Plane waves

Where ε and σ are constants independent of the coordinates of the medium, Equation (1.17) has a simple solution of the form

$$\boldsymbol{E} = \boldsymbol{E}_{01} \exp(-\boldsymbol{\gamma}.\boldsymbol{r}) + \boldsymbol{E}_{02} \exp(+\boldsymbol{\gamma}.\boldsymbol{r}) \tag{1.34}$$

where $\boldsymbol{r}$ is the radius vector of an arbitrary point, $\boldsymbol{\gamma}$ is a constant vector whose modulus is equal to γ and $\boldsymbol{E}_{01}$, $\boldsymbol{E}_{02}$ are constant vectors perpendicular to $\boldsymbol{\gamma}$.

The significance of the direction of $\boldsymbol{\gamma}$ may be appreciated by noticing that the phase of the exponents in Equation (1.34) depends upon the scalar product $\boldsymbol{\gamma}.\boldsymbol{r}$. Since $\boldsymbol{\gamma}$ is a constant vector, the scalar product is constant for those values of $\boldsymbol{r}$ which lie in a plane perpendicular to $\boldsymbol{\gamma}$, as shown in Figure 1.1. Hence the phase is constant in that plane and we are entitled to refer to the solution as a plane wave, having a plane wavefront.

For the interpretation of the modulus of $\boldsymbol{\gamma}$ we shall, for simplicity, consider a one-dimensional model in which

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0 \tag{1.35}$$

That is, there is variation only in the x direction. Furthermore, we shall return to the notation which includes the temporal variation, and write the exponents of Equation (1.34) as

$$\exp(j\omega t \mp \gamma x) = \exp[j(\omega t - \beta x)] \exp(\mp \alpha x) \tag{1.36}$$

Choosing the upper sign, we may clearly see that the wave is attenuated in the positive x direction, with a coefficient α. Hence, we may term α the attenuation constant. The phase is constant when

$$\omega t - \beta x = \text{const.} \tag{1.37}$$

whence it may be deduced that the wave propagates in the positive x

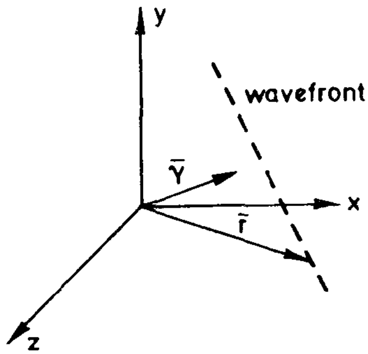

Figure 1.1 Diagram showing that the scalar product $\boldsymbol{\gamma}.\boldsymbol{r}$ represents a planar wavefront.

direction, and the speed of the wave is equal to

$$\frac{dx}{dt} = v = \frac{\omega}{\beta} = (\mu\varepsilon)^{-1/2} \tag{1.38}$$

In a vacuum, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$, and the speed of the wave is

$$c = (\mu_0\varepsilon_0)^{-1/2} = 3{\cdot}00 \times 10^8 \mathrm{m\,s^{-1}} \tag{1.39}$$

For the materials of interest, the magnetic permeability is always equal to μ_0, but we may of course encounter a wide range of dielectric permittivities.

Introducing n, the refractive index, with the definition

$$n = (\varepsilon_r)^{1/2} \tag{1.40}$$

the speed of the wave in a medium of permittivity ε_r is equal to

$$v = \frac{c}{(\varepsilon_r)^{1/2}} = \frac{c}{n} \tag{1.41}$$

The spatial period of the wave, called the wavelength, is denoted by λ which is related to β by the simple formula

$$\lambda = \frac{2\pi}{\beta} \tag{1.42}$$

Since β characterizes the propagation of the wave, it is usually referred to as the propagation constant. In vectorial form, $\boldsymbol{\beta}$ (pointing in the same direction as $\boldsymbol{\gamma}$) is called the wave vector.

In free space we shall use the notation $\lambda = \lambda_f$ and $\beta = \beta_f$.

Returning now to the lower sign in Equation (1.36), it may be shown by the preceding arguments that it represents a wave propagating in the negative x direction.

1.7 Refraction of plane waves at plane boundaries

We shall assume in this section that both media are lossless ($\sigma = 0$) and a monochromatic wave of frequency ω, with electric polarization parallel to the interface, is incident at an angle θ_1 from medium 1 upon medium 2, as shown in Fig. 1.2. Part of the incident wave will be reflected and part of it transmitted into medium 2, where it will propagate at an angle θ_2.

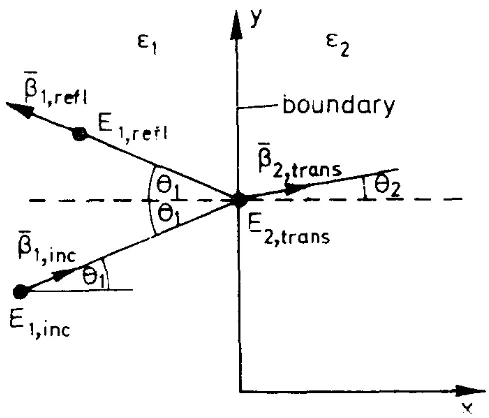

Figure 1.2 Schematic representation of the reflection of plane waves (electric field vector parallel to the interface) at the boundary of two different media.

Assuming that the angle of reflection is the same as the angle of incidence, the three wave vectors may be expressed as follows:

$$\boldsymbol{\beta}_{1,inc} = \omega(\mu_0\varepsilon_1)^{1/2}(\boldsymbol{i}_x \cos\theta_1 + \boldsymbol{i}_y \sin\theta_1) \tag{1.43}$$

$$\boldsymbol{\beta}_{1,refl} = \omega(\mu_0\varepsilon_1)^{1/2}(-\boldsymbol{i}_x \cos\theta_1 + \boldsymbol{i}_y \sin\theta_1) \tag{1.44}$$

$$\boldsymbol{\beta}_{2,trans} = \omega(\mu_0\varepsilon_2)^{1/2}(\boldsymbol{i}_x \cos\theta_2 + \boldsymbol{i}_y \sin\theta_2) \tag{1.45}$$

where $\boldsymbol{i}_x$ and $\boldsymbol{i}_y$ are unit vectors in the x and y directions respectively. The