Experimentation

An Introduction to Measurement Theory and Experiment Design
2nd Edition

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Preface

The first edition of this book was written to support the suggestion that, regardless of the chosen objectives for an introductory physics laboratory, the basic principles of experimenting should not be neglected and could in fact become the principal topic. Introductory laboratories in physics are particularly suited to this purpose since the systems and theories found there are usually simple enough that the basic characteristics of measurement and experimenting can easily be made visible and understandable. Such an approach to physics laboratory work can, therefore, be beneficial for a wide range of students, not only those who will proceed to professional work in physics.

That purpose on which the 1962 edition was based seems still to be valid. Many changes have taken place in the practice of experimenting, partly through the introduction of new instrumentation, but mostly because of the revolutionary impact of computing. Not only can we easily attain a level of post-experiment data analysis that would have been completely impracticable twenty-five years ago but the possibilities for the conduct of the experiment itself have been enormously expanded by the availability of on-line data analysis or computer-based control of the apparatus.

Revolutionary though such changes have been in the actual conduct of experiments, there has, nevertheless, been little or no change in the basic principles underlying the experimenting, and training in these basic principles is still required. Indeed, emphasis on these basic principles may be even more necessary today than it was twenty-five years ago on account of the present-day possibility that an experimenter can be completely insulated from the phenomena under study by an almost impenetrable barrier of data processing equipment and procedures. Under these circumstances, wholly invisible defects can produce final results with little or no meaning. Unless we have complete and clear understanding of all phases of our experiment and data analysis, we turn over our experiment wholly to the computer at our peril.

The plan of the book is largely the same as in the earlier edition but the text has been almost completely rewritten. Chapter 1 gives an outline of an approach to introductory physics laboratory work that facilitates contact with the basic nature of experimenting. Chapters 2, 3, and 4 provide the basic information on measurement, statistics and scientific procedures on which experiment design is based. Chapter 5 treats in a step-by-step way the practical requirements in designing an experiment, and Chapter 6 provides the corresponding procedures for evaluating the results of the experiment after the measurements have been made. At the end of the main text Chapter 7 contains some suggestions for writing laboratory reports.

The appendices contain material which, although desirable in itself, would have interrupted the development within the main text This includes mathematical derivation of some of the equations quoted in the main text. In addition, a sample experiment is described in extensive detail, starting at the beginning of the experiment design, continuing through the conduct of the actual experiment and the evaluation of the results, and ending with the final report.

The material in the text has been the basis of many years of teaching in our First Year Physics Laboratory and I am grateful to the generations of students whose sometimes painful experience with it provided the opportunity for continued refinement. Finally I wish to thank Mrs. Jill Hodgson and Mme. Rachel Desrosiers for their generously-provided and invaluable assistance in preparing the manuscript.

D. C. Baird

Contents

1 Approach to Laboratory Work

This book is intended for use in introductory physics laboratories. It was written, however, in the hope that it will serve a much wider purpose—that of providing an introduction to the study of experimenting in general, irrespective of the area in which the experimenting is carried out. Some of those studying in an introductory physics laboratory may pursue careers in physics research, and it is hoped that the book will serve as a suitable introduction to their continued studies. Many others will pursue careers in completely different areas, perhaps in other sciences. Whatever the need, the introductory physics laboratory can provide a useful introduction to the fundamental principles that underlie experimenting of any kind.

For our purposes experimentation has a very broad definition: by experimentation we mean the whole process of identifying a portion of the world around us, obtaining information from it, and interpreting that information. This definition covers a very wide range of activities—from a biologist in a white coat splicing DNA molecules to a manufacturer taking a poll to determine individual preferences in toothpaste. This book is intended to meet the needs of all who find themselves engaged in any kind of study of the world around us.

That includes those who may not themselves be actually involved in experimenting. Even if we are not actively engaged in generating it, we are all faced frequently with the requirement of at least passing judgment on experimental information offered by others. For example, our professional work may require us to make a choice between competitive bids on equipment having certain specifications, or, as members of the general public, we may be called upon to form opinions on such issues as the possible health hazards of nuclear power plants, the safety of food additives, the impact of acid rain on the environment, or the influence of national monetary policy on unemployment. What all of these examples have in common is the prominent part played by experimental information. Such public issues impose on us the responsibility of reaching our own decisions, and these decisions should be based on our assessment of the reliability of the experimental information. Even in less important matters we hear repeatedly such assertions as that scientific tests have shown that we can control our tooth decay or headaches by $x\%$ using certain products. Our choice of a new automobile may depend on our assessment of the accuracy of claimed values for fuel consumption. We are all, scientists and nonscientists alike, faced daily with the requirement to be knowledgeable concerning the nature of experimental information and the ways in which it is obtained and to be appropriately skeptical about its reliability.

To return to our claim that a physics teaching laboratory can provide an introduction to the subject of experimentation in general, it is natural to wonder how the normal laboratory with its usual experiments can be used for such a purpose. The answer lies not so much in the experiments themselves as in the attitude with which we approach them. This will become clear as our studies of experimental methods proceed, but it may be helpful at this point to illustrate the proposal with a few examples.

In offering these we must anticipate a little the work of Chap. 4 and note that we shall be viewing everything susceptible to experimental investigation in terms of "systems." By a system we mean, in general, any isolated, defined entity that functions in a specific manner. We assume that we can influence or control the system, and we refer to the methods we have available to do this as "inputs." We also assume that the system will perform some identifiable function or functions, and we refer to these as "outputs." The various examples that follow will make clear the use of the terminology. An economist, for example, may view the economy of a country as a system with an extensive set of inputs and a correspondingly varied set of outputs. The system itself will include the whole productive capacity for goods and services, transportation facilities, supply of raw materials, inhabitants, opportunities for foreign trade, weather, and many other things. The inputs are those things that can be controlled by us—the money supply, tax rates, government spending, tariffs on imports, etc. The outputs are those things that we cannot control directly; their magnitudes are determined by the system, not by us. Outputs of an economic system would include the gross national product, unemployment rate, inflation rate, external trade balance, etc. It would be very comforting and convenient if we could secure the desired values of these outputs by simple manipulation, but we cannot. No matter how desirable it may be, we cannot instruct the country's gross national product or unemployment rate to have a certain value; we are restricted to controlling our inputs. Even there we have problems. In a system as complex as a national economy the linkages between the inputs and the outputs are tangled and indirect. A change in one input variable will likely have an effect on a number of output variables, instead of the single output in which we may be interested. For example, an attempt to increase the gross national product of a country by reducing taxation rates will possibly be at least partially successful, but the simultaneous effects on other outputs may be equally prominent and not nearly as desirable—for example, a possible increase in the rate of inflation. The methods available for handling such situations are sophisticated but, with a system of this complexity, the level of success achieved by the politicians and economists shows that substantial room for improvement still remains.

There are other systems that, although still complex, are simple enough that we can control them reasonably successfully. Consider, for example, a nuclear reactor. Here the system has a smaller number of input controls and outputs, and the situation is much more clearly defined. The inputs include the position of the control rods, amount and type of fuel, rate of coolant flow, etc. The outputs include such quantities as the neutron flux density, total power produced, useful life of the fuel elements, etc. In this case the linkage between the inputs and outputs is sufficiently simple (although still not directly one-to-one) that a reasonable level of control is possible. On a more familiar level every supermarket is a system with outputs and inputs whose manipulation constitutes an experiment on the system. Every time the supermarket manager alters the price of beans (one of his inputs) he is in fact performing an experiment, for he wishes to detect a consequent change in one or more of his outputs (for example, his end-of-week profit). If he is not able to perceive the desired alteration in his outputs, he may be prompted to revise his original decision and again alter the price of beans. In other words, he is continually testing the properties of his system through experiment, and his skill in interpreting experimental results may make all the difference between profit and loss.

Incidentally, we should note in passing that, in the example of the supermarket manager, some of his inputs and outputs will involve people—work schedules, pay rates, morale, productivity, etc.—and in case this use of a systems approach to all problems, human as well as mechanical, sounds like an overmechanistic approach to life, we should note that much of our subsequent work will be concerned with limits on the validity of experimental methods. We have all heard the phrase "it has been scientifically proved" offered as an irrefutable argument, and we must be alert to the dangers of misplaced faith in scientific infallibility.

But, to return to our systems, how does all this refer to the introductory physics laboratory? In fact, if we are to prepare people to enter a scientifically literate population, would it not be better to tackle the important problems right away, and start deciding whether the mercury content of fish makes it safe to eat? The trouble is, however, that these are extremely difficult problems. Evidence is hard to obtain and its interpretation is usually uncertain; even the experts themselves disagree, often vigorously and publicly. It is almost impossible to make a significant contribution to the solution of such complex problems without first developing our skills using simpler situations. To make a start on this, let us think about some of these simpler systems.

A gasoline engine is a system that is simple in comparison with any of the earlier examples. The system includes the engine, fuel supply, mounting, surrounding atmosphere, etc. The inputs may be the obvious controls like fuel supply, fuel-air ratio, ignition timing, etc. and the outputs, as always, are the factors whose values are set by the system—the number of rpm, the amount of heat produced, the efficiency, the composition of the exhaust gases, etc. This is still a somewhat complex system, but we can begin to see that relatively simple relationships between inputs and outputs can exist. For example, the input-output relation between throttle setting and rpm for a gasoline engine is sufficiently direct and predictable that most of us invoke it daily. Note, however, that the effect of that input is not restricted to the one output in which we are interested—rpm; other outputs like heat produced, exhaust-gas composition, and efficiency are also affected by that one input, even if we are prepared, generally, to ignore the coupling.

In this example we are beginning to reach the stage at which our system is simple enough that we can start working on our theory of experimenting. Let us go one stage further and consider the example of a simple pendulum. It, too, is a system. It is, however, a system that includes very little other than the string, bob, stand, and surrounding air. Furthermore, it has only two immediately obvious inputs—the length of the string and the initial conditions according to which the motion is started. The outputs, too, are few in number. Apart from small, secondary effects, they include only the frequency of vibration and the amplitude of oscillation. Lastly, the connection between the inputs and outputs is relatively direct and reproducible. Altering the length of the pendulum's string will offer few surprises as we measure the frequency of vibration. Here, then, is a system in which the principles of experimenting will be clearly visible. If we use it to develop ability to control systems and evaluate their outputs, we shall develop the competence to tackle the more important but more complex problems later on. This gives us the key to at least some constructive uses of the introductory physics laboratory. There is real point in working with a pendulum—but only if we view it properly. If we look at it as just a pendulum, which we have all "done" before, our only reaction will be total boredom. If, however, we view it as a system, just like a supermarket, an airport, a nuclear reactor, or the national economy, but differing from them only in that it is simple enough that we can understand it relatively well, it will supply excellent simulation of the problems of the real world.

Here we have the justification for using the introductory physics laboratory to teach experimentation. The systems involved are sufficiently simple that they are close to being understandable, and practice with them will equip us to proceed later to our real work on complicated and important systems. We must be careful, however, about the ways in which we practice on these simple systems. We shall derive only very limited benefit if we confine ourselves to sets of instructions which tell us how to do particular experiments. If it is our intention to provide a base for proceeding to *any* type of information analysis in science, technology, business, or any of the social sciences, we shall have to provide preparation for a wide variety of experimental circumstances. In some areas random fluctuation dominates, as in the biological sciences; in others measurement may be precise, as in astronomy, but control over the subject matter is limited. The range is enormous. As we have said before, we shall try to identify general principles of experimenting, in the hope that they will be valid and useful, regardless of the future subject matter or type of experimenting. The remainder of this text will be concerned with those principles, and we shall assume henceforth that laboratory experiments will be regarded as exercises to illustrate the principles.

It may now be obvious that many of the traditional procedures in introductory laboratories are inappropriate for our purpose. For example, we must avoid thinking of an experiment as a procedure to reproduce some "correct" answer, deviation from which makes us "wrong." Instead, we simply assess the properties of our particular system dispassionately and take the results as they come. Also, there is no point in seeking some "procedure" to follow: that is nothing more than asking someone else to tell us how to do the experiment. In real life there is rarely someone waiting to tell us what to do or what our result should be; our usefulness will depend on our ability to make our *own* decisions about handling the situation. It takes a great deal of practice and experience to develop confidence in our own decisions about the conduct of any experimental procedure, and the elementary laboratory is not too early to start. We shall, therefore, place a great deal of emphasis on experiment planning, for this is the stage at which much of the skill in experimenting is needed. It is important to avoid the temptation to regard preliminary planning as a waste of time or a distraction from the supposedly more important task of making the measurements. Time must be explicitly set aside for adequate analysis and planning of the experiment before a start is made on the actual measuring process.

It is necessary, also, to learn to work within the framework of the apparatus available. All professional experimenting is subject to limits on resources, and much of the skill in experimenting lies in optimizing the experimental yield from these resources. Restrictions on time, too, merely simulate the circumstances of most actual experimenting. The apparatus itself will never be ideal. This should not, however, be regarded as a defect but as a challenge. The real work of evaluating experimental results consists of separating the grain of useful results from the chaff of error and uncertainty. The experimenter must learn to identify sources of error or uncertainty for himself, and, if possible, eliminate them or correct for them. Even with the greatest care, however, there will always be an irreducible residuum of uncertainty, and it is the experimenter's responsibility to evaluate the precision of the final answer, a quantity which is just as important as the answer itself. The ability to cope with such requirements can be acquired only by actual contact with realistic working conditions, and it is a common injustice to students in introductory physics laboratories to provide apparatus that is too-carefully adjusted, or to give, in other ways, the impression that the experiments are ideal. This is unfortunate, because the foundation of future expertise lies in constructive response to experimental limitations.

In summary, the most fruitful use of laboratory time will result when the experiments are accepted as problems to be solved by the student himself. Certainly, errors in judgment will be made, but we can learn more effectively from personal experience of the consequences of our decisions than we can by following rigidly some established, "correct" procedure. What we learn is more important than what we do. This is not to say, however, that we should show complacent indifference to the outcome of the experiment. Development of our experimenting skills will come about only if we take seriously the challenge of obtaining the best possible result in every experiment.

The writing of laboratory reports should be tackled in the same constructive spirit. In professional life there is very little point in spending time and trouble on an experiment unless we can adequately convey the outcome to others. We have an obligation to our readers to express ourselves as lucidly, if not elegantly, as possible. It is wrong to regard this as the responsibility of departments of English, and report writing in the introductory science laboratory should be accepted as an opportunity for exercise in descriptive composition. Report writing that degenerates into a mere indication that the experiment has been performed is little more than a waste of time and a loss of opportunity for necessary practice. Report writing at the level suggested here is almost pointless without adequate review and criticism. Opportunities for improvement become much more obvious in hindsight, and such detailed review should be regarded as an indispensable part of the work in a teaching laboratory.

2

Measurement and Uncertainty

2–1 BASIC NATURE OF MEASURING PROCESS

Measurement is the process of quantifying our experience of the external world. The nineteenth-century Scottish scientist, Lord Kelvin, once said that "when you can measure what you are speaking about and express it in numbers, you know something about it; but, when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the stage of science." While this may be a slight overstatement, it remains true that measurements constitute one of the basic ingredients of experimenting. We shall not reach a satisfactory level of competence in experimenting without knowledge of the nature of measurement and the significance of measurement statements.

It is obvious that the quantifying process will almost invariably involve comparison with some reference quantity (how many paces wide is my back yard?). It is equally obvious that the good order of society requires extensive agreement about the choice of reference quantities. The question of such measurement standards, defined by legislation and subject to international agreement, is extensive and important. No one seriously interested in measurement can ignore the question of defining and realizing standards in his area of work. A discussion of this important topic here would, however, distract us from our chief concern, the process of measuring. We shall, therefore, leave the topic of standards without further mention except reference to the texts listed in the Bibliography, and take up the study of actual measuring processes.

Let us start at the most basic level with an apparently simple measurement; let us try to find out what kind of process is involved and what kind of statement can be made. If I give the notebook in which this is being written to someone and ask him to measure its length with a meter stick, the answer is absolutely invariable—the length of the notebook is 29.5 cm. But that answer must make us wonder: are we really being asked to believe that the length of the book is exactly 29.50000000 . . . . cm? Surely not; such a claim is clearly beyond the bounds of credibility. So how are we to interpret the answer? A moment's thought in the presence of the notebook and a meter stick will make us realize that, far from determining the "right" or "exact" value, the only thing we can realistically do is approach the edge of the notebook along the scale, saying to ourselves as we go: "Am I sure the answer lies below 30 cm? Below 29.9 cm? Below 29.8 cm?" The answer to each of these questions will undoubtedly be "Yes." As we progress along the scale, however, we shall eventually reach a point at which we can no longer give the same confident reply. At that point we must stop, and we identify thereby one end of an interval that will become our measured value. In a similar way we can approach the edge of the notebook from below, asking ourselves at each stage: "Am I sure that the answer lies above 29.0 cm? 29.1 cm," and so on. Once again we shall reach a value at which we must stop, because we can no longer say with confidence that the answer lies above it. By the combination of these two processes we identify an interval along the scale. It is the smallest interval that, as far as we can be certain, does contain our desired value; within the interval, however, we do not know where our answer lies. Such is the only realistic outcome of a measuring process. We cannot look for exact answers, and we must be content with measured values that take the form of intervals. Not only does this example illustrate the essential nature of a measuring process, it also provides guidance for actually making measurements. The process of approaching the value we seek from each side separately reminds us of the necessity of stating the result as an interval, and also makes it easier to identify the edges of that interval.

The final outcome of our discussion is a most important one. As we make measurements and as we report the results we must keep in mind constantly this fundamental and vital point—measurements are not exact, single numbers but consist of intervals, within which we are confident that our desired value lies. The act of measurement requires us to determine both the location and width of this interval, and we do it by the careful exercise of visual judgment every time we make a measurement. There are no rules for determining the size of the interval, for it will depend on many factors in the measuring proc-