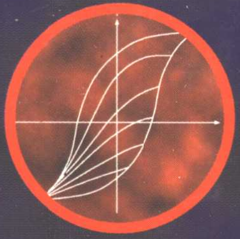
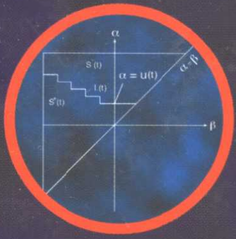
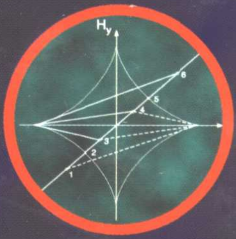
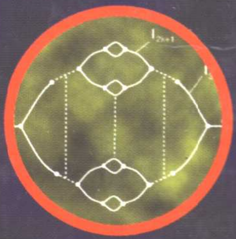
ISAAK MAYERGOYZ

MATHEMATICAL MODELS OF HYSTERESIS AND THEIR APPLICATIONS









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Mathematical Models of Hysteresis and Their Applications

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Preface

"But I have lived, and have not lived in vain:
My mind may lose its force, my blood its fire,
And my frame perish even in conquering pain,
But there is that within me which shall tire
Torture and Time, and breathe when I expire..."
Lord Byron

This book is a greatly expanded, revised and updated version of the previous book "Mathematical Models of Hysteresis" (Springer-Verlag, 1991). This book deals with mathematical models of hysteresis nonlinearities with "nonlocal memories". The distinct feature of these nonlinearities is that their future states depend on past histories of input variations. It turns out that memories of rate-independent hysteresis nonlinearities are quite selective. Indeed, only some past input extrema (not the entire input variations) leave their marks upon the future states of rate-independent hysteresis nonlinearities. Thus, special mathematical tools are needed to describe nonlocal selective memories of such hysteresis nonlinearities. The origin of such tools can be traced back to the landmark paper of Preisach.

The first three chapters of this book are primarily concerned with Preisach-type models of hysteresis. All these models have a common generic feature: they are constructed as superpositions of the simplest hysteresis nonlinearities—rectangular loops. The discussion in these chapters is by and large centered around the following topics: various generalizations and extensions of the classical Preisach model of hysteresis (with special emphasis on vector generalizations); finding of necessary and sufficient conditions for the representation of actual hysteresis nonlinearities by various Preisach-type models; solution of identification problems for these models, their numerical implementation and extensive experimental testing. Our exposition of Preisach-type models of hysteresis has two salient features. The first is the strong emphasis on the universality of the Preisach models and their applicability to the mathematical description of hysteresis phenomena in various areas of science and technology. The second is the accessibility of the material in the first three chapters to a broad audience of researchers, engineers and students. This is achieved through the deliberate use of simple mathematical tools. The exception is the discussion of the identification problems for the vector Preisach models in the third chapter, where some machinery of integral equations and the theory of irreducible representations of the group of rotations are occasionally used.

The book contains three new chapters that deal with applications of the Preisach formalism to the modeling of thermal relaxations (viscosity) in hysteretic materials as well as to the modeling of superconducting hysteresis and eddy current hysteresis. In Chapter 4, Preisach models driven by stochastic inputs are used for the description of thermal relaxations in hysteretic systems. This approach explicitly accounts for the hysteretic nature of materials, their past histories and stochastic characteristics of internal thermal noise. In this sense, this approach has certain advantages over traditional thermal activation type models of viscosity. This approach also reveals the origin of universality of intermediate $\ln t$-type asymptotics for thermal relaxations. Some results of experimental testing of thermal decay in magnetic materials are presented and the phenomenon of scaling and "data collapse" for viscosity coefficients is reported. The chapter also presents the modeling of temperature dependent hysteresis within the framework of randomly perturbed fast dynamical systems and the discussion of functional (path) integration models of hysteresis and their connections with Preisach-type models.

Chapter 5 covers the modeling of superconducting hysteresis. It starts with the discussion of the critical state (Bean) model for superconductors with ideal (sharp) resistive transitions. It is demonstrated that this model is a very particular case of the Preisach model of hysteresis and, on this basis, it is strongly advocated to use the Preisach model for the description of superconducting hysteresis. The results of extensive experimental testing of the Preisach modeling of superconducting hysteresis are reported and the remarkable accuracy of this modeling is highlighted. The case of gradual resistive transitions described by "power laws" is treated through nonlinear diffusion equations and analytical solutions of these equations are found for linear, circular and elliptical polarizations of electromagnetic fields.

Chapter 6 deals with eddy-current hysteresis in magnetically nonlinear conductors. It is demonstrated that in the case of sharp magnetic transitions (abrupt saturation), the eddy current hysteresis can be represented in terms of the Preisach model. This representation reveals the remarkable fact that nonlinear (and dynamic) eddy current hysteresis can be fully characterized by its step response. Eddy current hysteresis for gradual magnetic transitions is studied by using nonlinear diffusion equations and analytical solutions of these equations are reported for linear and circular polarizations of electromagnetic fields. The developed techniques are used to study "excess" eddy current and hysteresis losses as well as rotational eddy current losses.

In this book, no attempt is made to refer to all relevant publications. For this reason, the lists of references are not exhaustive but rather suggestive. The presentation of the material in the book is largely based on the publications of the author and his collaborators.

I first heard about the Preisach model during my conversation with Professor K. M. Polivanov. This was about thirty years ago, and at that time I lived in Russia. Shortly thereafter, my interest in the Preisach model was strongly enhanced as a result of my discussions with Professors M. A. Krasnoselskii and A. Pokrovskii. When I came to the United States, my work on hysteresis modeling was encouraged by Dr. O. Manley from the U.S. Department of Energy. My research on the Preisach models has benefited from many penetrating discussions I have had with Professor D. Fredkin (University of California, San Diego). I was also fortunate to have such wonderful graduate students as G. Friedman, C. Korman, and A. Adly, who assisted me at different times in my work on hysteresis and who became important contributors in this field in their own right. I am very grateful to my collaborators Professor M. Freidlin, Drs. G. Bertotti, C. Serpico and C. Krafft for the gratifying experience I have had working with them. I acknowledge with gratitude the numerous stimulating discussions I had with Professors A. Visintin, M. Brokate, J. Sprekels and P. Krejci over the past twenty years. I am very thankful to Mrs. P. Keehn who patiently, diligently and professionally typed several versions of the manuscript. In the preparation of the manuscript, I have also been assisted by my students Chun Tse and Mihai Dimian. Finally, I gratefully acknowledge the financial support for my research on hysteresis from the U.S. Department of Energy, Engineering Research Program.

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Introduction

The topic being discussed in this book is mathematical models of hysteresis. Special emphasis is placed on the mathematical exposition of these models which makes them quite general and applicable to the description of hysteresis of different physical nature. There are, however, two additional reasons for this emphasis. As was pointed out by A. Einstein [1], "...mathematics enjoys special esteem, above all other sciences, [because] its laws are absolutely certain and indisputable... ." Mathematics has achieved and maintained this exceptional position because its results are derived from a few (more or less self-evident) axioms by a chain of flawless reasonings. Since it is based on impeccable logic, mathematics can provide some level of security (and clarity) for natural sciences which is not attainable otherwise. For this reason, the rigorous mathematical treatment of natural sciences is highly desirable and should be attempted whenever is possible. In addition, mathematics more and more often serves as a vehicle of communication between scientists and engineers of different specializations. As a result, if some area of science is represented in a rigorous mathematical form, its accessibility is strongly enhanced. With these thoughts in mind, it is hoped that the mathematical exposition of hysteresis models undertaken here will bring much needed clarity into this area and will make it appealing to the broader audience of inquiring researchers.

This monograph has been written by an engineer for engineers. For this reason, mathematics is largely used in the book as a tool rather than a topic of interest in its own right. As a result, many mathematical subtleties of hysteresis modelling are omitted. These subtleties are by and large related to the fact that hysteresis operators are naturally defined on sets of piece-wise monotonic functions that do not form complete function spaces. This leads to the problem of continuous extension of hysteresis operators from the above sets to some complete function spaces. The reader interested in this type of mathematical problems is referred to the study by the Russian mathematicians M. Krasnoselskii and A. Pokrovskii [2] as well as to the books of A. Visintin [3] and M. Brokate and J. Sprekels [4].

FIGURE 1

The phenomenon of hysteresis has been with us for ages and has been attracting the attention of many investigators for a long time. The reason is that hysteresis is ubiquitous. It is encountered in many different areas of science. Examples include magnetic hysteresis, ferroelectric hysteresis, mechanical hysteresis, superconducting hysteresis, adsorption hysteresis, optical hysteresis, electron beam hysteresis, economic hysteresis, etc. However, the very meaning of hysteresis varies from one area to another, from paper to paper and from author to author. As a result, a stringent mathematical definition of hysteresis is needed in order to avoid confusion and ambiguity. Such a definition will serve a twofold purpose: first, it will be a substitute for vague notions, and, second, it will pave the road for more or less rigorous treatment of hysteresis.

We begin with the definition of scalar hysteresis and, for the sake of generality, we adopt the language of control theory. Consider a transducer (see Fig. 1) that can be characterized by an input $u(t)$ and an output $f(t)$. This transducer is called a hysteresis transducer (*HT*) if its input-output relationship is a multibranch nonlinearity for which branch-to-branch transitions occur after input extrema. This multibranch nonlinearity is shown in Fig. 2. For the most part, the case of rate-independent hysteresis nonlinearity will be discussed. The term "rate-independent" means that

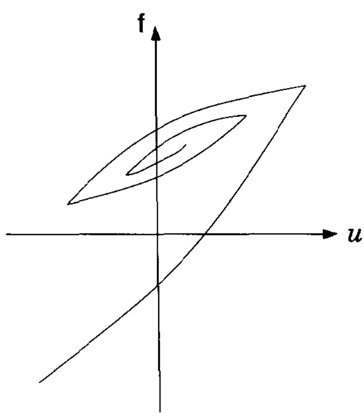

FIGURE 2

branches of such hysteresis nonlinearities are determined only by the past extremum values of input, while the speed (or particular manner) of input variations between extremum points has no influence on branching. This statement is illustrated by Figs. 3a, 3b and 3c. Figures 3a and 3b show two different inputs $u_1(t)$ and $u_2(t)$ that successively assume the same extremum values but vary differently between these values. Then, for a

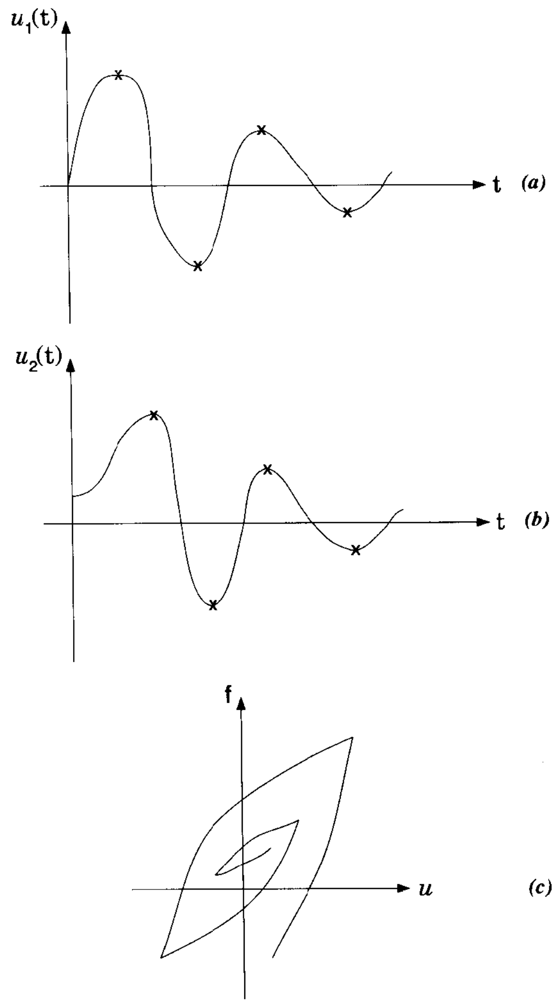

FIGURE 3

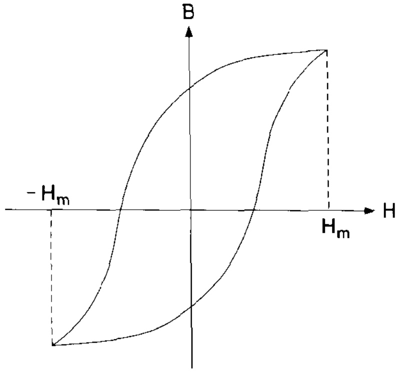

FIGURE 4

rate-independent *HT*, these two inputs will result in the same f–u diagram (see Fig. 3c), provided that the initial state of the transducer is the same for both inputs.

The given definition of rate-independent hysteresis is consistent with existing experimental facts. Indeed, it is known in the area of magnetic hysteresis that a shape of major (or minor) loop (see Fig. 4) can be specified without referring to how fast magnetic field H varies between two extremum values, $+H_m$ and $-H_m$. This indicates that time effects are negligible and the given definition of a rate-independent hysteresis transducer is an adequate one. It is worthwhile to keep in mind that, for very fast input variations, time effects become important and the given definition of rate-independent hysteresis fails. In other words, this definition (as any other definition) has its limits of applicability to real life problems.

It is also important to stress that the notion of rate-independent hysteresis implies three distinct time scales. The first is the time scale of fast internal dynamics of the transducer. The second is the time scale on which observations (measurements) are performed. This time scale is much larger than the time scale of internal transducer dynamics so that every observation can be identified with a specific output value of the transducer. The third is the time scale of input variations. This time scale is much larger than the observation time scale so that every measurement can be associated with a specific value of input.

In the existing literature, the hysteresis phenomenon is by and large linked with the formation of hysteresis loops (looping). This may be misleading and create the impression that looping is the essence of hysteresis. In this respect, the given definition of hysteresis emphasizes the fact that history dependent branching constitutes the essence of hysteresis, while looping is a particular case of branching. Indeed, looping occurs when the input varies back and forth between two consecutive extremum values, while branching takes place for arbitrary input variations.

From the given definition, it can also be concluded that scalar hysteresis can be interpreted as a nonlinearity with a memory which reveals itself through branching.

In the given definition of hysteresis, the physical meanings of the input $u(t)$ and the output $f(t)$ were left unspecified. It was done deliberately, for the sake of mathematical generality. However, it is not difficult to specify the meanings of $u(t)$ and $f(t)$ in particular applications. For instance, in magnetism $u(t)$ is the magnetic field and $f(t)$ is the magnetization, in mechanics $u(t)$ is the force and $f(t)$ is the displacement (length), in adsorption $u(t)$ is the gas pressure and $f(t)$ is the amount of material adsorbed. The notion of hysteresis transducer may have different interpretations as well. For instance, in magnetism the *HT* can be construed as an infinitesimally small volume of magnetic material, and the corresponding input-output hysteresis nonlinearity can be interpreted as a constitutive equation for this material.

All rate-independent hysteresis nonlinearities fall into two general classifications: (a) hysteresis nonlinearities with local memories, and (b) hysteresis nonlinearities with nonlocal memories. The hysteresis nonlinearities with local memories are characterized by the following property. The value of output $f(t_0)$ at some instant of time t_0 and the values of input $u(t)$ at all subsequent instants of time $t \geqslant t_0$ uniquely predetermine the value of output $f(t)$ for all $t > t_0$. In other words, for hysteresis transducers with local memories the past exerts its influence upon the future through the current value of output. This is not the case for hysteresis transducers with nonlocal memories. For such transducers, future values of output $f(t)$ $(t \geqslant t_0)$ depend not only on the current value of output $f(t_0)$ but on past extremum values of input as well.

Typical examples of hysteresis nonlinearities with local memories are shown in Figs. 5, 6, and 7. Figure 5 shows the simplest hysteresis nonlinearity with local memory. It is specified by a major loop which is formed by ascending and descending branches. These branches are only partially reversible (their vertical sections are not reversible). This type of hysteresis nonlinearity is characteristic, for instance, of single Stoner–Wolhfarth magnetic particles [5]. For this type of hysteresis, branching occurs if extremum values of input exceed $+u_m$ or $-u_m$.

A more complicated type of hysteresis nonlinearities with local memories is illustrated by Fig. 6. Here, there is a set of inner curves within the major loop and only one curve passes through each point in the f–u diagram. These curves are fully reversible and can be traversed in both

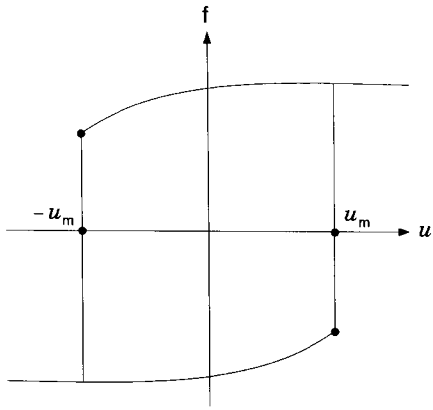

FIGURE 5

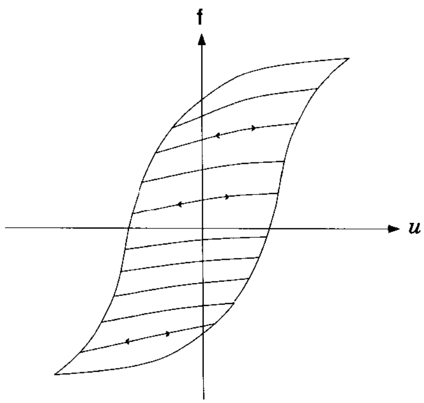

FIGURE 6

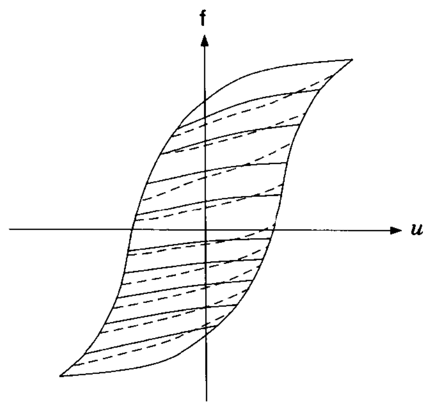

FIGURE 7

directions, for a monotonically increasing and decreasing input $u(t)$. For this type of hysteresis, branching may occur only when ascending or descending branches of major loops are reached.

A hysteresis nonlinearity with local memory that has two sets of inner curves (the ascending and descending curves) is shown in Fig. 7. This type of hysteresis was probably first described by Madelung [6] in the beginning of the century, and afterwards it was independently invented by many authors time and time again (see, for instance, [7] and [8]). For this hysteresis nonlinearity, only one curve of each set passes through each point in the f–u diagram. If the input $u(t)$ is increased, the ascending curve

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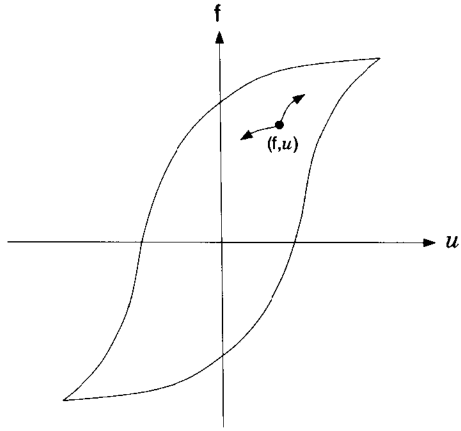

FIGURE 8

is followed; if it is decreased, the descending curve is traced. Thus, branching occurs for any input extremum. However, in general, minor loops are not formed; if $u(t)$ varies back and forth between the same two values, the output usually exhibits a continued upward drift.

It is clear from the above examples that all hysteresis nonlinearities with local memories have the following common feature: every reachable point in the f–u diagram corresponds to a uniquely defined state. This state predetermines the behavior of HT in exactly one way for increasing $u(t)$ and exactly one way for decreasing $u(t)$. In other words, at any point in the f–u diagram there are only one or two curves that may represent the future behavior of HT with local memory (see Fig. 8). This is not true for hysteresis transducers with nonlocal memories. In the latter case, at any reachable point in the f–u diagram there is an *infinity* of curves that may represent the future behavior of the transducer (see Fig. 9). Each of these curves depends on a particular past history, namely, on a particular sequence of past extremum values of input. By analogy with the random process theory, hysteresis nonlinearities with local memories can be called Markovian hysteresis nonlinearities, while hysteresis nonlinearities with nonlocal memories are non-Markovian. It is clear that hysteresis nonlinearities with nonlocal memories are much more complicated than those with local memories.

Mathematical models of hysteresis nonlinearities with local memories have been extensively studied by using differential and algebraic equations. These models have achieved high level of sophistication that is reflected, for instance, in publications [9–12]. However, the notion of

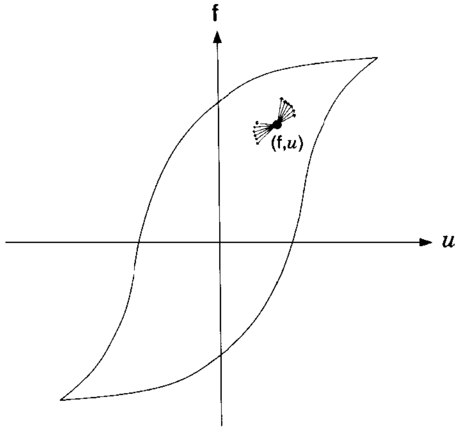

FIGURE 9

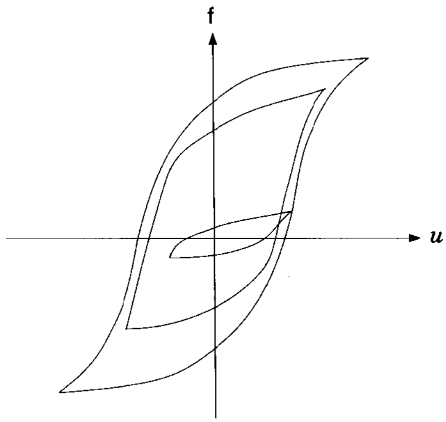

FIGURE 10

hysteresis nonlinearities with local memories is not consistent with experimental facts. For instance, it is reported in [13] that crossing and partially coincident minor loops have been experimentally observed. These loops are schematically shown in Figs. 10 and 11, respectively. The existence of crossing minor loops attached to a major loop is more or less obvious, while the presence of partially coincident minor loops is a more subtle phenomenon. The existence of crossing and partially coincident minor loops clearly suggests that the states of the corresponding hysteresis

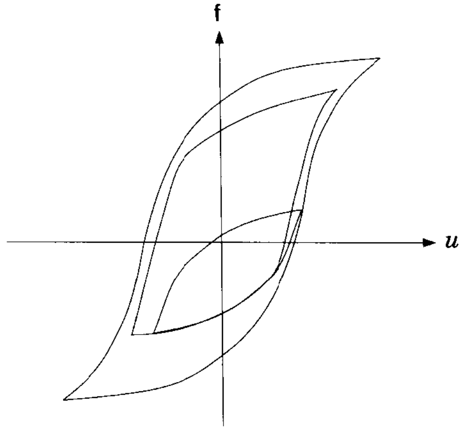

FIGURE 11

transducers are not uniquely specified by their inputs and outputs. Thus, hysteresis of this transducer does not have a local memory.

This book is solely concerned with mathematical models of hysteresis with nonlocal memory. The question arises, why are these models needed? The answer is that the hysteresis transducer is usually a part of a system. As a result, its input is not known beforehand, but is determined by the interaction of the transducer with the rest of the system. Since the input of *HT* is not predictable a priori, it is impossible to specify ahead of time the branches of hysteresis nonlinearity which will be followed in a particular regime of the system. This is the main impediment as far as self-consistent mathematical descriptions of systems with hysteresis are concerned. To overcome the difficulty mentioned above, mathematical models of hysteresis are needed. These models represent new mathematical tools that themselves (due to their structure) will detect and accumulate input extrema and will choose appropriate branches of the hysteresis nonlinearity according to the accumulated histories. Coupled together with mathematical description of the rest of the system, these models will constitute complete and self-consistent mathematical descriptions of systems with hysteresis. Without such models, the self-consistent mathematical descriptions of systems with hysteresis are virtually impossible.

We next turn to the discussion of vector hysteresis. This hysteresis can be characterized by a vector input $\vec{u}(t)$ and vector output $\vec{f}(t)$ (see Fig. 12). Two- and three-dimensional vector inputs and vector outputs are most relevant to practical applications. That is why only two- and three-dimensional vector hysteresis models are discussed in the book. However,

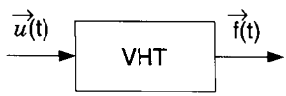

FIGURE 12

the formal mathematical generalization of these models to n dimensions ($n > 3$) is straightforward. It is believed that such a generalization will be performed by the reader if it is needed.

The most immediate problem we face is how to define vector hysteresis in a mathematically rigorous as well as physically meaningful way. To do this, it is important to understand what constitutes in the case of vector hysteresis the essential part of past input history that affects the future variations of output. In the case of scalar rate-independent hysteresis, experiments show that only past input extrema (not the entire input variations) leave their mark upon future states of hysteresis nonlinearities. In other words, the memories of scalar hysteresis nonlinearities are quite selective. There is no experimental evidence that this is the case for vector hysteresis. As a result, we must resign ourselves to the fact that all past vector input values may affect future output variations. The past input variations can be characterized by an oriented curve L traced by the tip of the vector input $\vec{u}(t)$ (see Fig. 13). Such a curve can be called an input "hodograph." Vector rate-independent hysteresis can be defined as a vector nonlinearity with the property that the shape of curve L and the direction of its tracing (orientation) may affect future output variations, while the speed of input hodograph tracing has no influence on future output variations. Next, we demonstrate that scalar rate-independent hysteresis can be construed as a particular case of vector rate-independent hysteresis. This case is realized when the vector input is restricted to vary along only one direction (one line). In fact, it can be successfully argued (at least in the area of magnetics) that there is no such a thing as scalar hysteresis. Whenever we talk about scalar hysteresis, we are actually dealing with some specific properties of vector hysteresis that have been observed

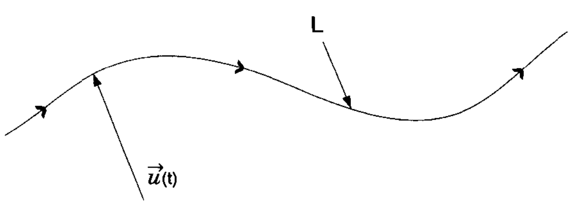

FIGURE 13

FIGURE 14

for vector input variations restricted to some fixed directions. It is apparent that, for unidirectional input variations $\vec{u}(t) = \vec{a}u(t)$, input hodographs (see Fig. 14) are uniquely determined by current values of $u(t)$ as well as by past extrema of $u(t)$. In this sense, vector rate-independent hysteresis is reduced to scalar rate-independent hysteresis with the input $u(t)$.

Next, we shall give another equivalent definition of rate-independent vector hysteresis in terms of input projections. This definition will be convenient in the design of mathematical models of vector hysteresis. Consider input projection along some arbitrary chosen direction. As the vector $\vec{u}(t)$ traces the input hodograph, the input projection along the chosen direction may achieve extremum values at some points of this hodograph. In this sense, the extrema of input projection along the chosen direction samples certain points of the input hodograph. If the projection direction is continuously changed, then the extrema of input projections along the continuously changing direction will continuously sample all points on the input hodograph. In this way, the past extrema of input projections along all possible directions reflect the shape of input hodograph and, consequently, the past history of input variations. Thus, we arrive at the definition of vector rate-independent hysteresis as a vector nonlinearity with the property that past extrema of input projections along all possible directions may affect future output values. It is clear that mathematical models of vector hysteresis are imperative for self-consistent descriptions of systems with vector hysteresis. These models should be able to detect and store past extrema of input projections along all possible directions and choose the appropriate value of vector output according to the accumulated history.

This book deals exclusively with the mathematical models of hysteresis that are purely phenomenological in nature. Essentially, these models represent the attempt to describe and generalize experimental facts. They provide no insights into specific physical causes of hysteresis. Nevertheless, they have been and may well continue to be powerful tools for device design. There are, however, fundamental models of hysteresis which attempt to explain experimental facts from first principles. For instance, in micromagnetics, these principles require that the equilibrium distribution of magnetization should correspond to free energy minimum. The minimized energy basically includes the exchange energy, the anisotropy energy, the energy of interaction with an applied field, the magnetostatic self-energy, and possibly some other terms. It turns out that there are many (at least two) different local minima of the total energy for a given applied field. Since only one of these energy minima corresponds to the thermodynamic equilibrium state, the others must be metastable. They may persist for a very long time. These persisting metastable states are responsible for the origin of hysteresis.

Although the above micromagnetic approach is fundamental in nature, its implementation encounters some intrinsic difficulties.

First, in order to carry out this approach, the detailed information of microscopic material structure is needed. Only on the basis of this information can the above-mentioned terms of minimized energy be specified. However, the detailed knowledge of material microstructure is often not available.

Second, the micromagnetic approach leads to nonlinear differential (or integrodifferential) equations which are quite complicated to solve even using sophisticated numerical techniques. In part, this is because the solution of these equations may exhibit highly irregular behavior. Indeed, domains and their walls should emerge from the micromagnetic approach. The domain walls are small regions where the direction of magnetization changes quite rapidly, from some particular direction in one domain to a different direction in an adjacent domain. In a way, these domain walls can be mathematically construed as interior layers. This suggests that micromagnetic problems may well belong to the class of singularly perturbed problems. (This fact has not been appreciated enough in the existing literature). To resolve the fast variations of magnetization over the domain walls, very fine meshes are needed. But, the domain walls usually move when the applied field is changed. Thus, it is not clear a priori where the fine meshes should be located. This may seriously complicate the numerical analysis.

Finally, the detailed domain structure which can be produced by the micromagnetic approach may be irrelevant to some practical problems. This is the case, for instance, in the design of devices for which the average value of magnetization over regions with dimensions much larger than domain dimensions is of interest.

Summarizing the above discussion, it can be concluded that the phenomenological approach is more directly connected with macroscopic experimental data. For this reason, it is of a great value to device designers. The fundamental micromagnetic approach, on the other hand, is intimately related to material structure and, therefore, it can be useful in the design of new materials.

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