FUNDAMENTALS OF DIGITAL SIGNAL PROCESSING

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PREFACE

In the early and mid-1960s, computer simulations for problems in speech, seismology, medical research, oceanography, and radar were prominent. The area was scattered, however, and some of the techniques, although used, were not well understood. At that time, Bernard Gold and Charles M. Rader of the Massachusetts Institute of Technology realized the importance of providing a unified and comprehensive coverage of this important area. They presented in their book, *Digital Processing of Signals* (1969), foundational material that could be used by engineers in the design of special-purpose digital hardware and general-purpose digital structures to solve signal processing problems. In the control systems area, excellent pioneering presentations in sampled data systems were given by E. I. Jury and A. J. Monroe. At that time, real-time implementation using the techniques presented bordered on feasibility. However, within the last decade integrated circuit technology and hardware developments have gradually made those digital signal processing techniques not only feasible in real time but almost required in everyday engineering applications.

The books in 1975 by Oppenheim and Shafer and Gold and Rabiner entitled *Digital Signal Processing and Theory* and *Applications of Digital Signal Processing*, respectively, were monumental efforts in clarifying, expanding, and organizing the fundamentals and tools of digital signal processing. The texts provided a strong foundation for further efforts; however, they were written with the graduate student in mind and were sometimes considered "rough sledding" for undergraduates and practicing engineers.

During those same years of development it became apparent that discrete-time techniques implemented by digital hardware were fundamental in attacking many problems of engineering and basic science, especially those in electrical and computer engineering. The educational community gradually began making discrete-time systems a required part of their electrical engineering undergraduate programs, and the impetus is now on to include digital signal processing as a basic required undergraduate course as well.

The main objective of this book is to provide background and fundamental material in: discrete-time systems, basic digital processing techniques, design procedures for digital filters, and the discrete Fourier transform. This book is written for advanced junior- and senior-level students in electrical and electrical and computer engineering programs, as well as for practicing engineers.

STRUCTURE OF TEXT

The theory of continuous-time linear systems has played a very important role in engineering and the physical sciences. The reason for this is that many natural physical components and systems can be described mathematically by linear differential equations. Measurable physical parameters such as voltage, current, force, velocity, etc., can be thought of as excitations or inputs to the system, with the resulting reactions or outputs being described by the solution of the proper differential equations. We are now faced with systems that are not exactly "physical" in nature but are constructed from computers, digital hardware, and even computer programs. To make matters even worse, these systems will be "bred" with the so-called physical systems mentioned above.

It would be noble, and certainly false, to say that this text will offer a unified approach to "modeling," designing, and analyzing such systems in general. It is hoped, however, that the text can present a logical and precise development for analyzing and designing certain specific structures of digital systems, namely, those in the A/D–digital filter–D/A structure.

In Chapter 1, definitions are presented for special types of discrete-time signals and systems with special emphasis on frequency response characteristics and the aliasing property. Further background material including fundamentals of the $\mathcal{Z}$ transform and relationships among linear constant coefficient difference equations, system functions, and impulse response, are presented in Chapter 2.

Chapters 3 and 4 provide design procedures for both classical analog filters and digital filters, while Chapter 5 gives some insight into the hardware realization of such digital filters.

The discrete Fourier transform and its inverse are presented along with algorithms for fast calculation in Chapter 6. Special attention is given to interpreting the results obtained by using the discrete Fourier transform on sampled continuous-time signals.

SUGGESTED COVERAGE

The text allows several options with respect to the selection and order of topics covered depending on the emphasis desired and the interests of the students. First, for those who wish to emphasize digital filters the suggested coverage is Chapters 0–5 in normal order, while those primarily interested in pursuing the discrete Fourier transform and fast Fourier transform may wish to cover Chapters 0–2 in regular order, followed by Chapter 6 and a return to either Chapter 5 or Chapters 3 and 4. Another convenient order, if both the discrete Fourier transform and digital filtering areas are to be presented, is Chapters 0–6 in regular order, skipping Chapter 5.

In any of the above orders Chapter 3 may be covered quickly or omitted if the students have a strong background in classical analog filter design involving Chebyshev, Butterworth, and elliptic filters as well as filter transformation theory.

ACKNOWLEDGMENTS

This book was born out of a fundamental need to present digital signal processing methods to undergraduate students and practicing engineers. There are many people and organizations that have contributed in a myriad of ways to the writing of this text, the list of whom and which would be larger than the text itself.

In particular, a great debt is owed to the pioneering and masterful text *Digital Signal Processing*, written by Alan V. Openheim and Ronald W. Schafer. Their book provided the framework, main source of references, and basic need for this text.

The author greatly appreciates the atmosphere in the Electrical and Computer Engineering Department of New Mexico State University, and the understanding of its faculty who supported him in many ways throughout all stages of the project. A sincere special thanks goes to the many students who endured the suffering of the initial and final stages of this project. Their constant encouragement, suggestions for improvement, and enthusiasm were a driving force.

The opportunity to present various parts of the manuscript as course material to engineers at: the Naval Undersea Warfare Engineering Station, Keyport, Washington; RAYTHEON, White Sands Missile Range; and the Central Inertial Guidance and Test Facility, Holloman AFB, was very important and helpful during the beginning stages of organization and writing. Thanks also are extended to the Centre d'Etude des Phénomènes Aléatoires et Géophysiques, Institut National Polytechnique de Grenoble and its researchers for providing assistance during the final polishing of the manuscript.

Appreciation is also expressed to Lynn Kirlin, University of Wyoming; Ernest Baxa, Jr., Clemson University; and the following reviewers for their helpful comments: Jan Allebach, Purdue University; Gary Ford, University of California at Davis; Ken Jenkins, University of Illinois; Russell M. Mersereau, Georgia Institute of Technology; David C. Munson, Jr., University of Illinois; and James T. Kajiya, California Institute of Technology. The book could never have been written without the magic performed by Lois Griese in the typing of the manuscript.

I will be forever grateful for the influence and instruction of one of the best high school educators of mathematics in the world, Florence Krieger, Rapid City High School, South Dakota.

Last, and most important, is the love, understanding, and encouragement provided by family and friends, especially my parents, who gave me a sound and healthy outlook on life and the skills necessary to mature in a changing society, and my two daughters Laurie and Miranda, who kept me from growing up.

LONNIE C. LUDEMAN

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Introduction

What is digital signal processing? This question will be answered by carefully defining each of the words and then presenting a precise example.

Examples of signals are: a voltage as a function of time, a potential as a function of position in a three-dimensional space, a force as a function of time and position, and an intensity as a function of x and y coordinates and time. More precisely, a *signal* is a function of a set of independent variables, with time being perhaps the most prevalent single variable. The signal itself carries some kind of information available for observation.

By *processing* we mean operating in some fashion on a signal to extract some useful information. In many cases this processing will be a nondestructive "transformation" of the given data signal; however, some important processing methods turn out to be irreversible and thus destructive.

The word *digital* shall mean that the processing is done with a digital computer or special purpose digital hardware.

In the following very simple example, analog and digital processing are illustrated and compared.

0.1 SIGNAL PROCESSING EXAMPLE

To explore the parallels and divergences between digital and analog filtering design methodology, a very simple problem will be approached by both methods.

Assume that a low-frequency signal $s(t)$, band limited to f_s Hz, is observed in an additive noisy environment to give a received signal $x(t)$ given by

$$x(t) = s(t) + n(t) \tag{0.1}$$

The noise signal $n(t)$ is assumed to be band-limited white noise, that is, its spectral content has equal per unit bandwidth power from dc to f_n Hz. The problem is to operate on $x(t)$ in some way to obtain an estimate $\hat{s}(t)$ of the signal $s(t)$. Two solutions are presented, the first being an analog signal processor, the second a digital signal processor. Both processors will be based on a direct approach of obtaining better estimates of $s(t)$ by using a low-pass filter to minimize the effect of the high-frequency components of the interfering noise. To be more precise, a set of frequency domain requirements is established, perhaps, in terms of specifying an acceptable passband attenuation, cutoff frequency, and minimum stopband attenuation. The analog and digital processors will be assumed to take the general forms shown in Figs. 0.1 and 0.2.

0.1.1 Analog Signal Processing

Using the given requirements above, classical techniques for filter design can be used to establish a transfer function $H(s)$—perhaps representing a Butterworth or Chebyshev low-pass filter. From the $H(s)$, a linear time invariant circuit could be synthesized; for example, the simple single pole filter consisting of an inductor and resistor as shown in Fig. 0.3 might be suggested. To actually build or construct the filter an approximation of the analytically determined L and R by actual available components would be required.

0.1.2 Digital Signal Processing

It can be established by methods described in later chapters that the prescribed filtering operation can be accomplished digitally using the structure shown in Fig. 0.2. The structure is composed of an analog prefilter, an analog-to-digital (A/D) converter, a digital filter represented by a transfer function, $H(z)$, a digital-to-analog (D/A) converter, and a reconstruction filter.

The analog prefilter specified by its transfer function $H_{pf}(s)$ in most cases is a low-pass or bandpass filter designed to reduce the effects of out of band interfering signals. Interfering signals could be extraneous noise or higher-fre-

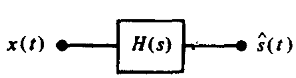

Figure 0.1 Analog signal processing of analog signals.

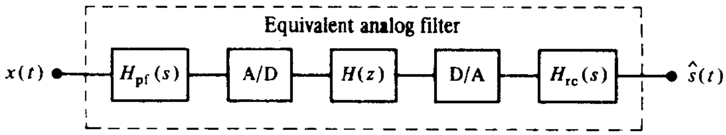

Figure 0.2 Digital signal processing of analog signals.

quency signals that when sampled produce lower-frequency signals (a phenomenon called "aliasing," which is fully described in the text). For this reason, the analog prefilter is sometimes called an "antialiasing" filter in that it combats the aliasing phenomenon.

The A/D converter is a device which will, upon command, give a binary code word corresponding to the quantized level of a continuous-time input signal at that time. Conceptually, it can be thought of as a combination of ideal sampler, quantizer, and encoder, as shown in Fig. 0.4. Normally the code words are strings of binary digits using ones-complement, twos-complement, offset binary, or sign and magnitude binary representations.

The digital filter represented symbolically by $H(z)$ is an algorithm that produces an output sequence $y(n)$ from the input sequence $x(n)$. A very widely used type of operation is one that produces the output $y(n)$ at time n as a weighted sum of the past input and output values.

The D/A converter is a device that operates on a sequence of input code words to produce a continuous-output signal usually of a staircase form. This staircase form is then smoothed by a reconstruction filter to produce the desired output signal $y(t)$.

An expanded version of the digital signal processor illustrating typical signals at various positions within the structure is shown in Figure 0.5. The input signal $x(t)$ shown on the left is filtered by the prefilter to provide the analog signal $x_a(t)$, which is sampled and coded to give the input sequence $x(n)$. This $x(n)$ is then operated on by the digital filter to produce an output sequence $y(n)$, which passes to the D/A converter producing a staircase function $y_a(t)$. Finally, $y_a(t)$ is smoothed by the reconstruction filter to produce the output $y(t)$, thus realizing an equivalent analog filter. For our example we desire $y(t)$ to be an estimate $\hat{s}(t)$ of the signal $s(t)$.

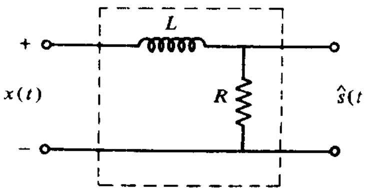

Figure 0.3 Simple analog processor.

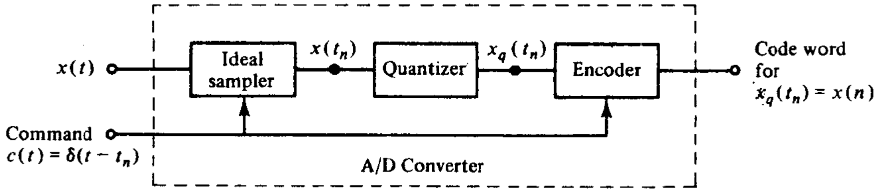

Figure 0.4 Block diagram of an analog-to-digital converter.

To complete the design of the processor for our example requires the specification of each block described above along with the sample rate for the A/D and D/A. Various techniques to be presented in this text can be used to determine the digital filter represented by $H(z)$. For the requirements of our example, it can be shown that the filter can be described by a difference equation as follows:

$$y(n) = b_0x(n) + b_1x(n-1) - a_1y(n-1) \qquad (0.2)$$

Later in the text it will be shown that this corresponds to a system function $H(z)$ given by

$$H(z) = \frac{b_0 + b_1z^{-1}}{1 + a_1z^{-1}}$$

Implementation of the filter described by (0.2) requires digital circuitry to perform the multiplications and additions; memory for the storage of present and past values of the input $x(n)$; memory for the storage of present and past values of the output $y(n)$; and memory for storage of the coefficients a_1, b_0, and b_1. A block diagram illustrating the required multiplications, additions, and storage is shown in Fig. 0.6.

The approximations necessary for actual implementation of the design are of two basic types. First, the coefficients b_0, b_1, and a_1 must be quantized because of finite memory requirements and, second, the corresponding multiplications and additions must be performed using finite representations. There is also an approximation error due to the fact that $x(n)$ is a finite representation of the sampled analog input $x(t)$.

Another way of imitating the same analog processor is shown in Fig. 0.7. In this implementation the special-purpose digital hardware is replaced by a digital computer that is programmed to perform the calculations given in Eq. (0.2).

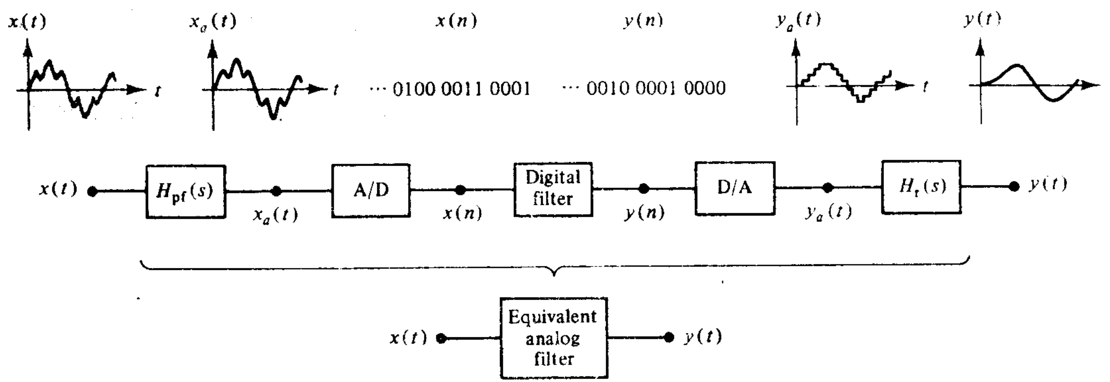

Figure 0.5 Expanded version of a digital signal processor.

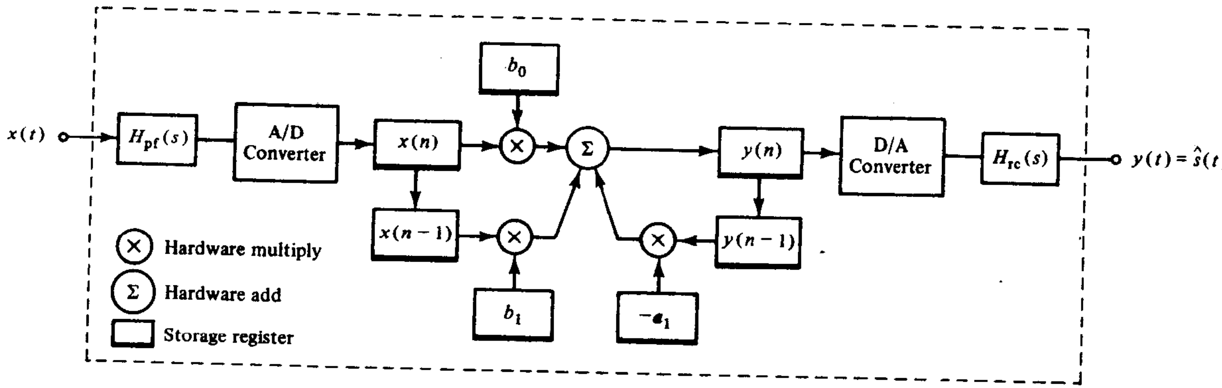

Figure 0.6 Digital signal processing equivalent to the analog processor of Figure 0.3.

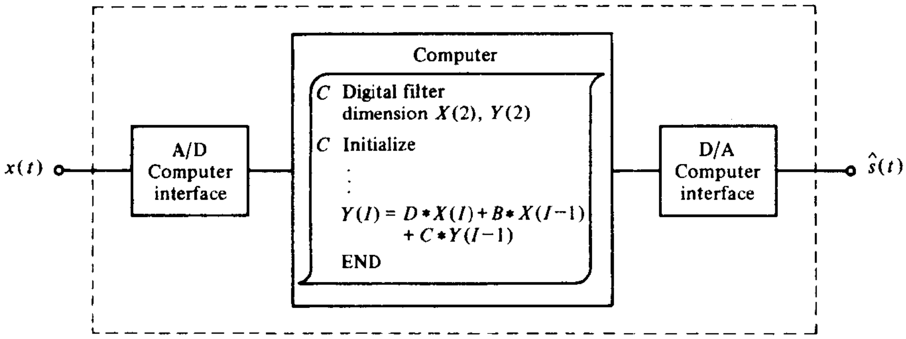

Figure 0.7 Implementation of digital filter with digital computer.

At this point you might, with good reason, ask "what have we gained?" We have managed to replace a resistor and inductor with an analog prefilter, A/D converter, a number of digital multipliers and adders, a D/A converter, and a reconstruction filter or, alternatively, a digital computer with A/D and D/A interfaces. For this simple example we have certainly lost in terms of overall complexity but, as we will see, have gained in terms of flexibility. A change of design simply requires a change in the constants b_0, b_1, and a_1 with no change in the actual components and structure. The advantage that analog signal processing has over digital signal processing concerning complexity fades when many complex filtering operations on many different inputs are required. Analog processing would require components for each filtering operation whereas the digital processor can perform all the operations within the same overall structure.

Since the same physical structure can be used to satisfy a wide variety of applications, the added complexity even for simple processing functions can be accepted because of the increased generality. Also, in many cases process operations difficult to obtain with conventional analog methods are easily performed using nonlinearities and time-varying parameters that allow adaption. Thus, digital signal processing can provide a simple fixed physical construction along with flexibility. The flexibility and structure are further explored in the following sections.

0.2 STRUCTURE OF SPECIAL DIGITAL SIGNAL PROCESSORS

The simplicity of the digital signal processing structure is now illustrated by beginning with a single input–single filtering operation and evolving to a multiple input–multiple filter structure. For ease in presentation the structures are given for a particular type of filtering where the output is a weighted sum of a finite number of previous inputs. Such a filter is called a finite impulse response (FIR) filter, since it can be shown that its response to an impulse is of finite duration.

0.2.1 Single Input–Single Filter

If we let $y(n)$ be the output and $x(n)$ the input, a special class of FIR filters can be described by a difference equation given by

$$y(n) = \sum_{k=0}^{L} b(k)x(n-k) \tag{0.3}$$

In this way the output at time n is determined by a weighted sum of past and present inputs as shown in Fig. 0.8. For its operation, the B register is first fixed with the coefficient values $b(k)$, $k = 0, 1, \ldots, L$ representing the filter coefficients.

The operation can be explained as follows: At time n, $x(n)$ is shifted into X, the input register, and $x(n - (L + 1))$ is pushed out of the X register. A term-by-term product of the contents of X and B is then completed and the sum of these products which gives the output $y(n)$ is placed in the output register Y. After $y(n)$ is calculated, another input value is shifted into the X register, with the last value on the right being shifted out and the multiplications and additions performed to obtain the next output value, etc. Thus, the determination of the

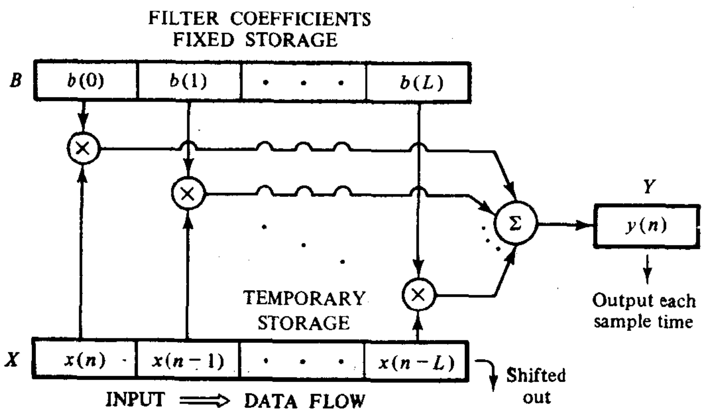

Figure 0.8 Single input–single FIR filter configuration.