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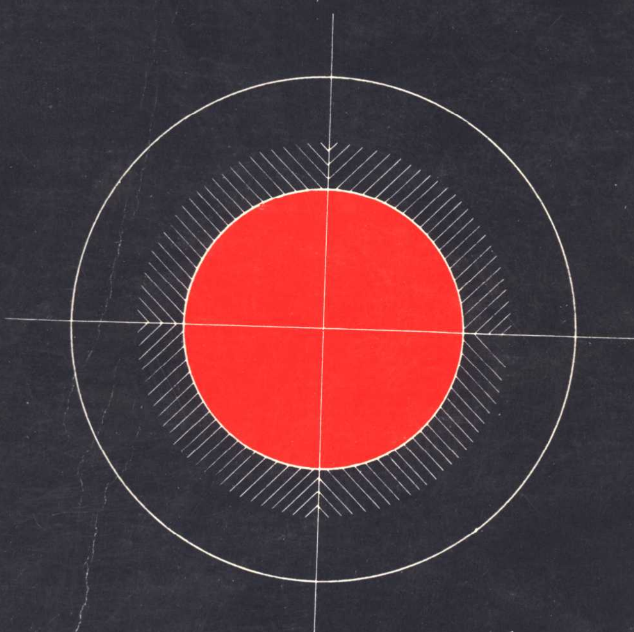
Gene F. Franklin and J. David Powell

DIGITAL CONTROL
OF DYNAMIC SYSTEMS



Digital Control of Dynamic Systems

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To

Gertrude, David, Carole

and

Lorna, Daisy, Annika

Preface

This is a book about the use of digital computers in the real-time control of dynamic systems such as servomechanisms, chemical processes, and vehicles which move over water, land, air, or space. The material requires some understanding of the Laplace transform and assumes the reader has studied a first course in linear feedback controls. The special topics of discrete and sampled-data system analysis are introduced, and considerable emphasis is given to the z-transform and the close connections between the z-transform and the Laplace transform.

The emphasis of the book is on the design of digital controls to achieve good dynamic response and small errors while using signals that are sampled in time and quantized in amplitude. Both transform (classical control) and state-space (modern control) methods are described and applied to illustrative examples. The transform methods emphasized are the root-locus method of Evans and the log-magnitude and phase–versus–log-frequency method of Bode; to aid in the use of Bode's method, the w-plane is introduced. The state-space methods developed are the technique of pole assignment augmented by an estimator (observer) with feed-forward or zero assignment included, and optimal quadratic-loss control. On the latter topic, the emphasis is on the steady-state constant-gain solution; the results of the separation theorem in the presence of noise are stated but not proved. The topic of model making is treated via statistical identification of parameters by least squares and maximum likelihood.

The material in the book which is new to the student is the treatment of signals which are discrete in time and amplitude and which must coexist with those that are continuous in both dimensions. The philosophy of presentation is that the new material should be closely related to material already familiar, and yet, by the end, a direction to wider horizons should be indicated. This approach leads us, for example, to relate the z-transform to the Laplace transform and to describe the implications of poles and zeros in the z-plane to those known meanings attached to poles and zeros in the s-plane. Also, in developing the design methods we relate the digital control design methods to those of continuous systems. And yet, pointing to more sophisticated methods, we present the elementary parts of quadratic-loss gaussian design with minimal proofs to give some idea of the use of this powerful method and to motivate the study of its theory more thoroughly later. The subject matter is particularly suitable for treatment in a laboratory setting and algorithms suitable for programming on a laboratory computer are frequently given.

To review the chapters briefly, the methods of linear analysis are presented in Chapters 1 to 4. Here are introduced the z-transform in Chapter 2, methods to generate discrete equations which will approximate continuous dynamics in Chapter 3, and combined discrete and continuous systems in Chapter 4. This last chapter introduces the sampling theorem and the phenomenon of aliasing. The basic deterministic design methods are presented in Chapters 5 and 6, the root-locus and Bode methods in Chapter 5, and pole placement and estimators in Chapter 6. The state-space material assumes no previous acquaintance with the phase plane or state space, and the necessary analysis is developed from the beginning. Some familiarity with simultaneous linear equations and matrix notation is expected, and a few unusual or more advanced topics such as eigenvalues, eigenvectors, and the Cayley-Hamilton theorem are presented in Appendix C. In Chapter 7 the nonlinear phenomenon of amplitude quantization and its effects on system error and system dynamic response are studied. These first seven chapters comprise the syllabus of a ten-week first course on digital control.

Chapter 8 introduces parametric identification by starting with deterministic least squares, introducing random errors, and completing with an algorithm for maximum likelihood. In Chapter 9 is introduced optimal quadratic loss control; first the control by state feedback is presented and then the estimation of the state in the presence of system and measurement noise is developed, based on the recursive least-squares estimation derived in Chapter 8. The final chapter, Chapter 10, presents methods of analysis and design guidelines for the selection of the sampling period in a digital control system. In such a work, the selection of notation is always critical to be sure our symbols aid rather than interfere with learning. We list at the front of the book a glossary of terms which we use and which we commend to those teachers who use this book.

At Stanford University, two courses based on this material are given. The first course covers Chapters 1 through 7 and follows a course in linear control which may have used Dorf (1980) or Ogata (1970). The second course covers Chapters 8 through 10. Both courses are heavily dependent on laboratory work as a supplement to the lectures for learning. A very satisfactory complement of laboratory equipment is a digital computer capable of running BASIC and having an A/D and a D/A converter, an analog computer with ten operational amplifiers, and a strip recorder.

As do all authors of technical works, we must acknowledge the vast array of contributors on whose work our own presentation is based. The list of references gives some small measure of those to whom we are in debt. On a more personal level, we wish to express our appreciation to those responsible for making Stanford an exciting place to work and to those students of E.207 and E.208 for whom this book was written. We hope its publication will contribute to the education of their successors at Stanford and elsewhere.

We would like especially to express our appreciation to Judy Clark, who aided us in so many ways in the preparation of the notes which became the manuscript which became this book.

Stanford, California G. F. F.
January 1980 J. D. P.

Glossaries

CONTROL GLOSSARY

Plant

Continuous case:

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{G}\mathbf{u}(t - \lambda) + \mathbf{G}_1\mathbf{w}$$

$$\lambda = \text{pure time delay}$$

Discrete:

$$\mathbf{x}_{k+1} = \mathbf{\Phi}\mathbf{x}_k + \mathbf{\Gamma}\mathbf{u}_k + \mathbf{\Gamma}_1\mathbf{w}_k$$

$\mathbf{x}$ = state = $N_s \times 1$ or $n \times 1$

$\mathbf{u}$ = control = $N_c \times 1$ or $m \times 1$

$\mathbf{w}$ = input disturbance or plant noise $N_w \times 1$

$\mathbf{F}$ = continuous system matrix

$\mathbf{\Phi}$ = discrete system matrix

$\mathbf{G}$ = continuous-control input matrix

$\mathbf{\Gamma}$ = discrete-control input matrix

$\mathbf{G}_1$ = plant-noise input matrix

$\mathbf{\Gamma}_1$ = discrete plant-noise matrix

$\overline{\mathbf{w}} = \mathscr{E}\mathbf{w}$ = average value of plant noise

$\mathbf{R}_\mathbf{w}$ = plant-noise spectral density matrix

$\mathscr{E}(\mathbf{w} - \overline{\mathbf{w}})(\mathbf{w} - \overline{\mathbf{w}})^T = \mathbf{R}_\mathbf{w}\delta(t)$ continuous

$= \mathbf{R}_\mathbf{w}$ discrete

$\lambda_i(\mathbf{F}) = p_i$ = open-loop poles = "x" on root locus

Plant output or sensor equations

Continuous system:

$$\mathbf{y} = \mathbf{Hx} + \mathbf{Ju} + \mathbf{v}$$

Discrete system:

$$\mathbf{y} = \mathbf{H}_d\mathbf{x} + \mathbf{J}_d\mathbf{u} + \mathbf{v}$$

$\mathbf{y}$ = output measurements = $N_0 \times 1$ or $p \times 1$

$\mathbf{v}$ = output noise or disturbance = $N_0 \times 1$ or $p \times 1$

$\mathbf{H}$ = continuous output matrix

$\mathbf{J}$ = continuous-plant direct transmission matrix

$\mathbf{H}_d$ = discrete-system output matrix

$\mathbf{J}_d$ = discrete-plant direct-transmission matrix

$\mathbf{v}_{\bar{v}} = \mathscr{E}\mathbf{v}$ = average value of output noise = sensor bias

$\mathbf{R}_v$ = measurement-noise spectral density matrix

$$\mathscr{E}(\mathbf{v} - \mathbf{v}_{\bar{v}})(\mathbf{v} - \mathbf{v}_{\bar{v}})^T = \mathbf{R}_v\delta(t) \quad \text{continuous}$$
$$= \mathbf{R}_v \quad \text{discrete}$$

Control law

$$\mathbf{u} = -\mathbf{Kx} \qquad \text{or} \qquad \mathbf{u} = -\mathbf{K\hat{x}}$$

Control characteristic polynomial

$$\alpha_c(s) \qquad \text{or} \qquad \alpha_c(z)$$

$\lambda_i(\mathbf{F} - \mathbf{GK}) = r_i$ = roots of closed-loop characteristic equation ("Δ" on root locus)

Controllability matrix ($n = N_s$)

$$\mathscr{C} = [\mathbf{G} \quad \mathbf{FG} \cdots \mathbf{F}^{n-1}\mathbf{G}] \qquad \text{or} \qquad [\mathbf{\Gamma} \quad \mathbf{\Phi\Gamma} \cdots \mathbf{\Phi}^{n-1}\mathbf{\Gamma}]$$

Estimator/observer

Continuous:

$$\dot{\hat{\mathbf{x}}} = \mathbf{F\hat{x}} + \mathbf{Gu} + \mathbf{L}(y - \hat{y})$$
$$\hat{y} = \mathbf{H\hat{x}} + \mathbf{Ju}$$

Discrete:

One-step prediction

$$\hat{\mathbf{x}}_{k+1} = \mathbf{\Phi}\hat{\mathbf{x}}_k + \mathbf{\Gamma u}_k + \mathbf{L}(\mathbf{y}_k - \hat{\mathbf{y}}_k)$$
$$\hat{y}_k = \mathbf{H}_d\hat{\mathbf{x}}_k + \mathbf{J}_d\mathbf{u}_k$$

Current estimator

$$\bar{\mathbf{x}}_{k+1} = \mathbf{\Phi}\hat{\mathbf{x}}_k + \mathbf{\Gamma u}_k \qquad \text{Time update}$$
$$\hat{\mathbf{x}}_{k+1} = \bar{\mathbf{x}}_{k+1} + \mathbf{L}(\mathbf{y}_{k+1} - \bar{\mathbf{y}}_{k+1}) \qquad \text{Observation update}$$

$$\bar{\mathbf{y}}_{k+1} = \mathbf{H}_d\bar{\mathbf{x}}_{k+1} \qquad (\mathbf{J}_d = 0)$$

$\mathbf{L}$ = estimator gain matrix

$\mathbf{P} = \mathscr{E}(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T = \mathscr{E}\tilde{\mathbf{x}}\tilde{\mathbf{x}}^T$ = state error covariance matrix

$\mathbf{R}_x = \mathscr{E}(\mathbf{x}\mathbf{x}^T)$

$\mathbf{R}_u = \mathscr{E}\mathbf{u}\mathbf{u}^T$

Controller

Continuous:

$$\dot{\mathbf{x}}_c = \mathbf{A}\mathbf{x}_c + \mathbf{B}\mathbf{y} + \mathbf{M}\mathbf{r}$$
$$\mathbf{u} = \mathbf{C}\mathbf{x}_c + \mathbf{D}\mathbf{y} + \mathbf{N}\mathbf{r}$$

Discrete:

$$\mathbf{x}_c(k+1) = \mathbf{A}\mathbf{x}_c(k) + \mathbf{B}\mathbf{y}(k) + \mathbf{M}\mathbf{r}(k)$$
$$\mathbf{u}(k) = \mathbf{C}\mathbf{x}_c(k) + \mathbf{D}\mathbf{y}(k) + \mathbf{N}\mathbf{r}(k)$$

$\mathbf{x}_c$ = controller state

$\mathbf{r}$ = reference input = $N_0 \times 1$

$\mathbf{A}$ = Controller system matrix

$\mathbf{B}$ = Controller input distribution matrix

$\mathbf{C}$ = Controller output matrix

$\mathbf{D}$ = Controller direct-transmission matrix

$\mathbf{M}$ = Controller reference-input distribution matrix

$\mathbf{N}$ = Controller reference-input direct transmission matrix

Optimal control

$$\mathscr{J} = \int_{t_0}^{t_f} l(\mathbf{x}, \mathbf{u}, t)\,dt + \psi(\mathbf{x}_f, \mathbf{t}_f)$$

Quadratic loss

$$\mathscr{J} = \mathscr{E}\left\{\int_{t_0}^{t_f} (\mathbf{x}^T\mathbf{Q}_1\mathbf{x} + \mathbf{u}^T\mathbf{Q}_2\mathbf{u})\,dt + \mathbf{x}_f^T\mathbf{Q}_0\mathbf{x}_f\right\}$$

Discrete quadratic loss

$$\mathscr{J} = \mathscr{E}\left\{\sum_{k=j}^{N} (\mathbf{x}^T\mathbf{Q}_1\mathbf{x} + \mathbf{u}^T\mathbf{Q}_2\mathbf{u}) + \mathbf{x}_N^T\mathbf{Q}_0\mathbf{x}_N\right\}$$

ALPHABETICAL GLOSSARY

A	Controller system matrix
B	Controller input matrix

$\mathbf{C}$	Controller output matrix
$\mathbf{D}$	Controller direct matrix
$\mathbf{F}$	Plant system matrix
$\mathbf{G}$	Plant input matrix
$\mathbf{G}_1$	Plant disturbance input matrix
$\mathbf{H}$	Plant output matrix
$\mathbf{H}_d$	Discrete plant output matrix
$\mathbf{J}$	Plant direct matrix
$\mathbf{J}_d$	Discrete plant direct matrix
$\mathbf{K}$	Control gain
$\mathbf{L}$	Estimator gain
$\mathbf{M}$	Controller-reference-input distribution matrix
$\mathbf{N}$	Controller-reference-input direct matrix
$\mathbf{P}$	$E\tilde{\mathbf{x}}\tilde{\mathbf{x}}^T$
$\mathbf{Q}_1$, $\mathbf{Q}_2$, $\mathbf{Q}_0$	Loss matrices, state, control, terminal, respectively
$\mathbf{R}_w$, $\mathbf{R}_v$, $\mathbf{R}_x$, $\mathbf{R}_u$	Spectral density matrices of $\mathbf{w}$, $\mathbf{v}$; covariance matrices of $\mathbf{x}$, $\mathbf{u}$
T	Sampling period
$\mathbf{u}$	Control
$\mathbf{v}$	Measurement noise
$\mathbf{w}$	Plant or input noise
$\mathbf{x}$	Plant state
$\mathbf{x}_c$	Controller state
$\mathbf{y}$	Output
α_c, α_e	Control and estimator characteristic polynomials
$\mathbf{\Gamma}$	Discrete plant control input matrix
$\mathbf{\Gamma}_1$	Discrete plant noise input matrix
λ	Plant delay time or transportation lag
$\mathbf{\Phi}$	Discrete plant system matrix

Contents

7 Quantization Effects

8 System Identification

9 Multivariable and Optimal Control

10 Sample Rate Selection

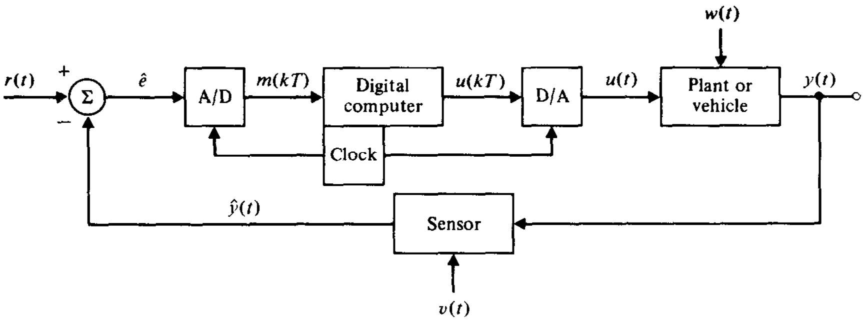
1 / Introduction

1.1 PROBLEM DEFINITION

The control of physical systems with a digital computer is becoming more and more common. Aircraft autopilots, mass transit vehicles, oil refineries, paper-making machines, and countless electromechanical servomechanisms are among the many existing examples. Furthermore, many new digital control applications are being stimulated by microprocessor technology including control of various aspects of automobiles and household appliances. Among the advantages of digital logic for control are the increased flexibility of the control programs and the decision-making or logic capability of digital systems which can be shared with the control function to meet other system requirements.

The digital controls studied in this book are for closed-loop (feedback) systems in which the dynamic response of the process being controlled is a major consideration in the design. A typical topology of the elementary type of system which will occupy most of our attention is sketched schematically in Fig. 1.1. This figure will help to define our basic notation and to introduce several features which distinguish digital controls from those implemented with analog devices. The process to be controlled is called the plant and may be any of the physical processes mentioned above whose satisfactory response requires control action.

By "satisfactory response" we mean that the plant output, $y(t)$, is to be forced to follow or track the reference input, $r(t)$, despite the presence of disturbance inputs to the plant [$w(t)$ in Fig. 1.1] and despite errors in the sensor [represented by $v(t)$ in Fig. 1.1]. It is also essential that the tracking succeed even if the dynamics of the plant should change somewhat during the operation. The process of holding $y(t)$ close to $r(t)$, including the case where $r \equiv 0$, is referred to generally as the process of *regulation*. A system which has good regulation in the presence of disturbance signals is said to have good *disturbance rejection*. A system which has good regulation in the face of changes in the plant parameters is said to have low *sensitivity* to these parameters. A system which has both good disturbance rejection and low sensitivity we call *robust*.



Notation:

r = reference or command inputs
u = control or actuator input signal
y = controlled or output signal
$\hat{y}$ = instrument or sensor output, usually an approximation to or estimate of y. (For any variable, say θ, the notation $\hat{\theta}$ is now commonly taken from statistics to mean an estimate of θ).
$\hat{e}$ = $r-\hat{y}$ = indicated error
e = $r-y$ = system error
w = disturbance input to the plant
v = disturbance or noise in the sensor
A/D = analog-to-digital converter
D/A = digital-to-analog converter

Fig. 1.1 Basic control system block diagram.

The means by which robust regulation is to be accomplished is through the control inputs to the plant [$u(t)$ in Fig. 1.1]. It was discovered long ago[1] that a scheme of feedback wherein the plant output is measured (or sensed) and compared directly with the reference input has many advantages in the effort to design robust controls over systems which do not use such feedback. Much of our effort in later parts of this book will be devoted to illustrating this discovery and demonstrating how to exploit the advantages of feedback. However, the problem of control as discussed thus far is in no way restricted to digital control. For that we must consider the unique features of Fig. 1.1 introduced by the use of a digital device to generate the control action.

We consider first the action of the analog-to-digital (A/D) converter on a signal. This device acts on a physical variable, most commonly an electrical voltage, and converts it into a stream of numbers. In Fig. 1.1, the A/D converter acts on the indicated error signal, $\hat{e}$, and supplies numbers to the digital computer. It is also common for the sensor output, $\hat{y}$, to be sampled and have the error formed in the computer. We need to know the times at which these numbers arrive if we are to analyze the dynamics of this system.

[1] See especially the book by Bode (1945).