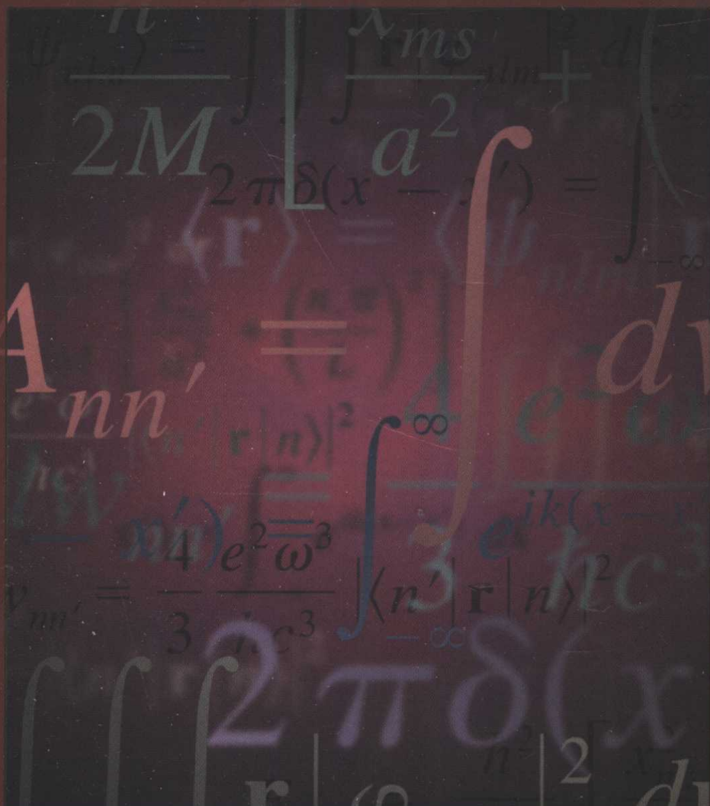
INTRODUCTORY QUANTUM MECHANICS

FOURTH EDITION



Richard L. Liboff

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Preface

Since earlier editions of this text, it remains the case that physics continues to evolve in esoteric and pragmatic directions. In the present edition, a new chapter addressing quantum computing has been added that well represents this theme, as quantum computing is founded on basic elements of quantum mechanics but is thought to represent a new concept for computers. Components of this chapter include: Binary Numbers; Logic Gates; Turing Machine and Complexity Classes; Qubits and Quantum Logic Gates. The chapter concludes with a description of Grover's algorithm, which comes into play in the 'search problem.'

Twenty seven problems are included in this new chapter, many of which, in the spirit of earlier editions, carry solutions. Some of the topics included in these problems are: Classical and quantum logic gates, Boolian relations, Factoring problems and Euclid's algorithm. There are a total of 870 problems in this edition.

A new appendix is included in this edition that describes the Harmonic Oscillator in Spherical Coordinates and a number of inserts are included in the appendix on Physical Constants and Equivalence Relations.

A total of nine new problems has been added to previous chapters that address in part: Properties of the commutator, reduced form of the square of angular momentum, parallel relations for the cubical quantum box, quantum confinement.

A number of corrections have been made throughout the text, mostly due to input from students and teachers throughout the world, whose suggestions I take pleasure in acknowledging.

I take this opportunity to express my deep gratitude to the many individuals throughout the world who have communicated with me regarding typos and suggestions for this text. More locally, in addition to all other colleagues who have contributed to the success of this book, the following individuals have proved to be of invaluable assistance in preparation of this new edition: Toby Berger, Bradley Minch, Rajit Manohar, Eric Sakk, Ian Rippke, Brian E. Moritti, Andy Martwick, and Igor Devetak. My special thanks goes to David Mermin for his expert assistance in the preparation of the new chapter.

It is my pleasure also to declare again my appreciation to the many individuals who have taught from prior editions of this work and the many who have learned from it. I trust that these kind individuals will find this new edition equally rewarding.

Ithaca, 2002
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R.L. Liboff

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PART
I

Elementary Principles and Applications to Problems in One Dimension

CHAPTER 1

Review of Concepts of Classical Mechanics

This is a preparatory chapter in which we review fundamental concepts of classical mechanics important to the development and understanding of quantum mechanics. Hamilton's equations are introduced and the relevance of cyclic coordinates and constants of the motion is noted. In discussing the state of a system, we briefly encounter our first distinction between classical and quantum descriptions. The notions of forbidden domains and turning points relevant to classical motion, which find application in quantum mechanics as well, are also described. The experimental motivation and historical background of quantum mechanics are described in Chapter 2.

1.1 ■ GENERALIZED OR "GOOD" COORDINATES

Our discussion begins with the concept of *generalized* or *good* coordinates.

A bead (idealized to a point particle) constrained to move on a straight rigid wire has *one degree of freedom* (Fig. 1.1). This means that only one variable (or parameter) is needed to uniquely specify the location of the bead in space. For the problem under discussion, the variable may be displacement from an arbitrary but specified origin along the wire.

A particle constrained to move on a flat plane has two degrees of freedom. Two independent variables suffice to uniquely determine the location of the particle in space. With respect to an arbitrary, but specified origin in the plane, such variables

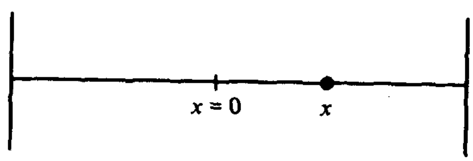

FIGURE 1.1 A bead constrained to move on a straight wire has one degree of freedom.

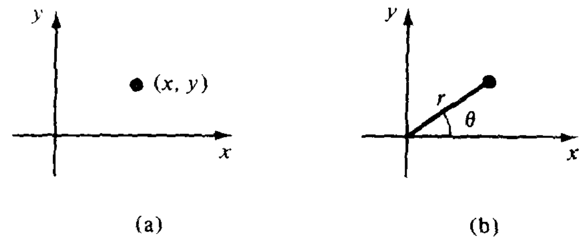

FIGURE 1.2 A particle constrained to move in a plane has two degrees of freedom. Examples of coordinates are (x, y) or (r, θ).

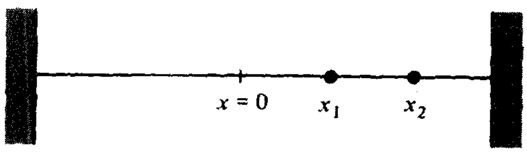

FIGURE 1.3 Two beads on a wire have two degrees of freedom. The coordinates x_1 and x_2 denote displacements of particles 1 and 2, respectively.

might be the Cartesian coordinates (x, y) or the polar coordinates (r, θ) of the particle (Fig. 1.2).

Two beads constrained to move on the same straight rigid wire have two degrees of freedom. A set of appropriate coordinates are the displacements of the individual particles (x_1, x_2) (Fig. 1.3).

A rigid rod (or dumbbell) constrained to move in a plane has three degrees of freedom. Appropriate coordinates are the location of its center (x, y) and the angular displacement of the rod from the horizontal, θ (Fig. 1.4).

Independent coordinates that serve to uniquely determine the orientation and location of a system in physical space are called *generalized* or *canonical* or *good* coordinates. A *system with N generalized coordinates has N degrees of freedom.*

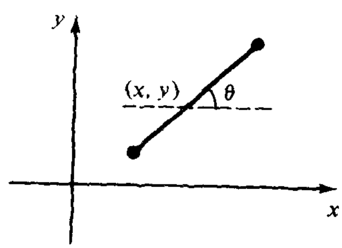

FIGURE 1.4 A rigid dumbbell in a plane has three degrees of freedom. A good set of coordinates are (x, y), the location of the center, and θ, the inclination of the rod with the horizontal.

The orientation and location of a system with, say, three degrees of freedom are not specified until all three generalized coordinates are specified. The fact that *good* coordinates may be specified independently of one another means that, given the values of all but one of the coordinates, the last coordinate remains arbitrary. Having specified (x, y) for a point particle in 3-space, one is still free to choose z independently of the assigned values of x and y.

PROBLEMS

1.1 For each of the following systems, specify the number of degrees of freedom and a set of good coordinates.

- **(a)** A bead constrained to move on a closed circular hoop that is fixed in space.
- **(b)** A bead constrained to move on a helix of constant pitch and constant radius.
- **(c)** A particle on a right circular cylinder.
- **(d)** A pair of scissors on a plane.
- **(e)** A rigid rod in 3-space.
- **(f)** A rigid cross in 3-space.
- **(g)** A linear spring in 3-space.
- **(h)** Any rigid body with one point fixed.
- **(i)** A hydrogen atom.
- **(j)** A lithium atom.
- **(k)** A compound pendulum (two pendulums attached end to end).

1.2 Show that a particle constrained to move on a curve of any shape has one degree of freedom.

Answer

A curve is a one-dimensional locus and may be generated by the parameterized equations

$$x = x(\eta), \qquad y = y(\eta), \qquad z = z(\eta)$$

Once the independent variable η (e.g., length along the curve) is given, x, y, and z are specified.

1.3 Show that a particle constrained to move on a surface of arbitrary shape has two degrees of freedom.

Answer

A surface is a two-dimensional locus. It is generated by the equation

$$u(x, y, z) = 0$$

Any two of the three variables x, y, z determine the third. For instance, we may solve for z in the equation above to obtain the more familiar equation for a surface (height z at the point x, y).

$$z = z(x, y)$$

In this case, x and y may serve as generalized coordinates.