
S. Unnikrishna Pillai

**Array Signal
Processing**



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C.S. Burrus
Consulting Editor



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To

**Professor Dante C. Youla
and the illustrious forefathers
Paṇini and Bādarāyaṇa**

Preface

This book is intended as an introduction to array signal processing, where the principal objectives are to make use of the available multiple sensor information in an efficient manner to detect and possibly estimate the signals and their parameters present in the scene. The advantages of using an array in place of a single receiver have extended its applicability into many fields including radar, sonar, communications, astronomy, seismology and ultrasonics. The primary emphasis here is to focus on the detection problem and the estimation problem from a signal processing viewpoint. Most of the contents are derived from readily available sources in the literature, although a certain amount of original material has been included.

This book can be used both as a graduate textbook and as a reference book for engineers and researchers. The material presented here can be readily understood by readers having a background in basic probability theory and stochastic processes. A preliminary course in detection and estimation theory, though not essential, may make the reading easy. In fact this book can be used in a one semester course following probability theory and stochastic processes. Concepts are explained and illustrated in detail along with important mathematical techniques. Since it is much easier to skip steps than to reconstruct them, complete proofs of all the major results are included and the book is essentially self contained. Problems at the end of each chapter have been chosen to extend the material presented in the book.

I wish to take this opportunity to thank my colleagues at Polytechnic University, especially Professor Dante C. Youla for his criticisms, encouragement and many useful comments through various stages of this manuscript. Working with Professor Youla on research problems has been a rewarding and quite satisfying experience. My students Youngjik Lee and Byung Ho Kwon have gone through the entire manuscript and worked out several computations in chapter 3. Y. Lee also helped me with major parts of the word processing work. Their willingness and enthusiasm to help are gratefully acknowledged.

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Finally a word about the Sanskrit scholars Pāṇini and Bādarāyaṇa mentioned in the dedication. The great grammarian Pāṇini (ca. 500 B.C.), and Bādarāyaṇa (ca. 350 B.C.) to whom the Bṛhadaranyaka Upanishad and Brahma Sūtra are attributed, are my all time heros. Of course, Professor Youla is contemporary and continues to be a source of good ideas.

Brooklyn, 1988

S. Uṇṇikrishṇa Pillai

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Array Signal Processing

Chapter 1

Introduction

1.1 Introduction

Sensor arrays have been in use for several decades in many practical signal processing applications. Such an array consists of a set of sensors that are spatially distributed at known locations with reference to a common reference point. These sensors collect signals from sources in their field of view. Depending on the sensor characteristics and the path of propagation, the source waveforms undergo deterministic and/or random modifications. The sensor outputs are composed of these source components and additive noise such as measurement and thermal noise.

In active sensing situations such as radar and sonar, a known waveform of finite duration is generated which in turn propagates through a medium and is reflected by some target back to the point of origin. The transmitted signal is usually modified both in amplitude and phase by the target characteristics, which by themselves might be changing with time and its position in space. These disturbances give rise to a random return signal. In the passive context the signal received at the array is self-generated by the target, such as propeller or engine noise from submarines in the case of sonar. Once again the signals are random in nature. In addition to these target-generated direct signals, there may also be spurious returns such as clutter in the case of radar. Moreover, signals from a target can undergo reflection, creating multiple returns that are delayed, amplitude-weighted replicas of the direct signal to the array. These, as well as intentional jamming signals, generate coherent interference. For example, in radar, multipath returns give rise to secondary signals that are completely coherent with the original signal. Similar phenomena occur in sonar, where reverberations from seabed of an incoming signal create multiple echos. In a satellite communication scenario, a smart jammer may artfully create such a signal to neutralize the incoming desired signal. In general, the signals may be uncorrelated, partially correlated or

completely coherent with each other. Similarly, the additive noise contained at the sensor outputs may also be uncorrelated or correlated with each other and may have equal/unequal noise variances. Since the natural causes responsible for signals and noise are often unrelated, it is customary to assume that the signals and noise are uncorrelated with each other.

Signals that can be adequately characterized by a single frequency are known as narrowband signals. In contrast to this, signals that occupy a significant frequency band constitute broadband or wideband sources. Physically, the signals may have originated far away from the array, or their point of origin can be quite close to the array. The former case is referred to as the far-field situation, and, by virtue of the distance between the sources and the array, the information carrying wavefronts associated with these far-field sources at the array may be assumed to be planar. In case of narrowband signals, if these plane waves advance through a nondispersive medium that only introduces propagation delays, the output of any other array element can be represented by a time-advanced or time-delayed version of the signal at the reference element.

The practical problems of interest in array signal processing are extracting the desired parameters such as the directions of arrival, power levels and crosscorrelations of the signals present in the scene from the available information including the measured data. Often one may also be specifically interested in the actual signal of one of these sources, and in that case it is necessary to estimate the actual waveform associated with the desired signal by improving the overall reception in an environment having several sources. To achieve this, ideally it should be possible to suppress the undesired signals and enhance the desired signal. The desired signal may correspond to a friendly satellite signal in presence of hostile jammers which may have time varying characteristics. This can happen because of physical motion or deliberate on-off jamming strategies of the opponent. In this case, quick adaptive learning capabilities of the changing scene are required to maintain an acceptable level of the desired signal characteristics at the receiver. At times, the desired signal structure might be partially known, and the objective in that case is to detect its presence in the available noisy data. This situation is often encountered in sonar where the data is analyzed at the receiver to detect the presence of the signature of a specific class of submarine. Though the signal

structure is known, it may still contain unknown parameters such as angle of arrival or random phase.

All these problems fall into one of the two categories: detection or estimation of signals and their parameters from multichannel information. For the signal and noise models discussed above, they form part of our study here.

1.2 Organization of the Book

Chapter 2 introduces the array concept and examines the signal and noise model it is expected to receive in detail. The advantages in using an array are illustrated for a single source scene, and the array structure and its impact on general performance is discussed.

The problem of detecting multiple signals is examined in great detail beginning with traditional techniques that estimate the signal parameters such as their arrival angles and power levels from direction dependent array output power measurements. This is followed by eigenstructure-based high resolution methods that exploit certain structural properties of the array output covariance matrix to achieve the same goal. The modifications of these methods to include coherent source scenes, and related schemes are also examined. The discussion in chapter 2 assumes exact knowledge of the required second order information, i.e., the array output covariance matrix is assumed to be completely known.

Chapter 3 analyzes the performance of the above mentioned techniques, when in the absence of the ensemble averages, these covariances are estimated directly from the available noisy data. Beginning with the maximum likelihood estimate of the covariance matrix in the case of zero mean, complex circular Gaussian data vectors, a detailed asymptotic analysis of the eigenvalues and a certain set of eigenvectors of this estimated covariance matrix is presented. This in turn is used to derive the mean and variance and in some cases the probability distribution function of the angle-of-arrival estimators discussed in chapter 2. In addition, using this analysis, asymptotic results for resolving two closely spaced sources are derived in terms of signal-to-noise ratio, number of array elements and the signal correlation. Comparisons are also presented to illustrate their performances relative to each other.

The signal acquisition problem discussed in chapter 4 first formulates the Wiener solution, which is optimum under the minimization of the mean square error criterion. This is followed by other optimality criteria such as maximization of signal-to-noise ratio and maximum likelihood performance measure. The realization of the Wiener solution through adaptive recursive procedures is outlined, and analysis of the least mean square technique is presented with details regarding its convergence properties. The chapter concludes with direct implementation techniques for realizing the Wiener solution.

1.3 Notations and Preliminaries

Throughout this book scalar quantities are denoted by regular lower or upper case letters. Lower and upper case bold type faces are used for vectors and matrices respectively. Thus a (or A), \mathbf{a} and \mathbf{A} stand for scalar, vector and matrix in that order. Similarly \mathbf{A}^* , \mathbf{A}^T , \mathbf{A}^\dagger , $\text{tr}(\mathbf{A})$ and $\det(\mathbf{A}) = |\mathbf{A}|$ represent the complex conjugate, transpose, complex conjugate transpose, trace and determinant of \mathbf{A} respectively. The symbol $\text{diag}[\lambda_1, \lambda_2, \dots, \lambda_M]$ is used sometimes to represent a diagonal matrix with diagonal entries $\lambda_1, \lambda_2, \dots, \lambda_M$. For two square matrices \mathbf{A} , \mathbf{B} of same size, $(\mathbf{AB})^\dagger = \mathbf{B}^\dagger \mathbf{A}^\dagger$, $\text{tr}(\mathbf{AB}) = \text{tr}(\mathbf{BA})$ and $|\mathbf{AB}| = |\mathbf{A}| |\mathbf{B}|$.

A square matrix \mathbf{A} of size $M \times M$ is said to be hermitian if $\mathbf{A} = \mathbf{A}^\dagger$, i.e., $a_{ij} = a_{ji}^*$ for all i, j , where a_{ij} represents the $(i, j)^{\text{th}}$ element of \mathbf{A} . It is said to be nonnegative definite if for any $M \times 1$ vector \mathbf{x} , $\mathbf{x}^\dagger \mathbf{A} \mathbf{x} \geq 0$. When strict inequality holds, i.e., $\mathbf{x}^\dagger \mathbf{A} \mathbf{x} > 0$ for $\mathbf{x} \neq 0$, \mathbf{A} is said to be positive definite.

Let λ_i denote an eigenvalue of \mathbf{A} . Then, there exists an eigenvector $\mathbf{u}_i \neq 0$, such that $\mathbf{A} \mathbf{u}_i = \lambda_i \mathbf{u}_i$. The eigenvectors are in general complex and for positive definite matrices, they can be made unique by normalization together with a constraint of the form $u_{ii} \geq 0$. The eigenvalues of a hermitian matrix are real. For, $\mathbf{u}_i^\dagger \mathbf{A} \mathbf{u}_i = \lambda_i \mathbf{u}_i^\dagger \mathbf{u}_i = \lambda_i$. Also $\mathbf{u}_i^\dagger \mathbf{A} \mathbf{u}_i = (\mathbf{A} \mathbf{u}_i)^\dagger \mathbf{u}_i = \lambda_i^* \mathbf{u}_i^\dagger \mathbf{u}_i = \lambda_i^*$. Thus $\lambda_i^* = \lambda_i$ or λ_i is real. If \mathbf{A} is also positive definite then it follows that its eigenvalues are all positive. Moreover, eigenvectors associated with distinct eigenvalues of a hermitian matrix are orthogonal. This follows by noticing that, if λ_i, λ_j and $\mathbf{u}_i, \mathbf{u}_j$ are any two such pairs, then $\mathbf{u}_i^\dagger \mathbf{A} \mathbf{u}_j = \mathbf{u}_i^\dagger \lambda_j \mathbf{u}_j = \lambda_j \mathbf{u}_i^\dagger \mathbf{u}_j$. However, we also have $\mathbf{u}_i^\dagger \mathbf{A} \mathbf{u}_j = (\mathbf{A} \mathbf{u}_i)^\dagger \mathbf{u}_j = (\lambda_i \mathbf{u}_i)^\dagger \mathbf{u}_j =$

$\lambda_i \mathbf{u}_i^\dagger \mathbf{u}_j$. Thus, $\lambda_j \mathbf{u}_i^\dagger \mathbf{u}_j = \lambda_i \mathbf{u}_i^\dagger \mathbf{u}_j$ or equivalently $\mathbf{u}_i^\dagger \mathbf{u}_j = 0$ provided $\lambda_i \neq \lambda_j$. Further, this implies that if an eigenvalue repeats (say L times), then the associated eigenvectors span an L -dimensional subspace that is orthogonal to the one spanned by the remaining set of eigenvectors. As a result, it is always possible to choose a new set of L orthonormal vectors from the above L -dimensional subspace to act as an eigenvector set for the above repeating eigenvalue. Thus, for an $M \times M$ hermitian matrix \mathbf{A} , if $\lambda_1, \lambda_2, \dots, \lambda_M$ and $\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_M$ represent its eigenvalues and an orthonormal set of eigenvectors, then $\mathbf{A} \mathbf{u}_i = \mathbf{u}_i \lambda_i$, $i = 1, 2, \dots, M$, or in a compact form $\mathbf{A} \mathbf{U} = \mathbf{U} \mathbf{\Lambda}$, where $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_M]$ and $\mathbf{\Lambda} = \text{diag}[\lambda_1, \lambda_2, \dots, \lambda_M]$. Clearly, $\mathbf{U} \mathbf{U}^\dagger = \mathbf{U}^\dagger \mathbf{U} = \mathbf{I}$, i.e., \mathbf{U} is a unitary matrix, and consequently $\mathbf{A} \mathbf{U} = \mathbf{U} \mathbf{\Lambda}$ gives $\mathbf{A} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^\dagger$. Thus, any hermitian matrix can be diagonalized by a unitary matrix whose columns represent a complete set of its normalized eigenvectors. Moreover $|\mathbf{A}| = |\mathbf{U}| |\mathbf{\Lambda}| |\mathbf{U}^\dagger| = \lambda_1 \lambda_2 \dots \lambda_M$ and $\text{tr}(\mathbf{A}) = \lambda_1 + \lambda_2 + \dots + \lambda_M$.

The number of linearly independent rows (or columns) of a matrix \mathbf{A} represents its rank $\rho(\mathbf{A})$. In case of a square matrix, its rank coincides with the total number of nonzero eigenvalues (including repetitions). For any two matrices \mathbf{A} and \mathbf{B} of dimensions $m \times n$ and $n \times r$, the rank of their product satisfies Sylvester's inequality [1] given by $\rho(\mathbf{A}) + \rho(\mathbf{B}) - n \leq \rho(\mathbf{AB}) \leq \min[\rho(\mathbf{A}), \rho(\mathbf{B})]$. A matrix of full rank is said to be nonsingular. Any nonnegative definite (hermitian) matrix may be factored into the form $\mathbf{A} = \mathbf{C}^2$ where \mathbf{C} is also hermitian and $\rho(\mathbf{A}) = \rho(\mathbf{C})$. A good knowledge of these results is essential to understanding the rest of this book.

Reference

- [1] F. R. Gantmacher, *The Theory of Matrices*. New York: Chelsea, 1977.

Chapter 2

Detection of Multiple Signals

2.1 Signals and Noise

In this chapter we will discuss the problem of detecting multiple signals using information from multiple sensors. To understand the advantages of using a sensor array over a single element in various aspects of detection and estimation it is necessary to understand the nature of signals and noise the array is desired to receive.

In active sensing situations such as radar and sonar, a known waveform of finite duration is generated which in turn propagates through a medium and is reflected by some target back to the point of origin. The transmitted signal is usually modified both in amplitude and phase by the target characteristics, which by themselves might be changing with time and its position in space. These disturbances give rise to a random return signal. In the passive context the signal received at the array is self-generated by the target, such as propeller or engine noise from submarines in the case of sonar. Once again the signals are random in nature. In addition to these target-generated direct signals, there may also be spurious returns such as clutter in the case of radar and reverberations from the ocean surface/bed or sea layers in the case of sonar. Moreover, signals from a target can undergo reflection, creating multipath returns that are delayed, amplitude-weighted replicas of the direct signal to the array. These as well as intentional jamming signals can generate coherent interference. In all these cases the signals that arrive at the array can be regarded as random, and at times the physical phenomena responsible for the randomness in the signal make it plausible to assume that the signals are Gaussian (normal) random processes.

Likewise thermal sensor noise and ambient noise are also random in nature. These additive components at the sensor outputs usually represent the totality of several small independent and identical sources, and application of the central limit theorem permits one to model the resulting noise as a Gaussian and (usually) stationary