SCHAUM'S OUTLINE OF

THEORY AND PROBLEMS

of

STRENGTH of MATERIALS

Second Edition

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BY

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Preface

This second edition of *Theory and Problems of Strength of Materials* adheres to the basic plan of the first edition, but with a considerable broadening of scope. As in the earlier edition, the contents are divided into chapters covering duly-recognized areas of theory and study. Each chapter begins with a summary of the pertinent definitions, principles, and theorems, followed by graded sets of solved and supplementary problems. Derivations of formulas and proofs of theorems are included among the solved problems. The problems have been chosen and solutions arranged so that the principles are clearly established. They serve to illustrate and amplify the theory, provide the repetition of basic principles so vital to effective teaching, and bring into sharp focus those fine points which are essential to a complete understanding.

Since publication of the first edition, course offerings in strength of materials have become more sophisticated and frequently tend to include topics in plastic analysis and design, treatments of shear centers, curved beams, and use of singularity functions to describe beam behavior. Also, it has become rather common to introduce the student to strain energy methods of analysis and the elements of the theory of elasticity. The present edition includes solved problems in all these areas, as well as in others not previously represented.

The author is deeply indebted to his wife, Verna B. Nash, and his children, Rebecca and Phillip, for their patience and understanding during the preparation of the manuscript.

WILLIAM A. NASH

Amherst, Massachusetts
March 1972

Preface to SI Edition

The SI edition of Strength of Materials by Nash is an adaptation, not a rewrite, of the second edition. All of the original examples have been retained and converted where necessary. Terminology, whether affected by the SI conversion or not, has been revised, and previously tabulated data given in the questions.

In practice it is difficult to obtain accurate material properties, and strength of material analyses are approximate. Therefore typical material properties are quoted, rounded-off so as not to overburden the text with unwieldy figures. Answers to problems are generally given to three significant figures, unless additional accuracy is required. Students should be aware of the inherent inaccuracies embodied in the calculations, and not be deluded by the accuracy of electronic calculators.

In the worked examples the arithmetic factors necessary to produce expressions having consistent units have been shown explicitly. A formalized method for checking units, such as "Unit Cancellation," should be considered mandatory in the SI system, as errors of orders of magnitude are easily made.

I would like to express my gratitude to the staff of McGraw-Hill (UK) for their helpful co-operation.

C. E. N. STURGESS

Birmingham
January 1977

Contents

Chapter 1

Tension and Compression

INTERNAL EFFECTS OF FORCES

In this book we shall be concerned with what might be called the *internal effects* of forces acting on a body. The bodies themselves will no longer be considered to be perfectly rigid as was assumed in statics; instead, the calculation of the deformations of various bodies under a variety of loads will be one of our primary concerns in the study of strength of materials.

AXIALLY LOADED BAR

The simplest case to consider at the start is that of an initially straight metal bar of constant cross-section, loaded at its ends by a pair of oppositely directed collinear forces coinciding with the longitudinal axis of the bar and acting through the centroid of each cross-section. For static equilibrium the magnitudes of the forces must be equal. If the forces are directed away from the bar, the bar is said to be in *tension*; if they are directed toward the bar, a state of *compression* exists. These two conditions are illustrated in Fig. 1-1.

Under the action of this pair of applied forces, internal resisting forces are set up within the bar and their characteristics may be studied by imagining a plane to be passed through the bar anywhere along its length and oriented perpendicular to the longitudinal axis of the bar. Such a plane is designated as a–a in Fig. 1-2(a). For reasons to be discussed later, this plane should not be "too close" to either end of the bar. If for purposes of analysis the portion of the bar to the right of this plane is considered to be removed, as in Fig. 1-2(b), then it must be replaced by whatever effect it exerts upon the left portion. By this technique of introducing a cutting plane, the originally internal forces now become external with respect to the remaining portion of the body. For equilibrium of the portion to the left this "effect" must be a horizontal force of magnitude P. However, this force P acting normal to the cross-section a–a is actually the resultant of distributed forces acting over this cross-section in a direction normal to it.

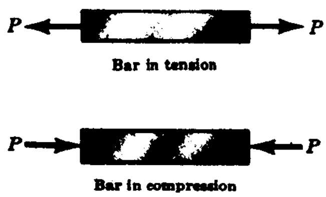

Fig. 1-1

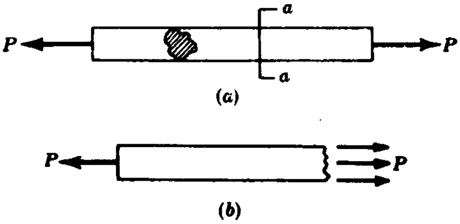

Fig. 1-2

DISTRIBUTION OF RESISTING FORCES

At this point it is necessary to make some assumption regarding the manner of variation of these distributed forces, and since the applied force P acts through the centroid it is commonly assumed that they are uniform across the cross-section. Such a distribution is probably never realized exactly because of the random orientation of the crystalline grains of which the bar is composed. The exact value of the force acting on some very small element of area of the cross-section is a function of the nature and orientation of the crystalline structure at that point. However, over the entire cross-section the variation is described with reasonable engineering accuracy by the assumption of a uniform distribution.

NORMAL STRESS

Instead of speaking of the internal force acting on some small element of area, it is better for comparative purposes to treat the normal force acting over a *unit* area of the cross-section. The intensity of normal force per unit area is termed the normal *stress* and is expressed in units of force per unit area, e.g. N m^{-2} ("Pascal"). The phrase *total stress* is sometimes used to denote the resultant axial force. If the forces applied to the ends of the bar are such that the bar is in tension, then *tensile stresses* are set up in the bar; if the bar is in compression we have *compressive stresses*. It is essential that the line of action of the applied end forces pass through the centroid of each cross-section of the bar.

TEST SPECIMENS

The axial loading shown in Fig. 1-2(*a*) occurs frequently in structural and machine design problems. To simulate this loading in the laboratory, a test specimen is held in the grips of either an electrically driven gear-type testing machine or a hydraulic machine. Both of these machines are commonly used in materials testing laboratories for applying axial tension.

In an effort to standardize materials testing techniques, various national bodies have issued specifications that are in common use throughout various countries. More than a score of different type specimens are prescribed for various metallic and nonmetallic materials for both axial tension and axial compression tests. For the present only two of these will be mentioned here, one for metal plates thicker than 5 mm and appearing as in Fig. 1-3, the other for metals over 37.5 mm thick and having the appearance shown in Fig. 1-4. The dimensions shown are those specified by the American Society for Testing Materials (ASTM), but the ends of the test specimens may be of any shape to fit the grips of the testing machine applying the axial load. As may be seen from these figures, the central portion of the specimen is somewhat smaller than the end regions so that failure will not take place in the gripped portion. The rounded fillets shown are provided so that no so-called stress concentrations will arise at the transition between the two lateral dimensions. The standard gage length over which elongations are measured is 200 mm for the specimen shown in Fig. 1-3 and 50 mm for that shown in Fig. 1-4.

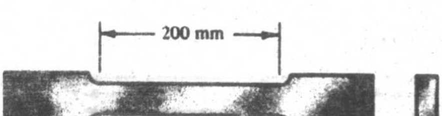

Fig. 1-3

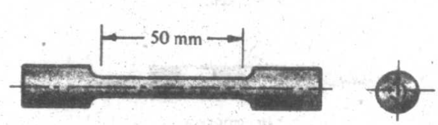

Fig. 1-4

The elongations are measured by either mechanical or optical extensometers or by cementing an electric resistance-type strain gage to the surface of the material. This resistance strain gage consists of a number of very fine wires oriented in the axial direction of the bar. As the bar elongates, the electrical resistance of the wires changes and this change of resistance is detected on a Wheatstone bridge and interpreted as elongation.

NORMAL STRAIN

Let us suppose that one of these tension specimens has been placed in a tension–compression testing machine and tensile forces gradually applied to the ends. The elongation over the gage length may be measured as indicated above for any predetermined increments of the axial load. From these values the elongation per unit length, which is termed *normal strain* and denoted by ϵ, may be found by dividing the total elongation Δ by the gage length L, i.e. $\epsilon = \Delta/L$, and consequently is dimensionless.

STRESS–STRAIN CURVE

As the axial load is gradually increased in increments, the total elongation over the gage length is measured at each increment of load and this is continued until fracture of the specimen takes place. Knowing the original cross-sectional area of the test specimen the *normal stress*, denoted by σ, may be obtained for any value of the axial load merely by the use of the relation

$$\sigma = \frac{P}{A}$$

where P denotes the axial load in newtons, and A the original cross-sectional area. Having obtained numerous pairs of values of normal stress σ and normal strain ϵ, the experimental data may be plotted with these quantities considered as ordinate and abscissa respectively. This is the *stress–strain curve* or *diagram* of the material for this type of loading. Stress–strain diagrams assume widely differing forms for various materials. Figure 1-5 is the stress–strain diagram for a medium-carbon structural steel, Fig. 1-6 is for any alloy steel, and Fig. 1-7 is for hard steels and certain nonferrous alloys. For nonferrous alloys and cast iron the diagram has the form indicated in Fig. 1-8, while for rubber the plot of Fig. 1-9 is typical.

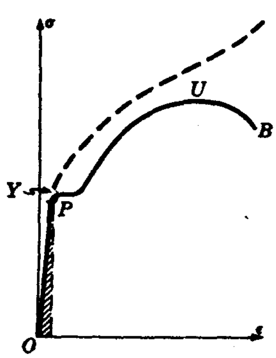

Fig. 1-5

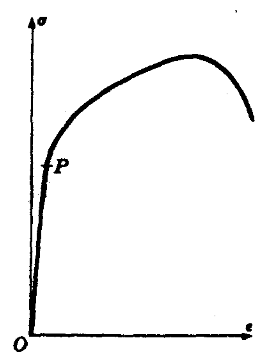

Fig. 1-6

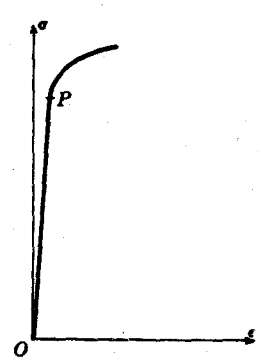

Fig. 1-7

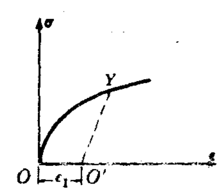

Fig. 1-8

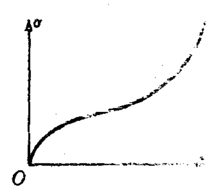

Fig. 1-9

DUCTILE AND BRITTLE MATERIALS

Metallic engineering materials are commonly classed as either *ductile* or *brittle* materials. A *ductile material* is one having a relatively large tensile strain up to the point of rupture (for example, structural steel or aluminum) whereas a *brittle material* has a relatively small strain up to this same point. An arbitrary strain of 0.05 is frequently taken as the dividing line between these two classes of materials. Cast iron and concrete are examples of brittle materials.

HOOKE'S LAW

For any material having a stress–strain curve of the form shown in Figs. 1-5, 1-6, or 1-7, it is evident that the relation between stress and strain is linear for comparatively small values of the strain. This linear relation between elongation and the axial force causing it (since these quantities respectively differ from the strain or the stress only by a constant factor) was first noticed by Sir Robert Hooke in 1678 and is called *Hooke's law*. To describe this initial linear range of action of the material we may consequently write

$$\sigma = E\epsilon$$

where E denotes the slope of the straight-line portion OP of each of the curves in Figs. 1-5, 1-6, and 1-7.

MODULUS OF ELASTICITY

The quantity E, i.e. the ratio of the unit stress to the unit strain, is the *modulus of elasticity* of the material in tension, or, as it is often called, *Young's modulus*. Since the unit strain ϵ is a pure number (being a ratio of two lengths) it is evident that E has the same units as does the stress, for example. For many common engineering materials the modulus of elasticity in compression is very nearly equal to that found in tension. *It is to be carefully noted that the behavior of materials under load as discussed in this book is restricted (unless otherwise stated) to the linear region of the stress–strain curve.*

The values of E used in the text are approximate to avoid unnecessary computation, although the quoted figures are within 5 percent of actual values. For particular materials E can be found from handbooks or, more precisely, from manufacturers' catalogs; in all real situations every effort should be made to ascertain accurate material data.

MECHANICAL PROPERTIES OF MATERIALS

The stress–strain curve shown in Fig. 1-5 may be used to characterize several strength characteristics of the material. They are:

PROPORTIONAL LIMIT

The ordinate of the point P is known as the *proportional limit*, i.e. the maximum stress that may be developed during a simple tension test such that the stress is a linear function of strain. For a material having the stress–strain curve shown in Fig. 1-8, there is no proportional limit.

ELASTIC LIMIT

The ordinate of a point almost coincident with P is known as the *elastic limit*, i.e. the maximum stress that may be developed during a simple tension test such that there is no permanent or residual deformation when the load is entirely removed. For many materials the numerical values of the elastic limit and the proportional limit are almost identical and the terms are sometimes used synonymously. In those cases where the distinction between the two values is evident the elastic limit is almost always greater than the proportional limit.

ELASTIC AND PLASTIC RANGES

That region of the stress–strain curve extending from the origin to the proportional limit is called the *elastic range*; that region of the stress–strain curve extending from the proportional limit to the point of rupture is called the *plastic range.*

YIELD POINT

The ordinate of the point Y, denoted by σ_{yp} at which there is an increase in strain with no increase in stress is known as the *yield point* of the material. After loading has progressed to the point Y, yielding is said to take place. Some materials exhibit two points on the stress–strain curve at which there is an increase of strain without an increase of stress. These are called *upper* and *lower yield points.*

ULTIMATE STRENGTH OR TENSILE STRENGTH

The ordinate of the point U, the maximum ordinate to the curve, is known either as the *ultimate strength* or the *tensile strength* of the material.

BREAKING STRENGTH

The ordinate of the point B is called the *breaking strength* of the material.

MODULUS OF RESILIENCE

The work done on a unit volume of material, as a simple tensile force is gradually increased from zero to such a value that the proportional limit of the material is reached, is defined as the *modulus of resilience.* This may be calculated as the area under the stress–strain curve from the origin up to the proportional limit and is represented as the shaded area in Fig. 1-5. The units of this quantity are N m m^{-3}. Thus, resilience of a material is its ability to absorb energy in the elastic range.

MODULUS OF TOUGHNESS

The work done on a unit volume of material as a simple tensile force is gradually increased from zero to the value causing rupture is defined as the *modulus of toughness.* This may be calculated as the entire area under the stress–strain curve from the origin to rupture. Toughness of a material is its ability to absorb energy in the plastic range of the material.

PERCENTAGE REDUCTION IN AREA

The decrease in cross-sectional area from the original area upon fracture divided by the *original* area and multiplied by 100 is termed *percentage reduction in area.* It is to be noted that when tensile forces act upon a bar, the cross-sectional area decreases, but calculations for the normal stress are usually made upon the basis of the original area. This is the case for the curve shown in Fig. 1-5. As the strains become increasingly larger it is more important to consider the instantaneous values of the cross-sectional area (which are decreasing), and if this is done the *true* stress–strain curve is obtained. Such a curve has the appearance shown by the dashed line in Fig. 1-5.

PERCENTAGE ELONGATION

The increase in length (of the gage length) after fracture divided by the initial length and multiplied by 100 is the *percentage elongation.* Both the percentage reduction in area and the percentage elongation are considered to be measures of the *ductility* of a material.

WORKING STRESS

The above-mentioned strength characteristics may be used to select a so-called *working stress.* Throughout this book all working stresses will be within the elastic range of the material. Frequently such a stress is determined merely by dividing either the stress at yield or the ultimate stress by a number termed the *safety factor*. Selection of the safety factor is based upon the designer's judgment and experience. Specific safety factors are sometimes specified in building codes. See Problems 1.4, 1.12, 1.13.

STRAIN HARDENING

If a ductile material can be stressed considerably beyond the yield point without failure, it is said to *strain harden*. This is true of many structural metals.

The nonlinear stress–strain curve of a brittle material, shown in Fig. 1-8, characterizes several other strength measures that cannot be introduced if the stress–strain curve has a linear region. They are:

YIELD STRENGTH

The ordinate to the stress–strain curve such that the material has a predetermined permanent deformation or "set" when the load is removed is called the *yield strength* of the material. The permanent set is often taken to be a strain of either 0.002 or 0.0035. These values are of course arbitrary. In Fig. 1-8 a set ϵ_1 is denoted on the strain axis and the line $O'Y$ is drawn parallel to the initial tangent to the curve. The ordinate of Y represents the yield strength of the material, sometimes called the *proof stress*.

TANGENT MODULUS

The rate of change of stress with respect to strain is known as the *tangent modulus* of the material. It is essentially an instantaneous modulus given by $E_t = d\sigma/d\epsilon$.

There are other characteristics of a material that are useful in design considerations. They are:

COEFFICIENT OF LINEAR EXPANSION

This is defined as the change of length per unit length of a straight bar subject to a temperature change of one degree kelvin (K). The value of this coefficient is independent of the unit of length but does depend upon the temperature scale used. Usually we will consider the Kelvin scale, in which case the coefficient denoted by α is given for steel, for instance, as 12×10^{-6}, K^{-1}. Temperature changes in a structure give rise to internal stresses just as do applied loads. See Problem 1.7.

POISSON'S RATIO

When a bar is subject to a simple tensile loading there is an increase in length of the bar in the direction of the load, but a decrease in the lateral dimensions perpendicular to the load. The ratio of the strain in the lateral direction to that in the axial direction is defined as *Poisson's ratio*. It is denoted in this book by the Greek letter μ. For most metals it lies in the range 0.25 to 0.35. See Problems 1.16–1.20.

GENERAL FORM OF HOOKE'S LAW

The simple form of Hooke's law has been given for axial tension when the loading is entirely along one straight line, i.e. uniaxial. Only the deformation in the direction of the load was considered and it was given by

$$\epsilon = \frac{\sigma}{E}$$

In the more general case an element of material is subject to three mutually perpendicular normal stresses σ_x, σ_y, σ_z, which are accompanied by the strains ϵ_x, ϵ_y, ϵ_z respectively. By superposing the strain components arising from lateral contraction due to Poisson's effect upon the direct strains we obtain the general statement of Hooke's law:

$$\epsilon_x = \frac{1}{E}[\sigma_x - \mu(\sigma_y + \sigma_z)] \qquad \epsilon_y = \frac{1}{E}[\sigma_y - \mu(\sigma_x + \sigma_z)] \qquad \epsilon_z = \frac{1}{E}[\sigma_z - \mu(\sigma_x + \sigma_y)]$$

See Problems 1.17 and 1.20.

ELASTIC VERSUS PLASTIC ANALYSIS

Stresses and deformations in the plastic range of action of a material are frequently permitted in certain structures. Some building codes allow particular structural members to undergo plastic deformation, and certain components of aircraft and missile structures are deliberately designed to act in the plastic range so as to achieve weight savings. Furthermore, many metal-forming processes involve plastic action of the material. For small plastic strains of low- and medium-carbon structural steels the stress–strain curve of Fig. 1-5 is usually idealized by two straight lines, one with a slope of E, representing the elastic range, the other with zero slope representing the plastic range. This plot, shown in Fig. 1-10, represents a so-called *elastic, perfectly-plastic material*. It takes no account of still larger plastic strains occurring in the strain hardening region shown as the right portion of the stress–strain curve of Fig. 1-5. See Problem 1.21.

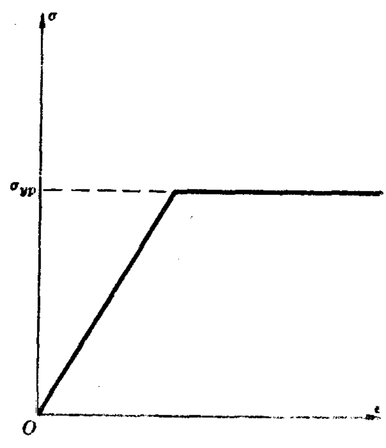

Fig. 1-10

CLASSIFICATION OF MATERIALS

This entire discussion has been based upon the assumptions that two characteristics prevail in the material. They are that we have a

HOMOGENEOUS MATERIAL, one with the same elastic properties (E, μ) at all points in the body, and an

ISOTROPIC MATERIAL, one having the same elastic properties in all directions at any one point of the body. Not all materials are isotropic. If a material does not possess any kind of elastic symmetry it is called *anisotropic*, or sometimes *aeolotropic*. Instead of having two independent elastic constants (E, μ) as an isotropic material does, such a substance has 21 elastic constants. If the material has three mutually perpendicular planes of elastic symmetry it is said to be *orthotropic*. The number of independent constants is 9 in this case. This book considers only the analysis of isotropic materials.

DYNAMIC EFFECTS

In determination of mechanical properties of a material through a tension or compression test, the rate at which loading is applied sometimes has significant influence upon the results. In general, ductile materials exhibit the greatest sensitivity to variations in loading rate, whereas the effect of testing speed on brittle materials, such as cast iron, has been found to be negligible. In the case of mild steel, a ductile material, it has been found that the yield point may be increased as much as 170 percent by extremely rapid application of axial force. It is of interest to note, however, that for this case the total elongation remains unchanged from that found for slower loadings.

Solved Problems

1.1. Determine the total elongation of an initially straight bar of length L, cross-sectional area A, and modulus of elasticity E if a tensile load P acts on the ends of the bar.

The unit stress in the direction of the force P is merely the load divided by the cross-sectional area, i.e. $\sigma = P/A$. Also the unit strain ϵ is given by the total elongation Δ divided by the original length, i.e. $\epsilon = \Delta/L$. By definition the modulus of elasticity E is the ratio of σ to ϵ, i.e.

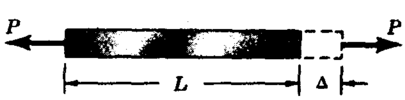

Fig. 1-11

$$E = \frac{\sigma}{\epsilon} = \frac{P/A}{\Delta/L} = \frac{PL}{A\Delta} \quad \text{or} \quad \Delta = \frac{PL}{AE}$$

Note that Δ has the units of length.

1.2. A surveyor's steel tape 30 m long has a cross-section of 6 mm by 1 mm. Determine the elongation when the entire tape is stretched and held taut by a force of 50 N. The modulus of elasticity is 200 GN m^{-2}.

$$\text{elongation } \Delta = \frac{PL}{AE} = \frac{(50)(30 \times 10^3)}{(6)(1)(200 \times 10^9 \times 10^{-6})} = 1.25 \text{ mm}$$

1.3. A steel bar of cross-section 500 mm^2 is acted upon by the forces shown in Fig. 1-12(*a*). Determine the total elongation of the bar. For steel, $E = 200$ GN m^{-2}

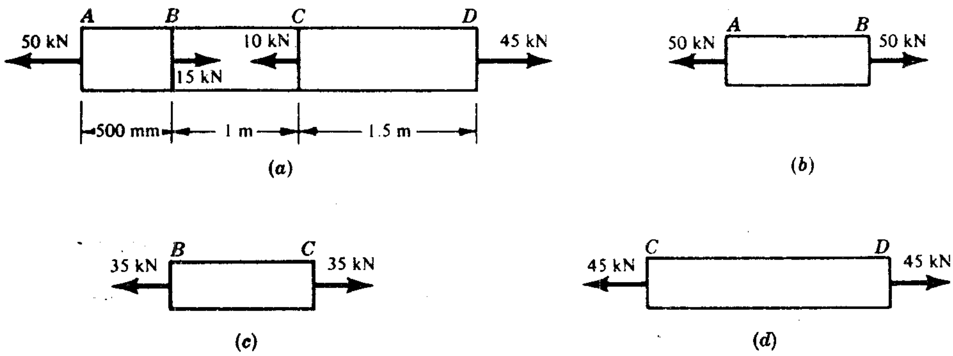

Fig. 1-12

The entire bar is in equilibrium, hence all portions of it are also in equilibrium. The portion of the bar between *A* and *B* has a resultant force of 50 kN acting over every cross-section, hence a free-body diagram of this 500 mm length appears as in Fig. 1-12(*b*) above. The force at the right end of this segment must be 50 kN to maintain equilibrium with the applied force at the left end. The elongation of this portion is

$$\Delta_1 = \frac{PL}{AE} = \frac{(50 \times 10^3)(500)}{(500)(200 \times 10^9 \times 10^{-6})} = 0.25 \text{ mm}$$

The force acting on the segment between *B* and *C* is found by considering the algebraic sum of the forces to the left of a section between *B* and *C*. This indicates that a resultant force of 35 kN acts to the left, i.e. the section has a tensile force acting upon it. This same result could of course have been obtained by considering the algebraic sum of the forces to the right of this section. Consequently, the free-body diagram of the segment *BC* appears as in Fig. 1-12(*c*). The elongation of this portion is

$$\Delta_2 = \frac{(35 \times 10^3)(1 \times 10^3)}{(500)(200 \times 10^9 \times 10^{-6})} = 0.35 \text{ mm}$$

Similarly, the force acting over any cross-section between *C* and *D* must be 45 kN to maintain equilibrium with the applied load at *D*. The free-body diagram of the segment *CD* appears as in Fig. 1-12(*d*). The elongation of this portion is

$$\Delta_3 = \frac{(45 \times 10^3)(1.5 \times 10^3)}{(500)(200 \times 10^9 \times 10^{-6})} = 0.675 \text{ mm}$$

The total elongation is consequently

$$\Delta = 0.25 + 0.35 + 0.675 = 1.275 \text{ mm}$$

1.4. The Howe truss shown in Fig. 1-13(*a*) supports the single load of 480 kN. If the working stress of the material in tension is taken to be 200 MPa, determine the required cross-sectional area of bars *DE* and *AC*. Find the elongation of bar *DE* over its 6 m length. Assume that the limiting value of the working stress in tension is the only factor to be considered in determining the required area. Take the modulus of elasticity of the bar to be 200 GN m^{-2}