

Mapping from Aerial Photographs

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CROSBY LOCKWOOD STAPLES

GRANADA PUBLISHING

London Toronto Sydney New York

Published by Granada Publishing Limited
in Crosby Lockwood Staples 1979

Granada Publishing Limited
Frogmore, St Albans, Herts AL2 2NF
and
3 Upper James Street, London W1R 4BP
1221 Avenue of the Americas, New York, NY 10020, USA
117 York Street, Sydney, NSW 2000, Australia
100 Skyway Avenue, Toronto, Ontario, Canada M9W 3A6
110 Northpark Centre, 2193 Johannesburg, South Africa
CML Centre, Queen & Wyndham, Auckland 1, New Zealand

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ISBN 0 258 97035 9

Filmset in 'Monophoto' Times 10 on 11 pt by
Richard Clay (The Chaucer Press), Ltd, Bungay, Suffolk
and printed in Great Britain by
Fletcher & Son Ltd, Norwich

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Preface

A more explicit title for this book would be 'An Introduction to the Main Theoretical Bases of Producing Topographical Maps from Aerial Photographs'. The text therefore is not concerned with providing technical instructions for the construction of maps but rather with the mathematical concepts on which their production is based. It has been written as an introduction to the main theoretical elements of photogrammetry. The restriction to the use of aerial photographs for the purposes of making topographical maps was introduced because it provided me with an obvious logical framework on which to develop the subject matter I had in mind. Of course, the mapping of other surfaces from other sources of photography is identical in theory and can often prove to be more simple in practice than the main application of photogrammetry discussed in this text.

The first two chapters are concerned with the nature of the basic data and examine the air survey camera and the procurement of suitable aerial photography. Having obtained the necessary photographs, the various ways in which map detail can be derived from them are examined systematically: from simple graphical constructions to sophisticated mathematical techniques. In this way, I have endeavoured to provide a text that will enable the reader to study the main topics of photogrammetric mapping in a manner that develops in a logical and systematic way.

A set of well-designed practical exercises are a highly desirable adjunct to any theoretical study of a subject of this nature and originally it was my intention to provide these in this volume. Unfortunately, it soon became apparent that their inclusion would expand the text to an unacceptable degree and also change the character of the book too much. However, it is realised that most students have a particular interest in the practical applications of photogrammetry. A series of exercises designed to illustrate various basic concepts can therefore do much to facilitate their appreciation and, at the same time, introduce the student to some of the practical aspects of the subject.

The text does not require any great familiarity with photogrammetric instruments and a minimum knowledge of these is assumed throughout. From this point of view, chapter eight, which is concerned with the design of analogue plotting instruments, was particularly difficult to write. A single chapter cannot hope to provide anything like a comprehensive treatment of the topic and it does not attempt to provide this. The intention has been to provide only a reasonably representative sample of instrument design features. Some detailed knowledge of photogrammetric equipment is most certainly necessary to any comprehensive study. Such information is however quite readily available through the medium of the excellent technical leaflets produced by many instrument manufacturers. A collection of these would make a valuable supplement to any text.

It may be noticed that many of the references given in the bibliography are to technical papers given at recent national and international conferences. By making use of these the reader should be able to make himself familiar with the latest developments in photogrammetry.

It will also be noticed that most of the diagrams in this book are simple two dimen-

sional line diagrams. These I hope will prove to be clear and intelligible but they should have an additional advantage: when one is trying to understand a concept or proof of the type most frequently encountered in this subject one usually resorts to a diagram; if this is highly complicated then one cannot readily remember it or reproduce it and so one's understanding of the concept is often thwarted. Consequently, an attempt has been made here to devise and use diagrams of the simplest nature only. Where required, two simple diagrams have been used instead of one of more complexity. In this connection I would like to acknowledge the great help of Mr P. Sorrell who drew all the diagrams for me and made many valuable suggestions as to their nature.

Rather than ask one unfortunate person to read the whole of my manuscript I asked a number of friends and colleagues to read various chapters. I am therefore most pleased to record my thanks to the following persons for their advice and criticisms; Dr A. L. Allen, Dr P. Dale, Mr G. B. Das, Mr B. Canzini, Mr J. R. Hollwey, Miss G. Sears and Mr S. Walker. Miss Sears and Mr Walker also gave me great assistance in the reading of the page proofs and so I am doubly grateful to them.

Finally I would also like to acknowledge the help given to me by Mr D. Wagstaffe who produced some of the photographs for me and also the following who so kindly and readily provided me with many of the photographs and diagrams included in the book:

Cartographic Engineering Ltd
The Editor, Photogrammetric Record
Survey & General Instruments Co. Ltd
Surveying & Scientific Instruments Ltd
Carl Zeiss (Jena) Ltd
Carl Zeiss (Oberkochen) Ltd
Wild Heerbrugg (U.K.) Ltd

September 1978

C. D. Burnside

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1: Some geometric properties of cameras and photographs

1.1 Introduction

For the purpose of making maps from aerial photographs it is necessary to examine certain characteristics of the cameras used and the photographs produced by them. Since map making is perhaps a geometric process (if we give the literal meaning to the word) then clearly the geometric properties of the camera are of prime importance. In addition, we must also be interested in the quality of the images produced by the photographic process. Factors such as sharpness of detail, tonal range and contrast will not only affect our estimated location of an image point position but also our interpretation of what that image represents on the ground. Photogrammetric mapping is therefore both an art and a science; the art being a subjective process of photo interpretation coupled with the more scientific process of geometric analysis. In topographic mapping, the degree of interpretive skill required is often not very high for the requirement is to show the locations of a selection of familiar ground features. In this case, therefore, the emphasis is very much on the correct location of these features on the map sheet. On the other hand, in other more specialised forms of mapping (soil, geological surveys and so on) the emphasis is, more often than not, the other way round; a correct recognition of certain features being more important than great accuracy in plotted positions. To some extent this change of emphasis is reflected in the relative importance given to the camera geometry and to the qualities of the images produced by the photochemical process.

1.2 The geometry of a simple camera

In this chapter we shall be concerned mainly with the photogrammetric aspects of camera geometry. Certain other factors will be mentioned but for a more detailed treatment of these the reader will be referred to other texts. In the first instance, therefore, we consider the case of image formation by a thin lens when the object is some considerable distance from the lens. In such a case the sharpest image will be produced in the focal plane of the lens as illustrated in Fig 1.1.

Light arriving at the lens from distant objects such as P and T is parallel on arrival. The bundles of rays are therefore in the form of thin cylinders of maximum diameter, $A \cdot \cos \theta$ where A is the aperture of the lens and θ is the angle of incidence. The axial ray of each cylindrical bundle can conveniently be taken as representative of that bundle and in this text is termed the *chief ray*. We note that all such chief rays pass through a single point N situated at the centre of the lens. This point is called a nodal point. The chief ray (Pp) that lies normal to the lens plane and the focal plane is in fact the optical axis of the lens. The two chief rays illustrated in the diagram form the angle θ at the node N and then carry on undeviated to intersect the focal plane in the two points p and t. The positions of the images p and t of the distant object points P and T are therefore geometrically defined in this way. We note that all other rays of a bundle are deviated to some degree and in the diagram all have been shown to intersect the focal plane at the same point as their chief ray. This is an ideal situation that cannot be achieved with real lenses in practice. All lenses and combinations of lenses must exhibit to some slight degree certain aberrations

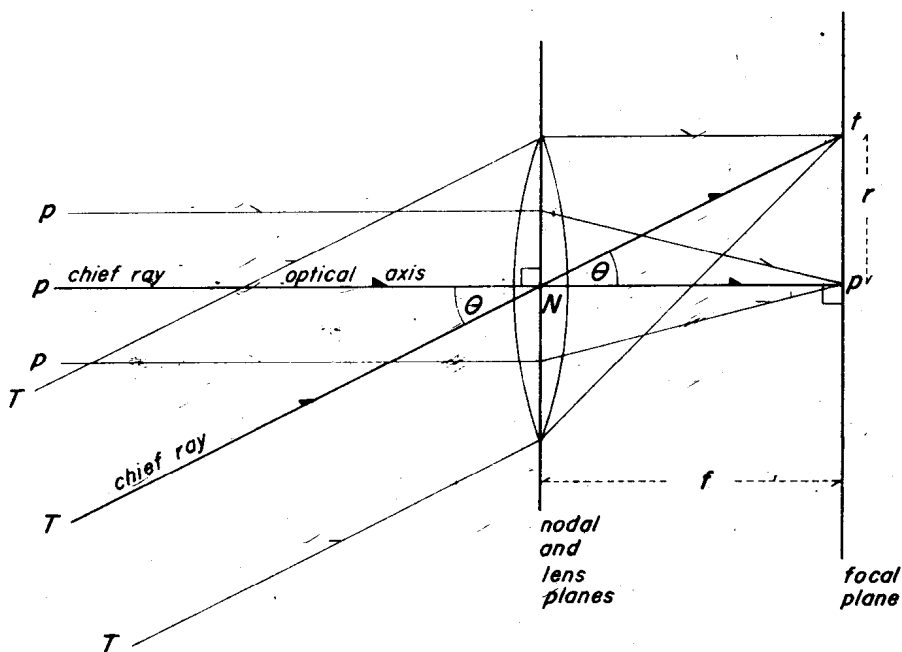


Figure 1.1. The Geometric Optics of a Thin Lens

(spherical aberration, coma, chromatic aberration, etc.) the effects of which are noted later in this chapter. At this point we note that their overall effect is to degrade the image from a sharp, well-defined point to a less well-defined area of confusion, the estimated centre of which may not be the point image defined by the geometry of our diagram. In the particular case of the air survey camera the object points are always at a considerable distance from the camera and so the plane of sharpest focus is the focal plane of the lens. The camera is therefore constructed so that the plane of the lens and the plane of sharp focus are parallel to one another and rigidly maintained at the required focal distance of separation. The point p is of special significance and is called the principal point of the photograph.

From an inspection of Fig 1.1 we see that the distant point subtends an angle θ at the nodal point N with the optical axis PNp where,

$$\tan \theta = \frac{r}{f} \quad (1.1)$$

Hence the value of this angle can be evaluated for the camera situation because r , the radial distance of t from p , is recorded on the photograph. However, in order to carry out this calculation we must know the values of the focal distance f and the position of p the principal point on the photograph. These data are therefore the essential parameters of the camera geometry that must be known to a high degree of accuracy for most mapping processes. Furthermore, it is also important that these parameters should remain constant over long periods of time under normal working conditions. That is to say, the structural geometry of the camera should be as stable as possible. The process of determining the parameters is called the camera calibration process.

From what has been said above we will now realise that the camera is in fact a recording goniometer. The data recorded by the photograph enable us to calculate the angles subtended at the station N by distant points such as T at the instant of exposure; the

reference direction being the optical axis pNP of the camera lens. Point N is taken to be the camera (or air) station.

1.3 The field method of camera calibration

There are a number of ways of carrying out the process of calibration and some of these are described later in this chapter. At this point we introduce the idea of field calibration because it requires no specialised equipment other than a theodolite and it reinforces the concept that calibration determines the relationship between angles in the object space and linear measurements taken off a plane photograph. In essence, we have to find the position of the node N with respect to the focal plane.

In the first instance we will take an ideal case in which all chief rays, regardless of their angle of incidence, are assumed to pass through a single nodal point situated somewhere on the optical axis of the lens. In practice this would suggest a thin lens devoid of any radial or tangential distortion and producing such image sharpness that the image point positions are not in any doubt. Later on, we will see what these terms imply in respect to the position of the node. This concept of a fixed nodal point regardless of the angle of incidence provides us with a model which we can profitably use as a standard to gauge other less perfect optical units.

The various stages in the calibration process are as follows:

- (a) Set out a horizontal line of targets on the ground, the size and nature of which are such that they are capable of accurate bisection by the stadia of the theodolite selected, and that they also provide the most suitable photographic images for the accurate measurement of distances on the photograph.
- (b) A theodolite of suitable precision is set up over a station mark some distance from the line of targets and approximately normal to them at the centre target. A series of horizontal directions are then observed and angles using the centre target as the reference object are then derived from the observations.
- (c) The theodolite is now replaced by the camera, which is set up so that the nodal point is vertically over the station mark and with the optical axis horizontal so as to produce a line of image points across the centre of the photograph format. The location of the node will not be known with any great accuracy, say to a centimetre or so, hence the range should be such that this uncertainty introduces no appreciable error into the angular observations.
- (d) A photograph is now taken of the row of targets. The camera is then rotated through 90° about the optical axis and another photograph taken. This will produce a row of images at right angles to those of the first photograph. When a detailed examination of the lens performance is required, more exposures will be made with rows of targets at other orientations.
- (e) After careful processing, a series of radial distances are measured on each photograph using the centre target as a reference point. The results of this procedure are illustrated in Fig 1.2. To illustrate the method of calculation consider two targets T_i and T'_i symmetrically disposed about the centre target T_o . From the theodolite observations we have the angles θ_i and θ'_i and from the photograph we have the two associated radial distances $r_i (=t_o t_i)$ and $r'_i (=t_o t'_i)$ as shown in Fig 1.3.

From Fig 1.3 we see that the quantities we require are Np, which is the focal distance f , and Δr , which is the distance of the principal point p from the centre target point t_o . The evaluation of these two quantities is best done by first calculating values for the angles α_i and α'_i .

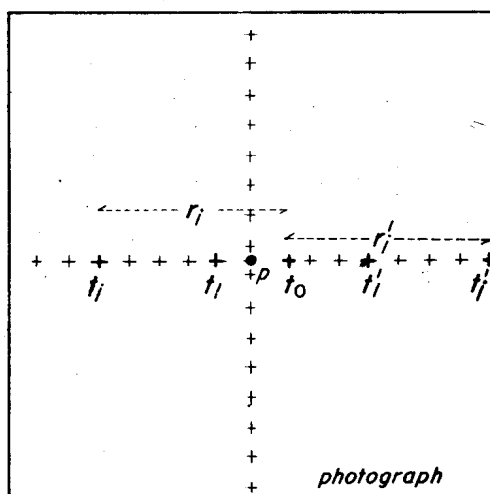
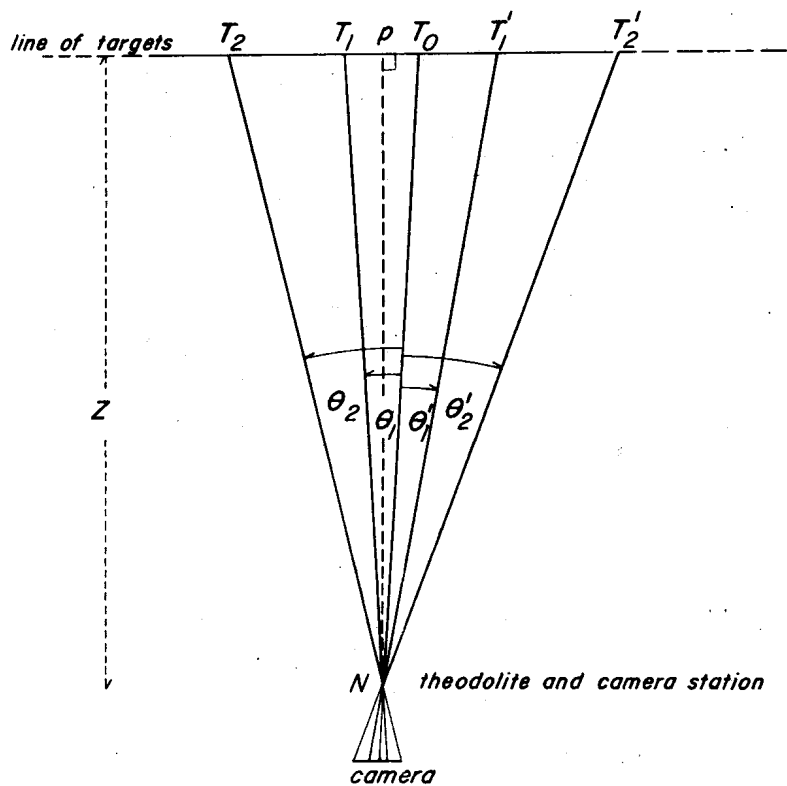


Figure 1.2. Field Method of Camera Calibration

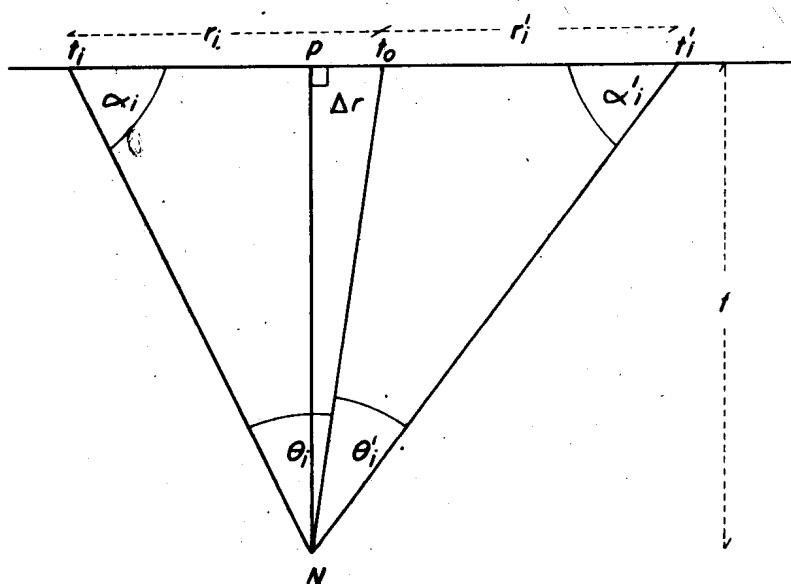


Figure 1.3. Calculation of Camera Parameters

We see immediately that

$$\begin{aligned} (\alpha + \alpha') &= 180 - (\theta + \theta') \\ &= k_1 \text{ (a constant)} \end{aligned} \quad (1.2)$$

By the application of the sine rule to the two large triangles we have

$$\begin{aligned} t_o N &= \frac{r \sin \alpha}{\sin \theta} \\ &= \frac{r' \sin \alpha'}{\sin \theta'} \end{aligned}$$

hence

$$\begin{aligned} \frac{\sin \alpha}{\sin \alpha'} &= \frac{r'}{r} \cdot \frac{\sin \theta}{\sin \theta'} \\ &= k_2 \text{ (a constant)} \end{aligned} \quad (1.3)$$

Students of surveying will recognise this geometry as a special case of the three-point problem in resection where the three points lie in a straight line in this case.

There are a number of ways of solving the two equations (1.2) and (1.3) simultaneously. A direct substitution for α' in equation (1.3) is probably as good as any, hence

$$\begin{aligned} \sin \alpha &= k_2 \sin (k_1 - \alpha) \\ &= k_2 (\sin k_1 \cos \alpha - \sin \alpha \cos k_1) \\ \therefore \tan \alpha &= k_2 \sin k_1 - k_2 \cos k_1 \cdot \tan \alpha \\ \text{i.e.} \quad \tan \alpha &= \frac{k_2 \sin k_1}{1 + k_2 \cos k_1} \end{aligned} \quad (1.4)$$

Having evaluated angle α and thence angle α' we are now able to calculate a value for the line Nt_o from either of the large triangles. Knowing this, Δr and f can then be calculated from the small triangle $Nt_o p$.

Using the same pair of targets on the other photograph at right angles to the first we can