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Preface

This text treats the fundamentals of optical and infrared detection in terms of the behavior of the radiation field, the physical properties of the detector, and the statistical behavior of the detector output. Both incoherent and coherent detection are treated in a unified manner, after which selected applications are analyzed, following an analysis of atmospheric effects and signal statistics. The material was developed during a one-semester course at M.I.T. in 1975, revised and presented again in 1976 at Lincoln Laboratory, and rewritten for publication in 1977.

Chapter 1 reviews the derivation of Planck's thermal radiation law and also presents several fundamental concepts used throughout the text. These include the three thermal distribution laws (Boltzmann, Fermi-Dirac, Bose-Einstein), spontaneous and stimulated emission, and the definition and counting of electromagnetic modes of space. Chapter 2 defines and analyzes the perfect photon detector and calculates the ultimate sensitivity in the presence of thermal radiation. In Chapter 3, we turn from incoherent or power detection to coherent or heterodyne detection and use the concept of orthogonal spatial modes to explain the antenna theorem and the mixing theorem. Chapters 4 through 6 then present a detailed analysis of the sensitivity of vacuum and semiconductor detectors, including the effects of amplifier noise. Thermal detectors are then treated in Chapter 7, thermal-radiation-field fluctuations being derived using the mode concept and a semiclassical approach originated by HANBURY BROWN and TWISS (1957a). Chapter 8 again uses the spatial-mode concept, as well as the stimulated- and spontaneous-emission relationships to determine the advantages of laser preamplification prior to detection. Atmospheric limitations on detection efficiency are briefly reviewed in Chapter 9 with special emphasis on the effects of turbulence. Following a detailed discussion of the significance of signal-to-noise ratios in terms of detection probabilities in Chapter 10, radar, radiometry, and interferometry are used as a framework to demonstrate the application of the previous results.

I have not tried to be encyclopedic in the treatment, either in terms of complete references or a discussion of every type of detector. References are included for more detailed background or, in some cases, historical interest. The most common photon and thermal detectors are analyzed; more specialized devices may be easily understood by extension of the treatments in the text. As an example the charge-coupled photon detector (CCD) recently reviewed by MILTON (1977) is basically a semiconductor photodiode with integrated readout and amplification. A detailed breakdown of the whole family of detectors may be found in KRUSE (1977). Another valuable reference is the annotated collection of papers by HUDSON and HUDSON (1975).

As basic sources for the fundamentals of incoherent detection and noise, I found SMITH et al. (1968) and VAN DER ZIEL (1954) invaluable, the latter especially helpful for an understanding of detector-noise mechanisms. The final version of the work profited by suggestions and comments from my colleagues and students and I especially thank Robert J. Keyes, my co-worker for many years, for his help and advice on detector behavior. I am indebted to Marguerite Ampolo for typing the original lecture notes, to Debra Brown for preparation of the final manuscript, and to Robert Duggan for the illustrations.

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R. H. KINGSTON

Contents

1. Thermal Radiation and Electromagnetic Modes

Before a discussion of the detection process, we first investigate the properties of the optical and infrared radiation in equilibrium with a cavity at temperature T. The results of this derivation are essential to an understanding of detection processes limited by thermal radiation from the vicinity of the detectors as well as from the background or other extraneous radiators near the desired signal source. The treatment also introduces the three forms of thermal statistics which will be used later in discussions of laser preamplification and thermal detectors. A up right final concept, introduced in this chapter and developed elsewhere in the text, is that of the allowed spatial modes of the electromagnetic field. The properties of these modes are key elements in the treatment of heterodyne detection and radiation field fluctuations.

1.1 The Nature of the Thermal Radiation Field

To treat the thermal radiation field we start with the applicable distribution statistics for "photons" and apply these statistics to the allowed modes of an electromagnetic field. First we consider the appropriate statistics, which are one of a class of three distinct types. These are Maxwell-Boltzmann, Fermi-Dirac, and Bose-Einstein. They are each the probability of occupancy of an allowed state and apply to different types of particles, as follows:

Maxwell-Boltzmann: Distinguishable particles, no exclusion principle. Limiting statistics for all particles for high energy or state occupancy much less than one,

$$f(E) = f_0 e^{-E/kT} , \qquad (1.1)$$

where E is the energy of the state and k = Boltzmann's constant = 1.38×10^{-23} J K^{-1}.

Fermi-Dirac: Indistinguishable particles obeying the exclusion principle, i.e., only one particle allowed per state (e.g., electrons),

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1} . \qquad (1.2)$$

Bose-Einstein statistics: Indistinguishable particles and unlimited state occupancy,

$$f(E) = \frac{1}{e^{(E-E_B)/kT} - 1}. \tag{1.3}$$

For the case of the modes of an electromagnetic field or "photons", the number of "particles" is unlimited and the distribution becomes

$$f(E) = \frac{1}{e^{E/kT} - 1} \tag{1.4}$$

and is usually referred to as "photon statistics". In both the Fermi-Dirac and Bose-Einstein statistics, the fixed energy parameter E_F or E_B is adjusted so that the total distribution in the system adds up to the total number of available particles. In the case of photons, the number is not limited and E_B becomes zero. The derivation of these statistics may be found in standard texts (see, for example, *Reif*, 1965) and is a subject in itself. We therefore shall not justify them at this time but later shall derive some relationships based on radiation theory which make them self-consistent and at least plausible.

1.2 Derivation of Planck's Radiation Law

We start with the following theorem:
In a large cavity (dimensions large compared to a wavelength), each allowed electromagnetic mode of frequency ν has energy $E = h\nu$, and the number of the modes excited is determined by the Bose-Einstein statistics applicable for photons. The temperature is determined by the temperature of the wall or of any absorbing particle in the cavity, under equilibrium conditions.

Consider a large cavity with slightly lossy walls, which for convenience in counting modes will be a parallelopiped of dimensions L_x, L_y, and L_z. To count the modes as a function of frequency, we write the standing-wave solution to Maxwell's equation as

$$E = E_0 \sin k_x x \sin k_y y \sin k_z z \sin 2\pi\nu t$$

subject to the boundary conditions that

$$k_x L_x = n\pi, \text{ etc}.$$

We also note that Maxwell's equation,

$$\nabla^2 E = (1/c^2)\partial^2 E/\partial t^2$$

requires that

$$k_x^2 + k_y^2 + k_z^2 = 4\pi\nu^2/c^2.$$

Using these relationships, we can construct the distribution of allowed modes in k space as an array of points occurring at $k_x = n\pi/L_x$, $k_y = n\pi L_y$, etc., where n is an integer 1 or greater. The density of points in k space, as seen from Fig. 1.1, becomes

$$\rho_k = L_x L_y L_z/\pi^3 = V/\pi^3 ,$$

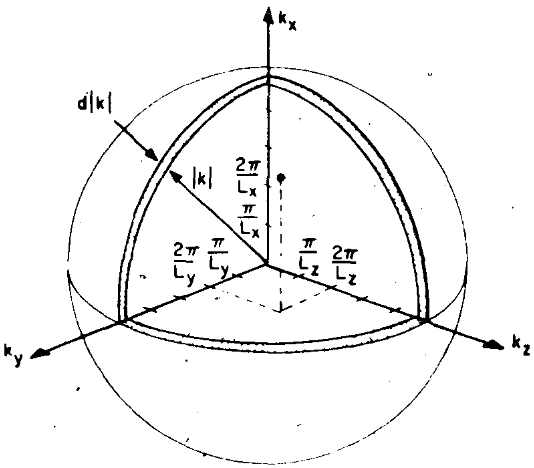

Fig. 1.1. Mode distribution in k space

where V is the volume of the cavity. To determine the density of modes versus frequency, we note that

$$|k| = \frac{2\pi\nu}{c},$$

as shown in Fig. 1.1. Therefore, a spherical shell in the first octant of k space contains points that represent all modes at frequency $\nu = |k|c/2\pi$. We then may solve directly for the number of modes dN in a frequency range $d\nu$

$$\begin{aligned} dN &= \frac{\pi}{2} \cdot \rho_k\, k^2 dk \\ &= \left(\frac{\pi}{2}\right)\left(\frac{V}{\pi^3}\right)\frac{4\pi^2\nu^2}{c^2} \cdot \frac{2\pi d\nu}{c} \\ &= \frac{4\pi V}{c^3} \cdot \nu^2 d\nu . \end{aligned} \tag{1.5}$$

To calculate the energy per unit frequency interval, we must take into account the allowed occupancy of the modes. This is given by (1.4) and also must include the fact that two separate orthogonal polarizations are allowed for each mode. Thus

$$du = dE/V = 2 f h\nu dN/V \tag{1.6}$$

and the final result is the energy per unit volume per unit frequency range,

$$du_\nu = \frac{8\pi h}{c^3} \cdot \frac{\nu^3 d\nu}{(e^{h\nu/kT} - 1)}, \tag{1.7}$$

where h = Planck's constant = 6.6×10^{-34} Js.

We now ask what is the power density that strikes the cavity wall or crosses any small surface area within the cavity. We know that the energy density arises from forward and backward travelling waves that produce standing wave modes, and that the waves and the energy are moving at the velocity of light c. If the surface has area A, the power striking it at an angle θ from the normal is given by

$$dP_\nu = \frac{c}{2} du_\nu \frac{A \cos\theta \, d\Omega}{2\pi}, \tag{1.8}$$

where, as shown in Fig. 1.2, the power density that flows in a small solid angle $d\Omega$ is one-half the energy density multiplied by the velocity of light, reduced by the ratio of the solid angle to a full hemisphere. The power collected by the surface is proportional to $A \cos\theta$ because of the angle of incidence of the flux.

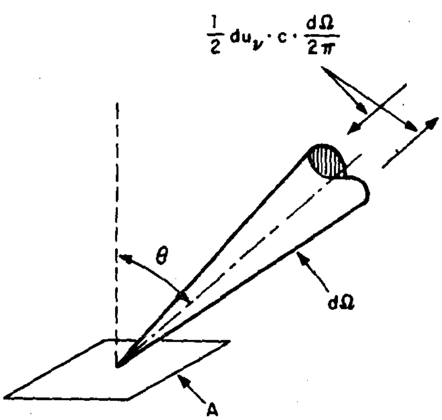

Fig. 1.2. Power flow in thermal-radiation field

This argument assumes that the radiation flow is isotropic, which could be proved by use of the cavity treatment but which we shall show later in a more general proof. Integrating over the hemisphere, with

$$d\Omega = 2\pi \sin\theta \, d\theta$$

yields

$$P_\nu = \frac{c}{2}\, du_\nu \int_0^{\pi} \frac{A\cos\theta\, 2\pi \sin\theta\, d\theta}{2\pi} = \frac{c}{4}\, du_\nu A \tag{1.9}$$

and

$$dI_\nu = \frac{2\pi h}{c^2}\, \frac{\nu^3 d\nu}{(e^{h\nu/kT} - 1)}, \tag{1.10}$$

where I_ν is defined as the irradiance or the total power per unit surface area. This last expression is the commonly used form of Planck's radiation law. The distribution for $T = 300$ K is plotted in Fig. 1.3 with ν^*, the reciprocal wavelength (wave numbers) as the variable.

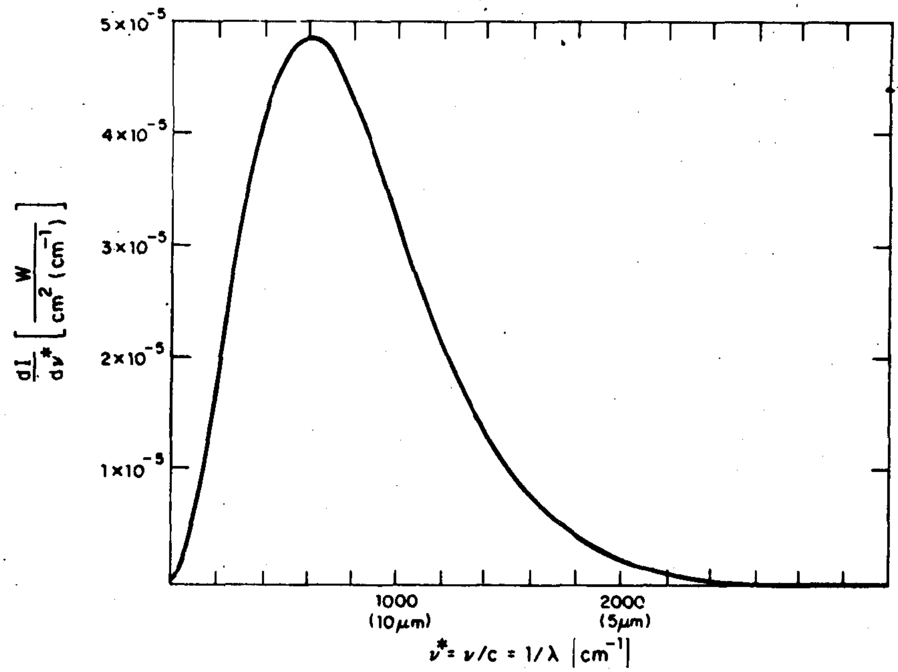

Fig. 1.3. Planck distribution for $T = 300$ K. For any temperature T, multiply the abscissa, ν^*, by $T/300$, and the ordinate $dI/d\nu^*$ by $(T/300)^3$

1.3 General Properties of Blackbody Radiation

In our derivation, we assumed an ideal cavity with slightly lossy walls in order to derive the radiation law. We now generalize and introduce the concepts of emissivity ε and absorptivity α. By use of a general principle known as detailed balance we may show that

1) The emissivity of a body is equal to its absorptivity at all frequencies and at all incidence angles.

2) The radiation is isotropic and independent of position in the cavity.

3) The radiation law is true for *any* enclosed region of arbitrary shape, provided that the temperature of all walls and enclosed objects is the same.

We first insert in the cavity a small particle that absorbs completely at all frequencies. If the particle is at the same temperature as the walls, then the energy it absorbs should equal the energy it radiates, otherwise its temperature would change. We imagine the particle to move throughout the cavity. Because the energy it radiates is a function of temperature alone, the energy incident on it must be independent of position, otherwise the temperature of the particle would change, in violation of the equilibrium requirement. Therefore, the radiation field is independent of position. We next assume that the particle has emissivity less than unity. In this case, the particle radiates less energy than the energy impinges on it. To maintain constant temperature, its reflectivity must balance its decreased emission. We therefore require that the emissivity be equal to (1 − reflectivity), which is equal to the absorptivity. In a similar manner, using arbitrary shields or frequency filters about the particle, we can show that the radiation in the cavity is isotropic and that the equality of emissivity to absorptivity holds at all frequencies and angles of incidence upon a surface. Finally, we imagine our sample cavity to be coupled through a small hole to a second cavity of arbitrary shape and with walls of any emissivity, all maintained at the equilibrium temperature T. By inserting frequency, angular, or polarization filters between the two cavities, we may show that the radiation in both cavities must have the same frequency, angular, and polarization behavior; otherwise the second cavity would absorb or lose energy, again violating the equal-temperature requirement. For further details the reader is referred to *Reif* (1965).

We now deduce two important consequences of the foregoing arguments. These are, first, that a surface with emissivity ε and temperature T emits energy, *even in the absence* of incident radiation, at a rate that is equal to the rate at which it absorbs energy from the radiation that is incident on it in a cavity that is at the same temperature. If we define the radiance in $\mathrm{Wm^{-2}}\cdot$steradian, $H_{\nu,\Omega}$, then the radiance from a surface of area A is

$$dH_{\nu,\Omega} = \varepsilon(\nu,\theta)\cdot\frac{2h}{c^2}\frac{\nu^3 d\nu\cos\theta}{(e^{h\nu/kT}-1)} \tag{1.11}$$

and the total emitted power for isotropic ε is

$$dP_\nu = \varepsilon\cdot\frac{2\pi h}{c^2}\frac{A\nu^3 d\nu}{(e^{h\nu/kT}-1)}, \tag{1.12}$$

which, when integrated over frequency, yields the Stefan-Boltzmann equation.

An extension of this rule may be applied to a partially absorbing medium distributed in a small volume of the cavity. In this case, we surround the absorbing or partially transparent medium by an imaginary boundary and, considering one direction of energy flow through the bounded region, require that the net flow into one side equals the net flow out. If the medium is to maintain constant temperature, then the radiation absorbed along the path must equal the radiation emitted. It then follows that the medium emits in the forward *and* backward directions with an effective emissivity equal to the absorptivity or fractional power loss along the path. If we remove this absorbing medium from its cavity but maintain its temperature, it will appear against a radiationless background as an emitter that radiates according to the same laws as an emissive surface at the same temperature.

1.4 A Plausibility Test of the Planck Distribution

Although we presented the three statistical distributions without proof, it is instructive to show the interaction between two of them to indicate their physical plausibility. We do this by calculating the interaction between a simple two-level Maxwell-Boltzmann system and the radiation field inside a blackbody cavity. This treatment was used in a reverse manner by Einstein to derive the Bose-Einstein statistics. We start by defining the Einstein A and B coefficients for atomic transitions, as follows. In a two-level system with low occupancy, the probability of transition from a higher energy state, level 2, is given by

$$P(\text{emission}) = (A + Bu)f_2\,, \tag{1.13}$$

where A represents the spontaneous-emission term and Bu is the induced emission, which is proportional to the energy density in the radiation field. f_2 is the Maxwell-Boltzmann factor for the upper state, $\exp(-E_2/kT)$. The absorption rate is

$$P(\text{absorption}) = Bu\,f_1\,; \tag{1.14}$$

if the atomic system is in equilibrium with the radiation field, the two rates should be equal. Thus we have

$$A/B = u(f_1 - f_2)/(f_2) = u(e^{-(E_2-E_1)/kT} - 1)\,.$$

Substituting the Planck radiation density for u yields

$$A/B = 8\pi h\nu^3/c^3\,,$$

which we note is completely independent of temperature. This is satisfying physically, because it indicates that the ratio of spontaneous emission *power* to induced power is proportional to ν^4, as would be expected from classical theory.

1.5 Numerical Constants and Typical Values

The physical constants used in typical calculations of the radiation field are

$$h = 6.6 \times 10^{-34}\ \mathrm{Js}$$
$$k = 1.38 \times 10^{-23}\ \mathrm{JK^{-1}}$$
$$c = 3 \times 10^{8}\ \mathrm{ms^{-1}}$$
$$e = 1.6 \times 10^{-19}\ \mathrm{C}\,.$$

During most calculations, it is more convenient to express photon energies and thermal energy in eV, which are defined as the voltage difference through which an electron would have to fall to gain the same energy. Thus, if the energy is 1 eV, the energy in joules is 1 V times the electron charge, or 1.6×10^{-19} J. The photon energy is

$$E = h\nu = hc/\lambda$$

which, expressed in eV is

$$(h\nu)_{\mathrm{eV}} = hc/e\lambda = 1.24/\lambda\,,$$

where the wavelength λ is expressed in μm. Similarly, the thermal energy in eV is

$$(kT)_{\mathrm{eV}} = \frac{kT}{e} = 0.026\ (T/300)\,,$$

where T is expressed in K.

On the basis of these relationships, it is instructive to calculate the value of the Bose-Einstein occupancy factor for typical temperatures and wavelengths. The result for $T = 300$ K, and for a wavelength of 1 μm (just into the infrared from the visible), is

$$\mathscr{f} = \frac{1}{e^{h\nu/kT} - 1} = \frac{1}{e^{48} - 1} \simeq 10^{-21}\,.$$

For 10 μm, about the longest infrared wavelength propagated through the atmosphere, the factor is

$$f = \frac{1}{e^{4.8} - 1} = \frac{1}{120}.$$

Thus, in either of these cases, the state occupancy is much less than unity; we shall use the following approximation in many of the treatments to follow:

$$f = \frac{1}{e^{h\nu/kT} - 1} = e^{-h\nu/kT} \qquad h\nu \gg kT.$$

Problems

1.1 Derive the Stefan-Boltzmann law by integrating over the Planck distribution. $I = \varepsilon\sigma T^4$. Show that $\sigma = (2\pi^5 k^4)/(15c^2h^3) = 5.67 \times 10^{-8}$ Wm^{-2} deg^4. [The integral $\int_0^\infty (x^3 dx)/(e^x - 1) = \pi^4/15$].

1.2 A surface is in equilibrium with a blackbody field of temperature 300K.

a) What is the total irradiance on the surface?

b) At what wavelength λ is the irradiance per unit frequency a maximum? Check your answer with Fig. 1.3.

1.3 The sun's diameter subtends an angle of 9.3 milliradians as seen from the earth. If the sun is a perfect blackbody at temperature $T = 5800$ K, find the solar constant, the irradiance at the top of the earth's atmosphere.

1.4 An optical receiving element of area A has diffraction-limited beamwidth given by $\Omega = \lambda^2/A$, where Ω is the solid angle of the beam. Find the power per unit frequency interval incident on the receiver within this solid angle Ω. Find the value for the limiting case where $h\nu \ll kT$.

1.5 In Section 1.3, we derived the ratio of the Einstein A and B coefficients by placing a Maxwell-Boltzmann distribution in equilibrium with the radiation field. Repeat the calculation using a Fermi-Dirac system, again in equilibrium with the radiation. Note that the transition probability is now proportional to the occupancy of the initial state times the "emptiness" of the final state. You will thus have terms in $f_2(1 - f_1)$, etc.

2. The Ideal Photon Detector

We now treat the optical and infrared detection process by postulating the ideal photon detector. This is a device that samples the incident radiation and produces a current proportional to the total power incident upon the the detector surface. We first consider the fundamental principles of detection, then the noise associated with the detection process, and, finally, analyze two limiting forms of detection, signal-noise limited and background-noise limited. In the latter case, we shall use the results of Chapter 1 to find the limiting sensitivity in the presence of thermal radiation. The treatment in this section ignores noise from the following amplifier stages as well as noise sources peculiar to real devices; however, the results set the absolute limits of sensitivity for a device that exhibits ideal behavior.

2.1 Event Probability and the Poisson Distribution

Here we start with a fundamental theorem of photodetection that is the basis of the whole treatment of detection theory. The theorem is: If radiation of constant power P is incident upon an ideal photon detector, then electrons will be produced at an average rate given by

$$\bar{r} = \eta P/h\nu$$

where η is defined as the quantum efficiency, that is, the fraction of the incident power effective in producing the emitted electrons. A second and most important part of the theorem is that the electron-emission events are randomly distributed in time. Because of the quantum nature of radiation, each photoevent or electron results from the extraction or loss of one "photon" or $h\nu$ of energy from the incident field. If the incident radiation is time varying, the *average* rate will vary with time in the same manner. For constant $\bar{r}$, such a random process obeys Poisson statistics, which state that the probability of the emission of k electrons in a measurement interval τ is

$$p(k,\tau) = \frac{(\bar{r}\tau)^k e^{-\bar{r}\tau}}{k!}. \tag{2.1}$$

A derivation of this distribution may be found in *Davenport* and *Root* (1958). In the case of time-varying power and thus $\bar{r}$, the Poisson distribution is still valid, provided that the sampling interval τ is short compared to any charac-