

GAS LUBRICATION

V. N. CONSTANTINESCU

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Technical Editor

Professor Robert L. Wehe

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EDITOR'S NOTE: *Numbers in brackets refer to the references listed at the end of each paper; numbers in parentheses denote equations.*

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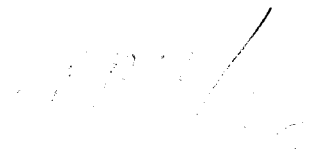
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FOREWORD

The publication of this translation of one of the earliest treatises on the subject of gas lubrication will be important to bearing engineers not able to read Romanian or Russian. Dr. Constantinescu has presented a very thorough theoretical study. In addition, he has related the theoretical results to the practical application of gas bearings in machinery. Two unique aspects of the work are his qualitative studies of bearing behavior where quantitative evaluation is difficult and his development of limiting solutions which give bounds to the performance of bearings. In this way, he is able to give a better understanding of the influence of compressibility and of inertia forces. While numerical solutions can be made using high speed digital computers, qualitative understanding can be of considerable aid to the bearing engineer.

The background material on gas properties and fluid mechanics which provide the starting point for the application to particular lubrication conditions in succeeding chapters is presented in Chapter I. Specific relationships which can be applied to externally pressurized bearings are then developed in Chapter II; these are used in Chapter III to develop relationships which can be used in the analysis and design of externally pressurized bearings. A designer interested in a particular bearing can find design relationships by turning directly to Section 3.3. Chapter IV covers bearings operating in the regime where compressibility effects are small enough so that bearing behavior can be predicted without considering them. In Chapters V and VI the author studies the lubrication of infinitely long thrust and journal bearings, respectively. The author uses the term two-dimensional lubrication to describe these cases, while most current literature designates them as one-dimensional, since variations are considered only in one dimension. Bearings with finite width are taken up in Chapter VII. A designer interested in self-acting bearings will find design relationships in Section 7.3, and additional information in Section 8.4.1.4. Bearings operating under nonsteady conditions of load or surface motion are covered in Chapter VIII. Lubrication in the turbulent regime has been covered by Dr. Constantinescu in another book which deals only with that subject.

The reader should be aware that since the equations and figures have been photographed directly from the original – both for accuracy and for economy – there will be some variation in style between the symbols in the equations and in the text. Where material has been added in Chapter III in Section 3.3.3.7, the symbols in the equations will not match in style the remainder of the book. The inconvenience should only be minor if the reader is alert to the situation. Material has also been added in Chapter VI to present important developments since the original publication.



A number of credits should be given, first to the Research Committee on Lubrication of the Society who contracted with National Science Foundation to cover translation, editing and production costs. Also to Scripta Technica, Inc., for their translation of the work which greatly simplified the technical editing, and to Dr. Douglas Muster who first suggested the project and edited Chapter I, but had to relinquish the task due to time constraints. Finally, and very important to me in presenting a faithful translation of his work, Dr. Constantinescu has been very generous with his time and energy in reviewing both the rough and final copies.

Professor Robert L. Wehe
Cornell University

PREFACE

This book is one of the first ever to be written on the subject of gas lubrication. It is only recently that gases and solids have come into use as lubricants. These developments are closely related and have contributed significantly to recent achievements in modern engineering such as automation, the innovation of high-speed and high-precision machines and instruments, the remote guidance of rockets, and the development of very low friction equipment. In part, they explain the ever-growing interest in this new branch of knowledge and the significance attached to the publication of the first major works in this field.

The author has earned a well-deserved international reputation in gas lubrication through his numerous and valuable contributions to the field in recent years. He obtained remarkable results in the study of two- and three-dimensional lubrication with compressible media and was the first to formulate a comprehensive theory of lubrication under turbulent conditions. In addition, he performed important research work on lubrication of nonstationary regimes at both high and very low speeds, and in the related fields of heat transfer and free molecular flow. The present book has benefited from this experience and contains in great part the results obtained by Constantinescu. The material used in the book stemming from the work of other investigators has been revised and further developed. As a consequence, the book is distinguished by a high degree of originality. Further, the presentation of the material is unified, comprehensive, and easy to follow. Although still young, the author has displayed scholarly knowledge in his chosen area of specialty, principally the fields of modern mechanics of solids and fluids, physics and applied mathematics. The treatment is on a high level, great attention being given to scientific rigor, clear exposition and complete results.

Modern science may be likened to a tree whose fruits become more succulent each year as its crown grows higher. The best fruits grow precisely on the highest branches, and considerable effort is sometimes required to reach them. The roots of the tree penetrate deep into the soil and spread over an ever greater area. This leads to a twofold difficulty: one for the reader who must seek to harvest the ripe fruit contained in a work such as the present one, the other for the author who must possess not only a comprehensive knowledge in his area of specialty, but a familiarity with the role played by phenomena in associated fields. We believe the author has accomplished brilliantly this difficult task. We hope that readers will be able to use the contents of this book in the solution of practical problems with which they are confronted or as background for new theoretical developments.

Bucarest, November 26, 1962

N. TIPEI

INTRODUCTION

From a theoretical viewpoint, gas lubrication is a branch of the mechanics of viscous compressible fluids. At the same time, it may be considered an integral part of hydrodynamic lubrication, although it exhibits the peculiar feature of compressing the lubricant.

From a practical viewpoint, gas lubrication has interesting and important technical applications, which are the primary reason for the expanded research programs of the recent past. Air- or gas-lubricated bearings have considerable advantage over oil-lubricated journal bearings. The heat produced by friction is, in most cases, negligible. The wear of gas-lubricated bearings is extremely low, a factor which contributes to their long life. To these advantages must be added the fact that gas-lubricated bearings can operate properly over a much greater temperature range than the conventional bearings and that they can operate in the presence of severe radiation. Finally, gas-lubricated bearings can be easily externally pressurized which makes it possible to operate them at very low or high speeds.

It is precisely for these reasons that gas-lubricated bearings are used successfully today, despite the technical difficulties they introduce and despite their small load capacity. We find them used in electric motors, ultracentrifuges, machine tools, textile machinery, equipment for gas liquefaction, turbines, turbo-compressors, apparatus and equipment for the nuclear industry, and high-precision instruments such as inertial guidance systems for rockets and space ships. Most of these applications are recent, and it is in their solution that gas-lubrication research has made great strides forward during the past few years.

The expanding research on the subject, as well as the relatively advanced stage of development already reached, suggest that the time has come to produce a treatise that would present the most important results obtained thus far. At this writing no such treatise has been published. The present work is based on research carried out by the author since 1954 and also on the results currently available in the literature. In a sense it is a continuation of two earlier Romanian publications.¹

¹N. Tîpei, *Hydro-aerodynamic Lubrication*, R.P.R. Academy Press, 1957; N. Tîpei, V. N. Constantinescu, Al. Nica, and O. Bita, *Slider Bearings*, R.P.R. Academy Press, 1961.

The general structure of the book is as follows. In an introductory chapter, the general gas-lubrication equations are developed. In Chapters II and III, the flow in the lubricant film of externally pressurized bearings is considered. The material in the remaining chapters is mainly concerned with self-acting bearings which operate under hydrodynamic conditions. Thus, in Chapter IV we examine the conditions of gas lubrication at low speeds for which the lubricant can be considered incompressible in a first approximation. In Chapters V and VI the simplified, two-dimensional problem of infinitely long bearings is developed, whereas in Chapter VII we examine the operation of actual bearings of finite dimension. Finally, in Chapter VIII we study the dynamic regimes and operational stability of gas-lubricated bearings. The effect of turbulence on the flow in the lubricant film has not been examined, since it will be the subject of another publication.

The book has a practical structure that is intended to interest both specialists in the fields of fluid mechanics and lubrication and industrial engineers. Since those who wish to design and construct air-bearings are primarily interested in actual methods of calculation and design and in precise construction details and in possible applications of air bearings, we have attempted to concentrate these topics in separate sections which could be read separately without undue reference to other more theoretical material. Thus, we suggest that those interested only in the design and construction of externally pressurized air-bearings should refer to Sections 3.3 (Chapter III) and 8.4.2 (Chapter VIII). Those interested in the design and construction of self-acting bearings should consult Sections 7.3 (Chapter VII) and 8.4.1.4 (Chapter VIII).

In addition, we have dealt with yet-unresolved or insufficiently-studied problems, hoping to make a useful contribution to the further development of the theory and practice of gas lubrication. In view of the inherent nonlinearity of the gas-lubrication equations, any analytic study of them can become difficult. The two-dimensional problem is most amenable to analytical treatment and has been examined in great detail. Beyond this, certain special problems have also been considered, including a qualitative study of the pressure equation and other studies of the effect of inertia forces (Chapter V), behavior at high speeds, the effect of the thermal regime and the molecular character of the flow (Chapter VI). For certain elementary cases of externally pressurized bearings (Chapter II), we likewise examine the effect of the forces of inertia, the influence of lubricant-film geometry on the pressure distribution. We utilized the results obtained for specific applications by numerical integration of the pertinent equations. The performance of gas-lubricated bearings under dynamic conditions and, in particular, the stability of operating bearings were also studied (Chapter VIII).

In conclusion, the author wishes to express his gratitude to the "Traian Vuia" Institute of Applied Mechanics of the R.P.R. Academy which is directed by Academician E. Carafoli, for permission to conduct his research. The author is grateful to Professor N. Tipei for his assistance, suggestions, and steady encouragement in the editing of this book. Finally, the author wishes to thank several foreign investigators for their courtesy in placing some of their unpublished results at his disposal. They include Professor A. K. Dyachikov and S. A. Scheinberg (USSR), Professor D. D. Fuller, Professor A. Saibel, J. H. Laub, A. A. Raimondi, and J. F. Osterlee (U.S.A.), S. Whitley (Great Britain), and especially

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V. N. CONSTANTINESCU

Bucarest, January 20, 1962

NOMENCLATURE

$$B_x = -\frac{h^2}{2\mu V} \frac{\partial p}{\partial x}, \text{ dimensionless parameter related to the derivative } \frac{\partial p}{\partial x}$$

$$B_z = -\frac{h^2}{2\mu V} \frac{\partial p}{\partial z}, \text{ dimensionless parameter related to the derivative } \frac{\partial p}{\partial z}$$

$$C_1 = p_M^* h, \text{ integration constant in pressure equation}$$

$$C_F^* = \frac{M h_m}{\mu V l b}, \text{ friction coefficient}$$

$$C_m^* = \frac{M h_m}{\pi \mu V l^2 b}, \text{ moment coefficient}$$

$$C_z^* = \frac{\mathcal{M}_z}{p_0 C V b}, \text{ flow coefficient}$$

$$C_{t,n} = \frac{P_{t,n}}{2p_0 r_1 b}, \text{ dimensionless coefficients associated with the force components } P_t \text{ and } P_n, \text{ respectively, in journal bearings}$$

$$F = \text{friction force}$$

$$F_{0,h} = \text{friction force on the two surfaces } (y = 0, y = h)$$

$$G = \frac{x_T T_0 l}{p_0 V c^2}, \text{ dimensionless parameter which occurs in the energy equation}$$

$$H = \frac{\mu V l}{p_0^* h_m^2} = \frac{\mu \omega}{p_0^* \psi^2}, \text{ Harrison's number (dimensionless parameter which occurs in the pressure equation)}$$

$$\bar{H} = \frac{\mu V l}{p_0 h_2^2} = \frac{\mu \omega r_1^2}{p_0 h_2^2}, \text{ Harrison's number referred to the minimum thickness}$$

$$H_* = \frac{\mu \omega}{p_0^* \psi^2}, \quad \bar{H} = \frac{\mu \omega r_1^2}{p_0^* h_2^2}, \text{ Harrison's numbers referred to the pressure } p_0^* \text{ at } \theta = 0 \text{ (journal bearings)}$$

$$\bar{H}_h = \frac{\mu V l}{p_0 (h_1 - h_2)^2}, \text{ Harrison's number for step bearings}$$

$$H_\epsilon = \frac{\mu V l}{p_0 \nu h_m} = \frac{\mu V l}{2 \epsilon p_0 h_m^2}, \text{ modified Harrison's number for bearings with inclined plane surfaces}$$

$$H_0 = \frac{\mu \Omega_0}{P_0 \psi^2} = \frac{\mu (\Omega_{10} + \Omega_{20} - 2\phi_0)}{P_0 \psi^2}, \text{ modified Harrison's number for unsteady flows (journal bearings)}$$

$$K_n = \text{Knudsen number} \left(K_n = \frac{l_p}{h} \text{ or } K_n = \frac{l_p}{h_m} \right)$$

$$\bar{K}_n^* = \frac{l_{p0}}{h_2}, \text{ Knudsen number defined as a function of minimum thickness } h_2$$

$$K_n^* = \frac{l_{p0}}{h_m}, \text{ Knudsen number defined as a function of free path } l_{p0}$$

$$K_3 = \frac{2\nu C_1}{3\mu V} = \frac{2K_1}{H_\epsilon}, \text{ integration constant (plane slider bearings)}$$

$$\bar{K}_1 = \frac{p_M h_0}{p_0 h_2} = \frac{h_m}{h_2} K_1, \text{ modified constant } K_1 \text{ (plane slider bearings)}$$

$$K = \frac{\sqrt{1 - \epsilon^2 K_1}}{6H}, \text{ modified constant } K_1 \text{ (journal bearings)}$$

$$K = \frac{\partial P}{\partial h}, \text{ bearing elastic constant}$$

$$K_1 = \frac{C_1}{\frac{1}{p_0^*} h_m} = \bar{p}^{\frac{1}{\kappa}} \bar{h}_0, \text{ dimensionless integration constant which occurs in the pressure equation}$$

$$K_1 = \text{shaft elastic constant}$$

$$L = \frac{\mu \omega^*}{P_0 \psi^2}, \text{ a dimensionless number analogous to Harrison's number for unsteady flow}$$

$$M = \text{friction moment}$$

$$M_* = \frac{U_m}{a_*}, \text{ modified Mach number (equal to Mach number if } \kappa = k \text{ and } \alpha = 1)$$

$$M = \frac{U_m}{a}, \text{ Mach number}$$

$$M_1, M_2 = \text{friction moments on journal and bearing}$$

$$\bar{M}_* = V/2a_*, \text{ Mach number modified with respect to velocity } V/2$$

$$M_{t,n} = \text{moment vector-component in the direction of center line and of normal to center line, respectively, in journal bearings}$$

$$O_1 = \text{journal center}$$

$$O_2 = \text{bearing center}$$

$$P_r = \text{Prandtl number} \quad P_r = \frac{\mu c_p}{\kappa T}$$

$$P = \text{bearing load}$$

$$P_1, P_2 = \text{loading of surfaces 1 and 2}$$

$$P_t, P_n = \text{components of force } P \text{ in the direction of center line and of normal to center line}$$

- Q_x, Q_z = volumetric flow-rates
 R = gas constant ($\text{m}^2/\text{s } ^\circ\text{C}$)
 R' = gas constant ($\text{kgm}/\text{kg } ^\circ\text{C}$)
 R = radius of curvature
 R_1, R_2 = vector resultant of external load $P_{1,2}$ and of pressure resultant $\mathcal{P}_{1,2}$
 $\text{Re} = \rho \frac{U_m}{\mu}$, Reynolds number; $\text{Re}^* = \rho \frac{VhM}{\mu M}$
 S = entropy
 S = bearing area (projected area for journal bearings)
 S_i = feed orifice cross-section
 T = absolute temperature ($^\circ\text{K}$)
 U_m = mean velocity $U_m = (U + V) / 2$
 U = mean velocity in the case $V = 0$
 V = velocity of mobile surface
 $\mathbf{V}_1, \mathbf{V}_2$ = velocities of surfaces 1 and 2
 $V_{1x}, V_{1y}, V_{1z}, V_{2x}, V_{2y}, V_{2z}$ = velocity components of surfaces 1 and 2 in x, y and z directions, respectively
 $\mathbf{V}_{1t}, \mathbf{V}_{2t}$ = projections of velocities \mathbf{V}_1 and \mathbf{V}_2 on a plane tangent to surface 1
 $\mathbf{V}_t^* = \mathbf{V}_{2t} - \mathbf{V}_{1t}$, difference between velocities \mathbf{V}_{2t} and \mathbf{V}_{1t}
 $\mathbf{V}_t^{**} = \mathbf{V}_{2t} + \mathbf{V}_{1t}$, sum of velocities \mathbf{V}_{2t} and \mathbf{V}_{1t}
 V_{1n}, V_{2n} , projections of velocities \mathbf{V}_1 and \mathbf{V}_2 on a normal to surface 1
 $V_n = V_{2n} - V_{1n}$, difference between velocities V_{2n} and V_{1n}
 W = power consumed by friction
 \bar{W} = mean velocity of side flow
 G = weight flow
 \mathcal{M} = mass flow
 $\mathcal{M}_x, \mathcal{M}_z$ = mass flow in x and z directions
 \mathcal{M}_r = radial mass flow (for axisymmetric motions)
 \mathcal{M}_{ri} = mass flow at bearing inlet
 \mathcal{M}_{re} = mass flow at bearing outlet
 \mathcal{P} = resultant pressure under dynamic operating conditions
 $\mathcal{P}_1, \mathcal{P}_2$ = resultant pressure on surfaces 1 and 2 (journal-bearing)
 $\mathcal{P}_{1t,n}, \mathcal{P}_{2t,n}$ = components of resultants pressures \mathcal{P}_1 and \mathcal{P}_2 in the direction of center line and of a normal to it
 $\tilde{\psi}$ = stream function
 $a_* = \sqrt{x \frac{p}{\rho}}$, critical gas velocity (velocity of sound for $x = k$)
 a = velocity of sound
 b = width of bearing
 b_1 = width of feed pocket (plenum)
 \bar{c} = mean velocity of random molecular motion
 c_v = specific heat at constant volume

- c_p = specific heat at constant pressure
 c = radial clearance ($c = r_2 - r_1$) for journal bearings
 c_* = critical velocity $\left(c_*^2 = \frac{2}{k+1} a_0^2 = \frac{k+1}{k-1} U^2 + \frac{2}{k+1} a_0^2 \right)$
 $\bar{e}_1, \bar{e}_2, \bar{e}_3$ = orthogonal versors
 e = eccentricity
 f_x, f_y, f_z = components of external force per unit mass
 $f = F/P$, friction factor
 h = enthalpy
 h = lubricant-film thickness
 h_1 = maximum lubricant-film thickness
 h_1 = thickness h at lubricant-film inlet
 h_2 = minimum lubricant-film thickness
 h_2^* = thickness h at lubricant-film outlet if $h_2^* \neq h_2$
 h_0 = thickness at the point at which the pressure is maximal or minimal
 $\bar{h} = 1 - \epsilon \cos \gamma$, modified thickness h (journal bearings)
 i, j, k = unit vectors in x, y and z directions
 k_b = Boltzmann's constant
 k = ratio of specific heats ($k = c_p/c_v$)
 k_p = permeability coefficient
 l = length of a thrust-bearing sector
 l_a = mixing length
 l_p = molecular free path (mean distance between two collisions)
 l_{p0} = molecular free path corresponding to an external pressure p_0
 m = molecular mass
 m = flow per unit length
 $m_d = S_i/S_a$, ratio of feed-orifice cross-section S_i to feed-duct cross-section S_a
 m^* = gas mass contained in lubricant film
 n = number of molecules per unit volume
 \mathbf{n} = versor of normal to a surface
 n_μ = exponent defining temperature-variation of viscosity
 p = pressure
 p_0 = pressure of ambient medium (atmospheric pressure)
 p_a = supply pressure
 p_1 = pressure in lubricant film at feed orifice or feed slot
 p_i = pressure immediately after feed orifice
 p_i^* = pressure in reservoir ($u = 0$)
 p_{0a} = stagnation pressure ($u = 0$) in front of feed orifices
 p_m = mean pressure
 p_M = maximum or minimum pressure
 p_0^* = pressure at $\theta = 0$ for journal bearings

- q_1, q_2, q_3 = orthogonal curvilinear coordinates
 q = exponent characterizing the dependence of the viscosity on the thickness
 h ($\mu = \mu_1 (h/h_1)^{-q}$)
 r = radius
 r_1 = journal radius
 r_2 = bearing radius
 r_1 = feed-pocket radius (thrust bearings)
 r_0 = outer radius (thrust bearings)
 t = relative temperature
 t = time
 u, v, w = velocity components along x, y and z axes
 $\bar{u}, \bar{v}, \bar{w}$ = mean velocities in time
 u', v', w' = turbulent fluctuations of velocities
 v_r, v_θ, w = velocity components in cylindrical coordinates
 v_r, v_θ, v_ϕ = velocity components in spherical coordinates
 $u_\delta, v_\delta, w_\delta$ = velocity components in a porous medium of thickness h
 v_r = radial velocity
 x, y, z = Cartesian coordinates
 $\bar{x}, \bar{y}, \bar{z}$ = dimensionless Cartesian coordinates
 $x_\delta, y_\delta, z_\delta$ = coordinates in a porous medium of thickness δ
 x^* = distance to point of application of resultant pressure (thrust bearings)
 $y = x + iz$, complex variable (Chapt. II)
 $p = (1 - \epsilon^2) p/K_1$, pressure function (journal bearings)
 $\bar{p} = p/a_1 f(x)$, pressure function
 \bar{p} = Weierstrass function
 v = volume
 v = specific volume
 Δ = feed-pocket depth
 Θ = pad angle of journal (partial journal bearings)
 Φ = dissipation function (Chapt. II)
 Φ = complex potential ($\Phi = \eta + i\eta'$) (Chapt. IV)
 Ω_1 = angular velocity of journal
 Ω_2 = angular velocity of bearing
 $\Omega = \Omega_1 + \Omega_2 - 2\dot{\phi} = \omega - 2\dot{\phi}$, equivalent velocity of rotation
 α_s = heat convection coefficient
 α = apex angle of cone (conical bearings)
 α_d = flow coefficient
 α = angle between loading and median cross-section of journal (partial journal bearings)
 γ = Sommerfeld's variable for journal bearings
 $\left(1 + \epsilon \cos \theta = \frac{1 - \epsilon^2}{1 - \epsilon \cos \theta}\right)$

- γ = angle between resultant pressure and a fixed direction
 γ^* = angle between external load and a fixed direction
 δ = thickness of boundary layer in the vicinity of lubricated surfaces
 δ = thickness of porous surface through which the gas is pressurized
 δ = angle between journal and bearing axes
 ϵ = relative eccentricity $\left(\epsilon = \frac{e}{c}\right)$
 ϵ' = angle between the vector radii of a point on the journal that connect this point with the bearing center and the journal center, respectively
 $\epsilon_1 = \nu l / h_1$, modified relative inclination
 ϵ_d = compressibility coefficient
 $\epsilon = \frac{h_1 - h_2}{h_1 + h_2}$, equivalent eccentricity (thrust bearings)
 ξ_1, ξ_2, ξ_3 = Lamé's coefficients
 ξ = loss coefficient
 $\zeta = P/p_0 S$, load coefficient (S being the projected area of the bearing)
 $\zeta_a = P/p_a S$, load coefficient referred to feed pressure
 $\zeta_{red} = P/S (p_a - p_0)$, load coefficient for pressurized bearings
 $\zeta^* = \zeta/H$, modified load coefficient
 $\zeta_* = P/p_0^* S$, coefficient ζ defined with respect to pressure p_0^* (journal bearings)
 $\sigma_x, \sigma_y, \sigma_z$ = normal stresses
 $\eta = p^{\frac{1}{\kappa} + 1}$, pressure function
 $\tilde{\eta} = h^{\frac{3}{2}} \eta$, modified η function
 $\bar{\eta} = \frac{r}{r_1} \left(\frac{p}{p_1} \right)$, function of pressure and of radius
 θ = angular coordinate
 $\bar{\theta} = x/r_1$, angle measured from line of centers (journal bearings)
 $\bar{\theta} = x/r_1$, ibidem (only in Chapt. II, paragraph 2.6.3)
 θ^* = angle between resultant pressure and line of centers (attitude angle)
 ϑ^* = angle between external load and line of centers
 κ = polytropic exponent of gas flow ($1 < \kappa < k$)
 κ_T = heat conduction coefficient
 κ_p = polytropic exponent of gas flow in a porous medium ($\kappa_p \cong 1$)
 $\lambda = b/2r_1$, aspect ratio (journal bearings)
 $\lambda = b/l$, aspect ratio (thrust bearings)
 $\lambda_v = \frac{U}{c^*}$, dimensionless velocity
 $\chi = \psi \text{Re}^*$, ratio of inertia- and viscous forces; $\psi = \frac{hm}{l}$
 μ = gas viscosity
 μ' = second viscosity coefficient

μ_d = contraction coefficient
 ν = kinematic viscosity coefficient ($\nu = \mu/\rho$)
 $\nu = -dh/dx$, inclination of surfaces in bearings consisting of plane surfaces
 ρ = density
 ρ_1 = density at lubricant-film inlet (at feed orifice or feed slot)
 ρ_0 = density of ambient medium
 $\tau_{xy}, \tau_{yz}, \tau_{xz}$ = shear stresses
 τ = shear stress due to relative motion of surfaces
 ϕ = angular coordinate
 ϕ = angle between line of centers and a fixed direction (journal bearings)
 $\dot{\phi} = d\phi/dt$, speed of rotation of line of centers
 $\psi = h_m/l$, ratio of mean thickness h to a characteristic length l
 $\psi = c/r_1$, clearance ratio (journal bearings)
 ω = angular speed of rotating axis
 $\omega = \Omega_1 + \Omega_2$, for journal bearings, when the journal rotates at a speed Ω_1 and the bearing at a speed Ω_2
 $\omega = \Omega_1$, speed of rotation of the journal when the bearing is fixed ($\Omega_2 = 0$)
 $\omega_0^* = 1/t_0$, characteristic reference frequency for unsteady flows
 ω_{*c} = natural frequency (thrust bearings)
 $\omega_{*c} = \sqrt{K_1/m_1}$, natural frequency (journal bearings)

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