Lecture Notes in
Control and
Information Sciences

Edited by M. Thoma and A. Wyner

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FOREWORD

The last decade has seen major advances in the theory and practice of control engineering. New algorithms such as self-tuning regulators have been accompanied by detailed convergence analysis; graphical work-stations allow a designer to explore a wide range of synthesis methods; microprocessors have enabled the practical realization of advanced control concepts. This growth of techniques has meant that only a few universities with large departments could train research students over the whole spectrum of control. Students in smaller departments could specialize in their research topics yet fail to appreciate developments in related areas.

The U.K. Science and Engineering Research Council (SERC) has for many years sponsored a set of six Vacation Schools designed to bring together research students working in control and instrumentation and to broaden their perspective of the field. The schools are all one week long and held at six-monthly intervals over a three-year cycle. Recently the scheme has been modified slightly to provide three 'basic' courses and three 'advanced' courses, the idea being that a student whose research topic is within a certain area would attend the advanced course relating to his topic and the basic courses outside his topic. Attendance at the schools is restricted to some 50 to 60 and industrial participants are allowed to take up spare places to encourage interaction between the students and practising engineers.

The introductory schools in the cycle are Deterministic Control I (state-space methods, classical control, elements of multivariable frequency-response design methods), Computer Control (sampled data theory, computer control technology and software, elements of instrumentation) and Signal Processing for Control. The advanced schools are Deterministic Control II (optimization, numerical methods, robustness and multivariable design procedures), Instrumentation (basic technology, sensor development and application studies) and Stochastic Control (stochastic systems, adaptive control, identification and pattern recognition). Each school has lectures, examples classes and experimental sessions. Case studies showing the application of the ideas in practice are presented, often by industrial engineers.

This volume consists of the lecture notes for the school on Signal Processing for Control. This school, held every three years at the University of Warwick, has proved to be popular with the students as it successfully combines the educational role of introducing many important ideas with the motivation provided by the wide range of interesting application examples. Whilst no multi-author book can ever be completely comprehensive and consistent, the editors are to be congratulated in providing an excellent introduction and overview of an increasingly important and practical discipline.

D.W. Clarke

Oxford University

(Chairman, Control and Instrumentation Subcommittee, SERC)

PREFACE

These lecture notes are from a Vacation School held at the University of Warwick (Coventry, England) from Sunday 15th to Friday 20th September 1985. The School, sponsored by the U.K. Science and Engineering Research Council (SERC), aimed to provide an introduction to the theory and application of signal processing in the context of control systems design.

There were 42 participants, 32 of whom were research students in the area of control engineering (the majority on SERC-funded studentships), the remaining 10 being industry-based engineers involved in control engineering and related topics. Some prior knowledge of classical control theory was assumed, involving familiarity with calculus, differential equations, Fourier series, Fourier and Laplace transforms, z-transforms, frequency domain methods of linear systems analysis, and basic matrix techniques.

The School was based on a complementary set of lectures, case studies and practical sessions covering the following topics:

(i) analytical and computational techniques for characterising random signals and their effect on dynamic systems;
(ii) system identification and parameter estimation;
(iii) digital filtering and state estimation;
(iv) state/parameter estimation in feedback control.

CURRICULUM OF THE SCHOOL

The School consisted of three Revision Lectures (R1 to R3), eleven further Lectures (L1 to L11) and four Case Studies (C1 to C4). The Revision Lectures were presented on the Sunday afternoon at the start of the School and contained material which most participants would have encountered at undergraduate level; attendance at these was optional. The "main-stream" Lectures (L1 to L11) were presented from Monday through to Friday. These covered the topics listed in (i) to (iv) above, building from the material in R1 to R3 through to more advanced techniques. The four Case Study lectures were designed to illustrate the practical application of the more theoretical material in L1 to L11. Outlines of R1 to R3, L1 to L11 and C1 to C4 are given later in this Preface.

Facilities for interactive dynamic data analysis were provided via the PRIME 550 computer system installed at the University of Warwick as a part of the SERC

Interactive Computing Facility. In addition, the MATRIX-X analysis and design package was available on a SYSTIME 8780 computer at the University. Students were able to perform a series of experiments involving the analysis of random data and the modelling of dynamic systems based on accessible data files chosen to illustrate representative applications and problems.

A hardware demonstration of data analysis techniques in both the time domain and frequency domain was given on a Hewlett Packard 5420B Digital Signal Analyzer. The demonstration was devised and run by Professor W.A. Brown of the Department of Electrical Engineering, Monash University, Australia, who was on sabbatical leave in the Department of Engineering at the University of Warwick at the time of the School.

On the Wednesday afternoon of the School, participants went on an industrial visit to the Lucas Research Centre at Shirley (near Birmingham) to hear presentations of relevant research and development projects in the area of automotive systems control and to tour the engine test and other experimental facilities.

The Vacation School Dinner was preceded by a keynote address given by Professor Thomas Kailath of the Electrical Engineering Department of Stanford University, California. Professor Kailath entitled his address "Signal Processing and Control" and dealt with numerical computation aspects of signal processing (in particular, square root algorithms) together with implementation considerations involving parallel processing and VLSI. Traditionally, keynote addresses at Vacation Schools of this type are intended as an up-to-date overview of some aspects of the topic of the School. As such, lecture notes are not sought and none are available for Professor Kailath's talk.

MATERIAL COVERED IN THE NOTES

A. Revision Lectures R1 to R3

In R1, *Signal Analysis I*, basic analytical and computational techniques that are available for the characterisation of dynamic signals and data are reviewed. These are Fourier series and the Fourier transform, the Discrete Fourier transform (including the Fast Fourier Transform algorithm), the Laplace transform, sampled data and the z-transform and a brief overview of random signal analysis and estimation errors.

Methods for characterising dynamic systems are discussed in R2, *Systems Analysis I*. These include differential equation representation, impulse response and convolution in the time domain, frequency response and methods of determining frequency responses, and the (Laplace) transfer function. Sampled data systems are also covered, with material on difference equations, pulse transfer functions, zero order hold elements, convolution sum and the estimation of unit pulse response using cross-correlation.

One of the primary aims of R3, *Matrix Techniques*, is to standardise notation and terminology of basic matrix concepts for subsequent lectures at the School. The use of vector-matrix concepts in studying dynamic systems is discussed, in particular the transfer function matrix and the state transition matrix. Vector-matrix difference equations for sampled data systems are described and the notes conclude with discussions of quadratic forms and diagonalisation, Taylor series, maxima and minima and multiple linear regression.

B. <u>Lectures L1 to L11</u>

In L1, *Relevant Probability Theory*, the main concepts of probability theory applied to the characterisation of scalar and vector random variables and random signals are outlined. Both discrete and continuous random variables are considered and, as well as single variable probability distributions and density functions, joint and conditional distributions are defined and illustrated with examples. Uses of the characteristic function are described and aspects of vector random variables are discussed, including marginal densities, vector moments and normal random vectors. The notes conclude with a brief discussion of stochastic processes, including aspects of stationarity.

Basic concepts in mathematical statistics and some of their applications in the analysis of signals and dynamic systems are described and illustrated in L2, *Relevant Statistical Theory*. Bias, variance, consistency and efficiency of an estimate are defined and methods of hypothesis testing and establishing confidence intervals are described, with illustrative examples. The Cramer-Rao bound and maximum likelihood estimation are discussed and the notes conclude with a discussion of optimal estimation techniques.

The emphasis in L3, *Systems Analysis II*, is on the use of autocorrelation and crosscorrelation in the time domain and the corresponding Fourier-transformed quantities, the power spectral density and cross-spectral density function in the frequency domain. The response of linear systems to stationary random excitation is considered, in particular methods for determining output power spectrum for a system with a specified (Laplace) transfer function excited by an input signal with a specified power spectrum. Corresponding quantities for discrete-time systems are also described.

An important problem in experiment planning is that of deciding in advance how much data must be collected to achieve a given accuracy. The considerations that affect the question are discussed in L4, *Signal Analysis II*, for a number of data analysis procedures and it is shown how a quantitative analysis leads to useful guidelines for the design of experimental procedures involving random data. The relationships between record characteristics and probable errors are described both for time domain and frequency domain analyses.

In L5, *Design and Implementation of Digital Filters,* both finite-impulse-response (FIR) filters (also known as moving average (MA) filters) and infinite-impulse-response (IIR) filters (also known as autoregressive-moving average (ARMA) filters) are considered. Impulse-invariant design of IIR filters is described. It is shown how aliasing can affect the frequency response of such designs and a method of avoiding this inaccuracy by use of bilinear transformation is discussed. The design of FIR filters by Fourier series and windowing is described and computer-optimised FIR filters are discussed. Problems of quantisation and rounding, which are of such practical importance in digital filtering, are also considered.

Statistical techniques for the estimation of parameters of dynamic systems from input-output data are described in L6 *Parameter Estimation.* In the section on non-recursive estimation, emphasis is placed on maximum-likelihood estimation and a problem in linear regression, that of estimating the pulse response sequence of a system, is considered in some detail. Recursive least squares is discussed, in particular, how to avoid direct matrix inversion. The notes conclude with a brief discussion of nonlinear regression.

The theme of recursive methods is continued in L7 *Recursive Methods in Identification.* Recursive forms of standard off-line techniques are described, in particular least squares and instrumental variables. Stochastic approximation and the stochastic Newton algorithm are discussed and this is followed by sections on the model reference approach and Bayesian methods and the Kalman filter. The problems with the various approaches when the system is time-varying are described and the convergence and stability of the different algorithms are considered.

Frequency domain analysis of dynamic systems is considered in L8, *Spectral Analysis and Applications.* In the first part of the notes, several examples of auto-correlation functions and corresponding (continuous) power spectra of waveforms are given and spectral relationships in closed loop systems are considered. The problems of digital spectral analysis are then reviewed. Some of the statistical properties of spectral estimates are discussed and the notes conclude with a brief description of cepstral analysis.

In the first part of L9, *Observers, State Estimation and Prediction,* the Luenberger observer is described in some detail, with asymptotic and reduced order observers being discussed. The closed loop properties of a system in which a stable asymptotic observer is applied to an otherwise stable control system design are considered. The Luenberger observer arose with regard to state estimation for deterministic, continuous-time systems; the emphasis of the notes now switches to discrete time systems, in which any noise that affects the system is directly taken into account. Successive sections of the notes deal with the Kalman filter, prediction and smoothing.

The problems introduced by nonlinearities are considered in L10, *Introduction to Nonlinear Systems Analysis and Identification*. Static nonlinearities are discussed in the first part of the notes. Nonlinear systems with dynamics are then considered, in particular the Volterra series representation. The inherent complexity of the analysis has led to the development of approximation methods based on linearisation techniques and these are described. Identification algorithms for nonlinear systems, considered next, can be categorised as functional series methods, algorithms for block oriented systems and parameter estimation techniques. Some of the ideas presented are illustrated by a practical application in which the relationship between input volume flow rate and level of liquid in a system of interconnected tanks is identified. The notes conclude by considering control of nonlinear sampled data systems.

The final lecture, L11, *An Introduction to Discrete-time Self-tuning Control*, provides a tutorial introduction to self-tuning control in its traditional discrete-time setting. The notes start by considering a slightly modified version of the self-tuning regulator of Åström and Wittenmark, the modifications including contro weighting and set-point following. A weighted model reference controller is then considered and finally a pole placement self-tuning controller is discussed. All three approaches are viewed within a common framework, namely that of emulating unrealisable compensators using a self-tuning emulator.

C. Case Studies C1 to C4

In C1, *Exploring Biological Signals*, some applications of systems techniques to biomedicine are described; in the examples described, signal processing and modelling are confined to one-dimensional time series. In the first part of the notes, the modelling of signals is considered. This is illustrated by the application of Fast Fourier Transforms, Fast Walsh Transforms, autoregressive modelling, phase lock loops and raster scanning to electrical signals from the gastrointestinal tract and by the analysis and subsequent modelling of the blood pressure reflex control system (part of the cardiovascular system). In the second part, the modelling of systems (as distinct from signals) is illustrated by two examples, the first the determination of lung mechanics and the second the identification of muscle relaxant drug dynamics. The latter is part of studies aimed at achieving on-line identification and control in the operating theatre.

Engineering surfaces have in their manufacture a large proportion of random events, and the study of surfaces, either for understanding of tribology or as a means of manufacturing control, provides a very interesting application of random process theory and spectral estimation. A range of such applications is illustrated in C2, *Stochastic Methods and Engineering Surfaces*. After a review of methods of modelling surfaces, subsequent sections deal with profile statistics, roughness parameters and profile filtering. Surface classification techniques are then described and these include the shape of autocorrelation functions, the first two even moments of the power spectral density and the skew and kurtosis of the amplitude probability density function. The notes conclude with a more detailed discussion of spectral analysis of surfaces.

Experiences gained in six applications of identification are described in C3, *Practical Problems in Identification*. The processes ranged from a steelworks blast furnace to a gas turbine engine, from an oil refinery distillation column to a human being. It is shown that while useful estimates of the dynamics of systems in industry can sometimes be obtained from simple step responses, noise is often at such a level that signals with impulse-like autocorrelation functions are needed, but that direction-dependent dynamic responses can then be a problem. If normal operating records are used, problems can arise if feedback is present and this may not be very obvious in some instances. For sampled records, the spacing of samples may mean that some parameters of a model are estimated with low accuracy. Finally, when trying to estimate the parameters of an assumed nonlinearity, it is essential that the data available adequately span the nonlinear characteristic.

The final Case Study, C4, *LQG Design of Ship Steering Control Systems*, is concerned with the control of course of a ship in the face of disturbances from ocean currents and sea waves. Modelling of the ship, wind, wave and steering gear and then the combined model of ship and disturbance are described. The cost function is formulated and the existence of the solution to the LQG (Linear, Quadratic, Gaussian) problem is investigated. The Kalman filter and controller design are then described and then simulation results are presented. It was found that one of the main problems was to design a Kalman filter which would estimate the ship motions; with the disturbance model changing significantly in different sea conditions, a fixed gain Kalman filter may not give an adequate estimation accuracy.

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NOMENCLATURE

One of the main drawbacks to the usefulness of many multiple-author texts is that different authors may use different symbols for the same quantity, which confuses most readers who are relatively new to the field and can even cause confusion among those familiar with the field. To remove this drawback from this text, the editors asked all authors to adhere to a specified nomenclature which is listed below.

1. GENERAL

$j = \sqrt{-1}$; F^* = complex conjugate of F

s = Laplace transform operator

F(s) = Laplace transform of f(t)

(alternatively (script) $\mathcal{L}[f(t)]$).

z = z-transform and shift operator

$F(z)$ = z-transform of $f(nT) = \sum_{n=0}^{\infty} f(nT).z^{-n}$ (or (script) $\mathcal{Z}(\ )$).

T = sampling interval

$t = 0^+$: time immediately after (but not including) t = 0.

$t = 0^-$: time immediately before t = 0.

$\delta(\ )$ = Dirac delta function.

u = system input (see also x)

y = system output

n = order of system

ω_0, ζ = 2nd. order system parameters

ϕ = phase angle

h(t) = unit impulse response of a system

x = state variable (also used for system input where no confusion with state variable is possible).

2. MATRICES

Matrices and vectors not usually underlined, except where confusion between these and scalar quantities might occur (e.g. in describing the Kalman filter).

Vectors are column vectors, except where specifically designated as row vectors.

Superscript T for transpose.

det A or $|A|$ = determinant of A

μ_{ij} = minor of element a_{ij}

γ_{ij} = cofactor of element a_{ij}

AdjA = Adjugate (adjoint) of A.

Tr(A) = Trace of A.

J = Jacobian matrix, with element $\{i,j\} = \frac{\partial y_i}{\partial x_j}$.

$\Phi(t)$ (or $\Phi(t,t_0)$ when appropriate)

= state transition matrix

λ = eigenvalue

Λ = diagonal matrix with eigenvalues along principal diagonal

v = column eigenvector

w = row eigenvector

Q = Quadratic form = $x^T Ax$

C = Curvature matrix, with element $\{i,j\} = \frac{\partial^2 f}{\partial x_i \partial x_j}$

Matrices for Kalman filter: See Section 6.

3. PROBABILITY AND ESTIMATION

P(A) = Probability of A.

P(A|B) = Conditional probability of A, given B

$P(X_i,Y_j)$ = joint probability distribution = $P(x = X_i$ and $y = Y_j)$.

Binomial distribution: p = probability of success in one trial

q = probability of failure in one trial

P(r) = Probability of r successes in n trials = $\frac{n!}{r!(n-r)!} p^r q^{n-r}$

Poisson distribution: ν = average number of events in unit time

P(r) = Probability of r events in time interval T = $\frac{(\nu T)^r}{r!} e^{-\nu T}$

$\bar{x}$ or μ_x = Mean value of x = E[x]

E[ ] = Expected value of quantity in square brackets.

Var[x] or σ_x^2 = Variance of x = $E[(x-\bar{x})^2]$

$f(X)$ = probability density function of x

$F(X)$ = cumulative distribution function = $\int_{x_{min}}^{X} f(X)dX$

$f(X|A)$ = Conditional probability density function

where $f(X|A)dX = P(X \le x < X+dX$, given the event A)

C_k = k'th central moment of p.d.f. about the mean

$= \int_{x_{min}}^{x_{max}} (X-\mu_x)^k f(X)dX = E[(x-\mu_x)^k]$

m_{kr} = joint moment = $E[x^k y^r]$

$Cov[x,y]$ or σ_{xy}^2 = covariance between x and y

$= E[(x-\mu_x)(y - \mu_y)]$.

ρ_{xy} = correlation coefficient = $\frac{\sigma_{xy}^2}{\sigma_x . \sigma_y}$

$\Phi(\omega)$ = characteristic function of a continuous variable x

$= E[\exp(j\omega x)] = \int_{x_{min}}^{x_{max}} f(X)\exp(j\omega X)dX$.

ε_t = Noise sequence

θ = Unknown parameter vector

$\hat{\theta}$ = estimate of θ

$L(z,\theta)$ = Likelihood function of observations z

(script) $\mathcal{L}(z,\theta)$ = Log. likelihood function of observations z.

4. TIME DOMAIN

τ = Time shift

$R_{xx}(\tau)$ or $R_x(\tau)$ = autocorrelation function of $x(t)$

$= E[x(t).x(t+\tau)]$.

$R_{xy}(\tau)$ = crosscorrelation function between $x(t)$ and $y(t)$

$= E[x(t).y(t+\tau)]$

$C_{xx}(\tau)$ or $C_x(\tau)$ = autocovariance function of $x(t)$

$= E[(x(t)-\mu_x)(x(t+\tau)-\mu_x)]$ for stationary $x(t)$

$\rho_{xx}(\tau)$ or $\rho_x(\tau)$ = normalised autocovariance function = $C_{xx}(\tau)/C_{xx}(0)$

$C_{xy}(\tau)$ = crosscovariance between x(t) and y(t)

$= E[(x(t)-\mu_x)(y(t+\tau)-\mu_y)]$

λ = Basic interval (bit interval, clock pulse interval) of discrete interval random binary signal or pseudo-random binary signal.

V = amplitude (i.e. ± V) of binary signal

$\oplus$ = modulo 2 addition

$Q_n(a)$ = n'th order Hermite polynomial

5. FREQUENCY DOMAIN

Fourier transform pair:

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \text{ or } F(jf) = \int_{-\infty}^{\infty} f(t)e^{-j2\pi ft}\, dt$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)e^{j\omega t}d\omega \text{ or } f(t) = \int_{-\infty}^{\infty} F(jf)e^{j2\pi ft}\, df.$$

Fourier transform can also be written as (script) $\mathcal{F}[f(t)]$.

N.B! Fourier transform taken as two-sided except where specifically stated otherwise.

Discrete Fourier transform (DFT):

$$F_D\,(k\Omega) = \sum_{n=0}^{N-1} f(nT)\exp(-j\Omega Tnk).$$

$S_{xx}(\omega)$ or $S_x(\omega)$ = Power spectral density function, i.e. Fourier transform of $R_{xx}(\tau)$ (or $R_x(\tau)$).

(or $S_{xx}(f)$ or $S_x(f)$)

$S_{xy}(j\omega)$ (or $S_{xy}(jf)$) = Cross-spectral density function

= Fourier transform of $R_{xy}(\tau)$.

$H(j\omega)$ = System frequency response = Fourier transform of h(t).

$\gamma_{xy}^2(\omega)$ = coherence function = $|S_{xy}(j\omega)|^2/[S_{xx}(\omega).S_{yy}(\omega)]$.

B = cyclic bandwidth (Hz).

6. KALMAN FILTERS AND SELF-TUNING REGULATORS

6.1. Continuous time Kalman filter

Plant model: $\dot{x}(t) = Ax(t) + Dw(t)$

$z(t) = Cx(t) + v(t)$

$E[x(0)] = m_0$; $Cov[x(0), x(0)] = \Sigma_0$

$Cov[w(t), w(t)] = Q$; $Cov[v(t), v(t)] = R$

Filter: $\dot{\hat{x}}(t) = A\hat{x}(t) + K(t)[z(t) - C\hat{x}(t)]$

$K(t) = P(t)C^T R^{-1}$

6.2 Discrete time Kalman filter

Plant model: $x(k+1) = F\,x(k) + hw(k)$

$z(k) = Hx(k) + v(k)$

$E[x(0)] = m_0$; $Cov[x(0), x(0)] = \Sigma_0$

$Cov[w(k), w(k)] = Q$; $Cov[v(k), v(k)] = R$

Filter: $\hat{x}(k) = F\hat{x}(k-1) + K(k)[z(k) - H\hat{x}(k-1)]$

$K(k) = P(k)H^T[R + HP(k)H^T]^{-1}$

6.3 Self tuning regulator

Plant model: $y = \frac{z^{-k}\,B(z^{-1})}{1+A(z^{-1})}\,u + \frac{C(z^{-1})}{1+A(z^{-1})}\,e$

Regulator: $u = \frac{G(z^{-1})}{1+F(z^{-1})}\,y.$