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ТРУДЫ

ордена Ленина

МАТЕМАТИЧЕСКОГО ИНСТИТУТА

имени В. А. СТЕКЛОВА

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Abstract. The papers comprising this collection are devoted to various present-day questions of algebraic and general topology, and their applications to different domains of mathematics.

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*The American Mathematical Society scheme for transliteration of Cyrillic may be found at the end of index issues of *Mathematical Reviews*.

FROM THE EDITOR

The International Topology Conference organized by the Academy of Sciences of the USSR was held at the Steklov Institute of Mathematics and Moscow State University from June 25 to 29, 1979.

About 170 mathematicians took part in the conference, including about 70 from abroad, and all the main directions in comtemporary topology and its applications were represented.

The organizing committee of the conference asked the authors of the most interesting reports to write papers based on these reports, with a view to publishing them later in a Soviet journal. Part of these articles have already been published (see Russian Math. Surveys **34** (1979), no. 6, and **35** (1980), no. 3; detailed information on the conference is also contained there), and part make up the present issue.

Below I reproduce (with minimal changes) the speech I gave at the opening of the conference (published in the cited 1979 number of Russian Math. Surveys).

The very first International Topology Conference took place in Moscow in August 1935. This was indeed a brilliant gathering of many of the best topologists in the world.

The main purpose of the 1935 conference was to represent topology as a whole, as a unified mathematical discipline, and to promote an active interpenetration of the two main directions of this discipline: the combinatorial-algebraic direction and the set-theoretic direction. The periods in which this interpenetration was especially intensive, the periods of synthesis of algebraic and set-theoretic topology, are in my opinion among the most productive in the development of our area of mathematics. It is to these periods that I want to devote a few words, only now and then touching lightly on certain other important moments in the development of topology.

The first of these periods is that of the immortal work of Brouwer in topology, mainly between 1909 and 1913. His striking geometric intuition, combined with his powerful set-theoretic thinking and set-theoretic imagination, enabled him to create in topology a new method, the famous Brouwer mixed method (méthode mixte), by repeated use of simplicial approximations and the degree of a mapping (first defined by him). Precisely this method led to the first synthesis of combinatorial-algebraic and set-theoretic topology.

Brouwer's work from 1909 to 1913 is, as it were, bracketed by two monumental mathematical creations which laid the foundation for general topology: the work of Fréchet in 1907, in which metric spaces were defined along with the properties of compactness and completeness for them, and Hausdorff's book in 1914, which provided the basis for the theory of topological spaces. Moreover, in 1913 Janiszewski published an article on irreducible continua which laid the groundwork for a large chapter of topology, the so-called topology of continua, which we have seen blossom in Poland and later in the USA. After the construction of the first indecomposable continua by Brouwer in 1909, I regard the construction of a hereditarily indecomposable continuum by Knaster and the proof by Bing that such continua are topologically unique (the "pseudoarc") as the highest achievements of this branch of topology. In 1921 the topology of continua closed ranks with dimension theory, which was constructed in the same year by Urysohn and Menger and for many years constituted one of the most remarkable and popular areas of topology. Between 1922 and 1924 general topology reached an essentially new level. Because of Kuratowski's definition of the most general topological spaces, the construction of the theory of compact spaces, and proofs of the first basic metrization theorems along with the propositions which border them (for example, Urysohn's lemma), 1922 is also notable for a proof of one of the most remarkable theorems in algebraic topology: the famous duality principle of Alexander, who discovered a number of duality theorems. Thus, by the end of the first half of the 1920's enough had happened both in set-theoretic topology and in combinatorial topology that the time had come for a new, second synthesis of the two main directions in topology.

The beginning of this new synthesis was the definition in 1925 of the nerve of a covering of a topological space. The finite canonical coverings of a (compact metric) space, considered together with their natural order, enable us to connect the nerves of these coverings by simplicial mappings and thereby obtain the so-called projective spectrum of the space.

The projective spectra make up a particular case of inverse spectra, and it was in this special particular case that one of the most important concepts in contemporary set-theoretic mathematics arose: the concept of an inverse spectrum.

The projective spectrum of a space made it possible to reduce the topology of the space to properties of simplicial complexes and their simplicial mappings, properties of a combinatorial nature in essence.

This made it possible, in particular, to determine the homology invariants of a (compact metric) space by reducing them to the corresponding invariants of the complexes that are the nerves of refining coverings of the space. This method of determining homology invariants was carried over to arbitrary spaces by Čech in 1932 in his famous paper "Théorie générale d'homologie". At about the same time as the determination of homology invariants of compact metric spaces with the help of nerves of coverings, Vietoris constructed his metric theory of homology in compact metric spaces, based on the concepts of an ε-cycle and of ε-homology and constituting a far-reaching development of ideas of Brouwer presented in the latter's brilliant note "Invarianz der geschlossenen Kurve".

The Alexander duality theorem and the availability of the homology invariants of compact metric spaces led in 1927 to the proof of the first duality theorem of Alexander type for all compact sets in Euclidean spaces. However, the first really fundamental progress in duality theory after Alexander's theorem was achieved only later in 1932 by the proof of the famous Pontryagin duality principle, which was epochal both in topology and in topological algebra. The homological theory of dimension for compact metric spaces was constructed at the same time (1930–1932).

The homological and, in general, combinatorial-algebraic topology of compact metric spaces that took shape as a result of these investigations infused the work relating to the second period of synthesis of algebraic and set-theoretic topology with a concrete geometric content. The conclusion of this second period was marked by the emergence, on the one hand, of a theory of homological properties of the disposition of complexes and closed sets (in compact Hausdorff spaces) in 1943, and, on the other hand, of a duality theory for nonclosed sets in Euclidean spaces, worked out at the end of the 1940's.

The algebraic topology of both polyhedra and topological spaces as a whole was raised to an essentially new level by the creation by Alexander and Kolmogorov in 1934 of the concept of cohomology and the subsequent construction of the theory of cohomology and cohomology operations.

Enormous progress in general topology was achieved in 1928 and from 1934 to 1936 by the work of Tychonoff and M. Stone and Cech, respectively.

Immediately after the end of the war there began a period of stormy development both of algebraic and differential topology and of purely set-theoretic topology. But I shall not go into all this.

There has also been a new, third period of synthesis of set-theoretic and algebraic topology. This period is continuing at present. It began with the creation by Borsuk of the theory of retracts, and continued with the creation, also by Borsuk, of the theory of shapes.

Both the theory of retracts and the theory of shapes relate to general topology in their subject matter, but both have an explicitly expressed geometric character. The theory of shapes is in essence a set-theoretic form of homotopic topology, and it is connected with cohomology theory and, consequently, with algebraic topology. On the other hand, there are exceedingly close connections between the theory of shapes and one of the most important parts of "infinite-dimensional" topology, namely, the theory of so-called Q-manifolds, i.e., compact metric spaces that are locally homeomorphic to the Hilbert cube.

The theory of inverse spectra, today one of the most powerful methods of investigation and construction in topology, has also penetrated essentially into the theory of shapes in its present form. The very substantial development of the theory of inverse spectra in the most recent years is due first and foremost to Shchepin and, first of all, to his spectral theorem asserting that under reasonable hypotheses two uncountable inverse spectra have homeomorphic limit spaces only when they contain isomorphic cofinal subspectra. This theorem makes it possible to solve the problem of whether two spaces are homeomorphic in a number of important concrete cases.

The spectral theorem enabled Shchepin to construct an "uncountable" version of the theory of infinite-dimensional manifolds, namely, the theory of so-called Tychonoff manifolds, i.e., compact Hausdorff spaces locally homeomorphic to a Tychonoff cube of a given uncountable weight τ. In some of its parts this theory is analogous to the theory of Q-manifolds, but in other parts it is quite unlike the latter.

Shchepin's theorem has an essentially "uncountable" character: there is no analogous theorem for countable spectra.

It follows from the foregoing that the substance of the synthetic interpenetration of the main directions in topology has changed with time in the most recent decades of its development and existence, yet has always been one of the basic conditions for real progress in our area of mathematics. Despite the heterogeneity of its content, topology as a mathematical discipline has always been unified, and permit me to express today my certainty that success in advancing it not only at present but also in the future resides precisely in this unity. Such advancement is stimulated by conferences like the first topology conference in 1935 and the present conference, because it is precisely in the unity, i.e., in the combination of different trends within a given area, that the basic meaning of large international scientific gatherings is found.

In conclusion allow me to express my wish and hope that this conference will be a significant new stage in the development of our area of knowledge.

P. S. Aleksandrov

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PARAMETRIC CANONICAL HOMOLOGY AND COHOMOLOGY GROUPS OVER PAIRS OF COPRESHEAVES AND PRESHEAVES, RESPECTIVELY

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D. O. BALADZE

ABSTRACT. Duality theorems are established for parametric canonical homology and cohomology groups of a locally compact metrizable space over pairs of copresheaves and presheaves, respectively.
Bibliography: 4 titles.

Let R be a locally compact metrizable space, C a closed subspace of R, and $\omega = \{(U_\alpha, V_\alpha)\}$ a system, directed by refinement, of canonical coverings of the pair of spaces (R, C) (see [1]). It is known (see [1]) that instead of a system $\omega = \{(U_\alpha, V_\alpha)\}$, directed by refinement, of coverings of the pair of spaces (R, C) one can take a system $\Omega = \{(\tilde{U}_\alpha, \tilde{V}_\alpha)\}$, directed by refinement, of open coverings $(\tilde{U}_\alpha, \tilde{V}_\alpha)$ of (R, C), in which the closed sets $u_\alpha \in U_\alpha$ and $v_\alpha \in V_\alpha$ in the coverings (U_α, V_α) are replaced by the open sets $(\tilde{u}_\alpha, \tilde{v}_\alpha)$, $\tilde{u}_\alpha \in \tilde{U}_\alpha$, $\tilde{v}_\alpha \in \tilde{V}_\alpha$, differing little from them and such that the relations of refinement of the coverings and the structures of the nerves of these coverings remain intact. By K_α we denote the nerve of the covering $\tilde{U}_\alpha$ of R, and by L_α the nerve of the covering $\tilde{V}_\alpha$ of C. Further, let K be an arbitrary locally finite complex, let (A, A') and (B, B') be conjugate pairs of copresheaves and presheaves, respectively, with base R (see [2]), and let p be an integer.

We consider the set $x = \{x_\tau\}$ of chains x_τ of the complex K_α over the pair of copresheaves (A, A') (see [2]), defined for each simplex $\tau \in K$ and possessing the property that $\dim x_\tau = p + \dim \tau$. We shall call such a set of chains $x = \{x_\tau\}$ a *p-dimensional parametric chain* of the complex K_α over the pair of copresheaves (A, A'), if for almost all simplexes $\tau \in K$ the coefficients of the chains x_τ lie in the corresponding subgroups $A'(|t|)$, $A'(|t|) \subset A(|t|)$. With respect to the operation of addition $(x + y)_\tau = x_\tau + y_\tau$, the set of all p-dimensional parametric chains of the complex K_α over the pair of copresheaves (A, A') is a group, which we shall denote by $C_p^K(K_\alpha; A, A')$ and which we shall call the *group of p-dimensional parametric chains* of the complex K_α over the pair of copresheaves (A, A'). We denote by $C_p^K(L_\alpha; A, A')$ the group of p-dimensional parametric chains of the complex L_α over the pair of copresheaves (A, A'). The factor group $C_p^K(K_\alpha; A, A')/C_p^K(L_\alpha; A, A')$ is denoted by $C_p^K(K_\alpha, L_\alpha; A, A')$ and will be called the *group of relative p-dimensional*

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parametric chains of the complex K_α modulo L_α over the pair of copresheaves (A, A'). As $\partial \cdot \partial = 0$, we obtain a chain complex $\{C_p^K(K_\alpha, L_\alpha; A, A')\}$, whose homology group is denoted by $H_p^K(K_\alpha, L_\alpha; A, A')$ and called the *p-dimensional parametric relative homology group* of the complex K_α modulo L_α over the pair of copresheaves (A, A').

Now let $C_K^p(K_\alpha, L_\alpha; B, B')$ be the group of relative parametric p-dimensional cochains of the complex K_α modulo L_α over the pair of presheaves (B, B') (see [2]), i.e. that consisting of those p-dimensional parametric cochains $y = \{y^\tau\}$, whose values on L_α are trivial and such that for almost all simplexes $\tau \in K$ the coefficients of the cochains y^τ lie in the corresponding subgroups $B'(|t|) \subset B(|t|)$. Here also, as $\delta \cdot \delta = 0$, we obtain a cochain complex $\{C_K^p(K_\alpha, L_\alpha; B, B'), \delta\}$, whose cohomology group is denoted by $H_K^p(K_\alpha, L_\alpha; B, B')$ and called the p-dimensional parametric relative cohomology group of the complex K_α modulo L_α over the pair of presheaves (B, B').

The following can be proved.

THEOREM 1. *If the pairs of copresheaves and presheaves (A, A') and (B, B') are conjugate, then the relative parametric homology and cohomology groups $H_p^K(K_\alpha, L_\alpha; A, A')$ and $H_K^p(K_\alpha, L_\alpha; B, B')$ of the complex K_α modulo L_α over the pairs of copresheaves and presheaves (A, A') and (B, B'), respectively, are dual, i.e.*

$$H_p^K(K_\alpha, L_\alpha; A, A') | H_K^p(K_\alpha, L_\alpha; B, B').$$

If $\alpha < \beta$; $\alpha, \beta \in \tau$, then the locally finite simplicial map $\rho_\alpha^\beta\colon K_\beta \to K_\alpha$ defines the homomorphisms

$$\rho_\alpha^{*\beta}\colon H_p^K(K_\beta, L_\beta; A, A') \to H_p^K(K_\alpha, L_\alpha; A, A')$$

and

$$\pi_{\alpha\beta}^*\colon H_K^p(K_\alpha, L_\alpha; B, B') \to H_K^p(K_\beta, L_\beta; B, B').$$

These groups and homomorphisms form an inverse spectrum

$$\{H_p^K(K_\alpha, L_\alpha; A, A'), \rho_\alpha^{*\beta}\}$$

and a direct spectrum

$$\{H_K^p(K_\alpha, L_\alpha; B, B'), \pi_{\alpha\beta}^*\}.$$

We call, by definition, the limit groups of these spectra the *canonical relative p-dimensional homology* and *cohomology groups* of the space R modulo C over the pairs of copresheaves (A, A') and presheaves (B, B'), respectively. We shall denote these groups by $H_p^K(R, C; A, A')$ and $H_K^p(R, C; B, B')$.

Here the following can be proved.

THEOREM 2. *If the pairs (A, A') and (B, B') are conjugate, then the relative parametric canonical p-dimensional homology and cohomology groups $H_p^K(R, C; A, A')$ and $H_K^p(R, C; B, B')$ of the space R modulo C over the pairs of copresheaves (A, A') and presheaves (B, B'), respectively, are dual, i.e.*

$$H_p^K(R, C; A, A') | H_K^p(R, C; B, B').$$

The proof relies on Theorem 1 and on the conjugacy of the homomorphisms $\rho_\alpha^{*\beta}$ and $\pi_{\alpha\beta}^*$.

In the case where the parameter K consists of a single point e, i.e. $K = e$, the canonical relative parametric p-dimensional homology group $H_p^K(R, C; A, A')$ of the space R modulo C over the pair of copresheaves (A, A') coincides with the relative canonical p-dimensional homology group $H_p(R, C; A, A')$ of the space R modulo C over the pair of copresheaves (A, A') defined by us in [3], and the relative canonical parametric cohomology group $H_K^p(R, C; B, B')$ of the space R modulo C over the pair of presheaves (B, B') coincides with the relative canonical cohomology group $H^p(R, C; B, B')$ of the space R modulo C over the pair of presheaves (B, B') (see [3]).

Let (K_α, L_α) be again the pairs of nerves of the pairs of coverings $(\tilde{U}_\alpha, \tilde{V}_\alpha)$, $(\tilde{U}_\alpha, \tilde{V}_\alpha) \in \Omega$, of the pair of spaces (R, C), and $C_p^K(K_\alpha, L_\alpha; A, A')$ the group of relative parametric p-dimensional chains of the complex K_α modulo L_α over the pair of discrete copresheaves (A, A'). Further, let f be the map of the pair (A, A') onto the discrete pair of copresheaves (B, B') for which $f_u(A(u)) = B(u)$ and $f_u(A'(u)) = B'(u)$. Denote $F(u) = \operatorname{Ker} f_u$ and $F'(u) = \operatorname{Ker}(f_u/A'(u))$. In this case the pair of copresheaves (F, F') with base R is obtained. Further, the exact sequence

$$0 \to (F, F') \to (A, A') \to (B, B') \to 0$$

determines the exact sequence

$$\begin{aligned} 0 \to C_p^K(K_\alpha, L_\alpha; F, F') &\to C_p^K(K_\alpha, L_\alpha; A, A') \\ &\to C_p^K(K_\alpha, L_\alpha; B, B') \to 0 \end{aligned} \tag{1}$$

of groups of relative parametric p-dimensional chains of the complex K_α modulo L_α over the pairs of copresheaves.

Let $H_p^K(K_\alpha, L_\alpha; A, A')$ denote the relative parametric p-dimensional homology group of the complex K_α modulo L_α over the pair of copresheaves (A, A'). The exact sequence (1) and the connecting homomorphism

$$\partial\colon H_{p+1}^K(K_\alpha, L_\alpha; B, B') \to H_p^K(K_\alpha, L_\alpha; F, F')$$

define the exact homology sequence

$$\begin{aligned} \cdots &\to H_{p+1}^K(K_\alpha, L_\alpha; B, B') \to H_p^K(K_\alpha, L_\alpha; F, F') \\ &\to H_p^K(K_\alpha, L_\alpha; A, A') \to H_p^K(K_\alpha, L_\alpha; B, B') \\ &\to H_{p-1}^K(K_\alpha, L_\alpha; F, F') \to \cdots \end{aligned} \tag{2}$$

of relative parametric homology groups of the complex K_α modulo L_α, taken relative to the pairs of copresheaves. The sequence (2) is called the *relative parametric canonical homology sequence* of the complex K_α modulo L_α, generated by the epimorphism $f\colon (A, A') \to (B, B')$.

Analogously to this, the exact parametric cohomology sequence

$$\begin{aligned} \cdots &\to H_K^{p-1}(K_\alpha, L_\alpha; B, B') \to H_K^p(K_\alpha, L_\alpha; F, F') \\ &\to H_K^p(K_\alpha, L_\alpha; A, A') \to H_K^p(K_\alpha, L_\alpha; B, B') \\ &\to H_K^{p+1}(K_\alpha, L_\alpha; F, F') \to \cdots, \end{aligned} \tag{3}$$

generated by the epimorphism $f\colon (A, A') \to (B, B')$, is constructed. Here it is assumed that the pairs (F, F'), (A, A'), and (B, B') are pairs of presheaves.

The system $\{(K_\alpha, L_\alpha)\}$ of pairs of nerves (K_α, L_α) defines an inverse spectrum of exact sequences (2) of relative parametric homology groups. The limit sequence of this spectrum

$$\begin{aligned} \cdots &\to H^K_{p+1}(R, C; B, B') \to H^K_p(R, C; F, F') \\ &\to H^K_p(R, C; A, A') \to H^K_p(R, C; B, B') \\ &\to H^K_{p-1}(R, C; F, F') \to \cdots, \end{aligned} \tag{4}$$

consisting of the relative canonical parametric homology groups of the space R modulo C, taken over the pairs of copresheaves, is semiexact (see [4]). The sequence (4) is called the *relative canonical parametric sequence* of the space R modulo C generated by the epimorphism $f\colon (A, A') \to (B, B')$. Further, again the system $\{(K_\alpha, L_\alpha)\}$ of pairs of nerves (K_α, L_α) defines a direct spectrum of exact sequences (3) of relative parametric cohomology groups, whose limit sequence

$$\begin{aligned} \cdots &\to H^{p-1}_K(R, C; B, B') \to H^p_K(R, C; F, F') \\ &\to H^p_K(R, C; A, A') \to H^p_K(R, C; B, B') \\ &\to H^{p+1}_K(R, C; F, F') \to \cdots \end{aligned} \tag{5}$$

is exact (cf. [4]). The sequence (5) is called the *relative parametric canonical cohomology sequence* of the space R modulo C, generated by the epimorphism $f\colon (A, A') \to (B, B')$.

If we now chose for B the system $\{A(|t|)/A'(|t|)\}$ of factor groups $A(|t|)/A'(|t|)$, and for B' the system $\{A'(|t|)\}$ of trivial subgroups of the factor groups $A(|t|)/A'(|t|)$, then from (4) and (5) we get the sequences

$$\begin{aligned} \cdots &\to H^K_{p+1}(R, C; A/A', A') \to H^K_p(R, C; A', A') \\ &\to H^K_p(R, C; A, A') \to H^K_p(R, C; A/A', A') \\ &\to H^K_{p-1}(R, C; A', A') \to \cdots \end{aligned} \tag{6}$$

and

$$\begin{aligned} \cdots &\to H^{p-1}_K(R, C; A/A', A') \to H^p_K(R, C; A', A') \\ &\to H^p_K(R, C; A, A') \to H^p_K(R, C; A/A', A') \\ &\to H^{p+1}_K(R, C; A', A') \to \cdots. \end{aligned} \tag{7}$$

From the exact sequence (7) the following result is obtained:

THEOREM 3. *If the space R has trivial $(p-1)$-dimensional relative modulo C canonical parametric cohomology group for finite parametric cocycles and trivial $(p+1)$-dimensional relative modulo C canonical parametric cohomology group for infinite parametric cocycles over the presheaves A/A' and A' respectively, then the p-dimensional relative canonical parametric cohomology group $H^p_K(R, C; A, A')$ of the space R modulo C over the pair of presheaves (A, A') is the extension of the p-dimensional relative canonical parametric cohomology group $H^p_K(R, C; A')$ for infinite parametric cocycles of the space R modulo C over the presheaf $A' = \{A'(|t|)\}$ by the p-dimensional relative canonical parametric cohomology group $H^p_K(R, C; A/A')$ for finite parametric cocycles of the space R modulo C over the presheaf $A/A' = \{A(|t|)/A'(|t|)\}$.*