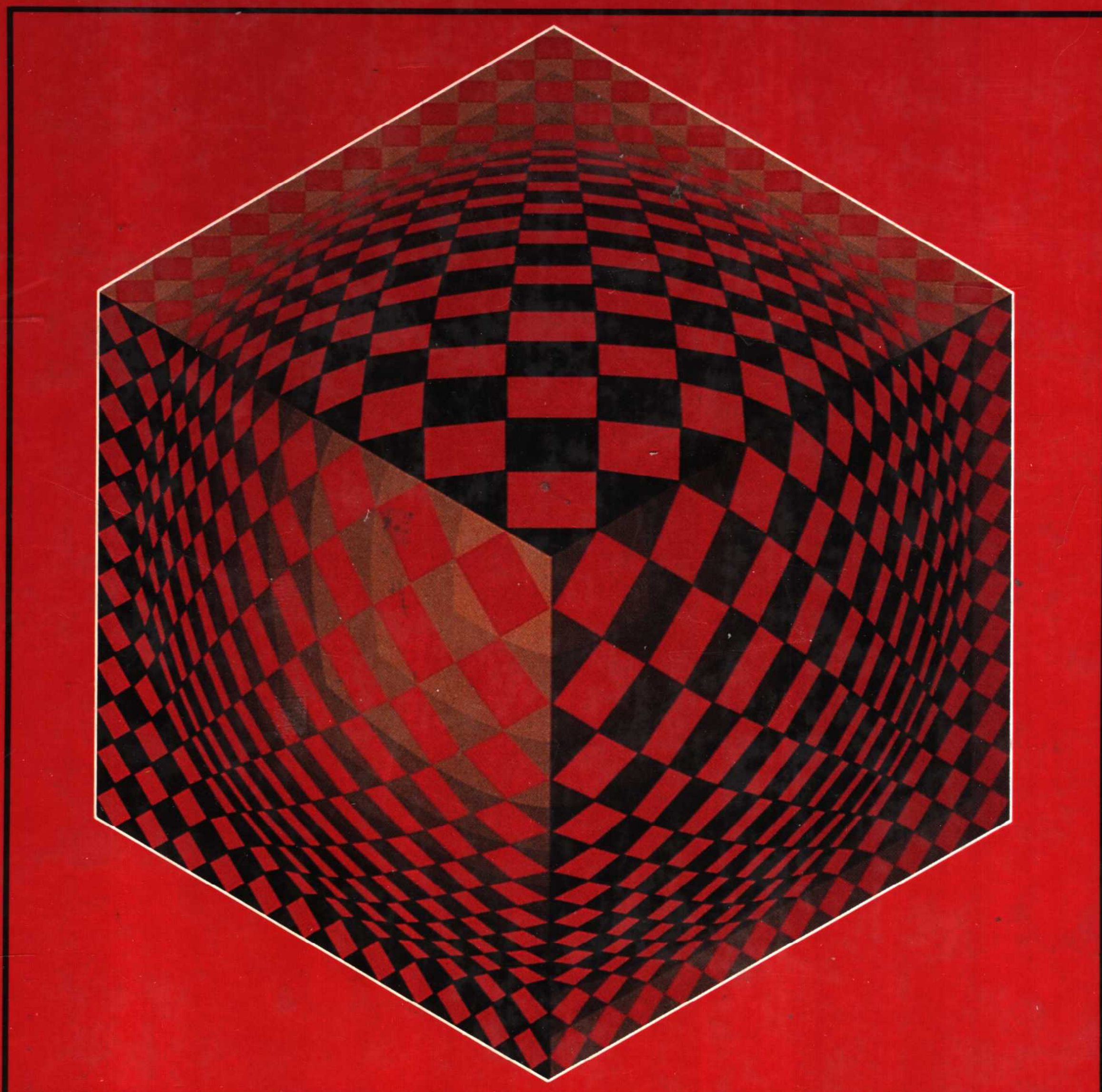
Second Edition

College Algebra



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COLLEGE ALGEBRA

Second Edition

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Preface

College Algebra has been written to provide the essential concepts and skills of algebra that are needed for further study in mathematics, with special emphasis given to direct preparation for the study of calculus. Thus an extensive review of the fundamentals of algebra is included in the first two chapters. However, much of the work with solving equations is delayed until the appropriate functions are studied. For example, quadratic equations are studied in the setting of quadratic functions. This allows for further building on algebraic skills as the course progresses while simultaneously showing how solutions to equations can be viewed as intercepts of related curves.

The text also contains a strong emphasis on graphing throughout. The objective is to have students become familiar with basic graphs and learn how to obtain new curves from these by using translations and reflections.

Improving the students' ability to read mathematics should be a major goal of a pre-calculus course. To assist students in this direction, the exposition here is presented in a relaxed style that avoids unnecessary mathematical jargon without sacrificing mathematical accuracy. Also, a number of highly successful pedagogical features that were commended in the first edition of this text have been maintained and expanded in this second edition. Included in the text are the following pedagogical features that are designed to assist the students in using the book for self-study and to reinforce classroom instruction:

Pedagogical features

Test Your Understanding: These are short sets of exercises (in addition to the regular section exercises) that are found within most sections of the text so that students can test their knowledge of new material just developed. Answers to these are given at the end of each chapter.

Caution Items: Where appropriate, students are alerted to the typical kinds of errors that they should avoid.

Illustrative Examples and Exercises: The text contains over 400 illustrative examples with detailed solutions. There are approximately 4700 exercises for the students to try. The answer to the odd-numbered section exercises are given at the back of the book.

Review Exercises: Each chapter has a set of review exercises that are the same as the illustrative examples developed in the text. Students can use these as a review of the work in the chapter and compare results with the worked-out solutions that can be found in the body of the chapter.

Marginal Notes: Marginal notes appear throughout the text and are designed to assist the students' understanding, and to provide additional insights into the topic under study.

Sample Test Questions: Two sets of sample test questions are given at the end of each chapter, with answers provided in the back of the book. One of these is of the multiple-choice format.

Boxed Displays: Boxed displays for important results, formulas, and summaries are used throughout the text.

Inside Covers: The inside of the cover contains summaries of useful information. The front cover contains a collection of basic graphs; in back are two pages of algebraic and geometric formulas.

The knowledge of the fundamentals of algebra is necessary for the study of calculus, but for many students it is not sufficient; they still have severe problems. All too often students are unable to adapt their knowledge of basic mathematics to calculus. Since it is a major objective of this book to help the students make a more comfortable transition from elementary mathematics to calculus, the authors have included exercises and material that can best be described as being *directly supportive* of topics in calculus. Some examples of these *supportive* items follow. Note that the calculus topics themselves are *not* included in the text.

Calculus topic Simplifying derivatives.

Pre-calculus support Procedures needed to simplify derivatives are included using the same algebraic forms as will be encountered in calculus. For example, the students learn to convert $x^{1/3} + \frac{x-8}{3x^{2/3}}$ (the derivative of $x^{1/3}(x-8)$) into the form $\frac{4(x-2)}{3x^{2/3}}$.

Calculus topic The concept of a derivative.

Pre-calculus support Work with difference quotients is introduced early and reinforced throughout as new functions are studied.

Calculus topic Using the signs of derivatives.

Precalculus support Inequalities are considered early in the book and are applied later to determine the signs of a variety of functions. The signs are then used as an aid in graphing the functions. A convenient tabular format is used throughout that can easily be extended for working with signs of derivatives when the students get to calculus.

Calculus topic The chain rule for derivatives.

Pre-calculus support In addition to the usual work in forming composites of given functions, special material is included that shows how to reverse this process. For example, the student learns how to view a *given* function, such as $f(x) = \sqrt{(2x - 1)^5}$, as the composition of other functions. Much of the difficulty students have later with the chain rule appears to be related to the inability to do this type of decomposition.

Calculus topic Applied or verbal problems.

Pre-calculus support A major difficulty that many students have throughout calculus with such problems deals with setting up functions related to practical or geometric situations. Students are introduced to this type of thinking in Section 3.5, and this experience is reinforced later via follow-up exercises as the course progresses.

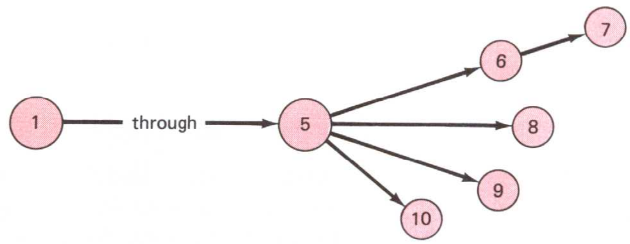
MAJOR CHANGES

The comments and suggestions made by users of the first edition have been incorporated into this second edition resulting in changes that are included in the following list.

- Numerous sections have been revised to enhance the pedagogy and in some places the subject matter has been extended. For example, the work on conic sections in Chapter 4 has been extended and compound interest has been incorporated into the study of exponential growth in Chapter 7.
- Each chapter now has an additional set of sample test questions of the multiple choice variety, with answers at the back of the book.
- Chapter 3 has been shortened by condensing some of the introductory work, and moving the work on linear systems into Chapter 8.
- An introduction to complex numbers has been included in Chapter 2 so that imaginary numbers will be available for subsequent work.
- The introductory work on inverse functions has been moved from Chapter 7 into Chapter 6.
- Many exercise sets have been enlarged to include additional routine and challenging exercises.
- New applications have been included where appropriate.

COURSE STRUCTURE

The text allows for considerable flexibility in the selection and ordering of topics. The first five chapters should be done in sequence. Thereafter the dependency of the chapters is displayed in the following diagram.



Note: Even though the material in Chapter 8, Matrices, Determinants, and Linear Systems, could be done directly after Chapter 3, Linear Functions and Equations, we have placed it later since the added maturity the students gain in the study of the first five to seven chapters will be helpful in the mastering of Chapter 8.

If the students' background is adequate, much of Chapters 1 and 2 can be assigned as self-study or covered quickly. Additional suggestions regarding course content are given in the Instructor's Manual, which is available upon adoption.

▪ This symbol is used to identify exercises that are more *directly* supportive of topics in calculus than the other exercises. Also, when this symbol appears next to a section heading it means that the section and its exercises fall into this support category. For such sections the exercises are not separately labeled with this symbol.

The subject matter labeled with ▪ may be treated as optional. It is not prerequisite to the subsequent developments.

* This symbol is used to identify exercises of a more challenging nature.

There are different points of view at this time as to the role that calculators should play in a mathematics course. The authors feel this is a choice that must be left to the judgment and discretion of the individual instructor. In this text students are advised occasionally to make use of a calculator to enhance the pedagogical development, as well as to complete certain exercises that involve cumbersome computations. However, use of a calculator is neither a prerequisite nor a requirement for the completion of the course.

SUPPLEMENTARY MATERIALS

1. *Instructor's Manual:* This manual contains suggestions for course outlines and answers to all even-numbered section exercises.
2. *Student's Solution Manual:* This manual contains completely worked-out solutions for every fourth exercise at the end of each section, as well as worked-out solutions for all the chapter tests.
3. *Test Bank:* This supplement provides the instructor with five tests for each chapter; three are in standard form, and two are of the multiple-choice variety. In addition, there is a large collection of final examination questions. Answers to all questions are included.
4. *Floppy Disk Testing:* The test bank material is also available, upon adoption, on a computer disk for both the IBM PC and the Apple II.

ACKNOWLEDGMENTS

The preparation of this second edition was significantly influenced by many individuals. We sincerely thank the many students who used the first edition of *College Algebra* and who contributed their comments and suggestions. For their detailed reviews, constructive criticisms, and suggestions as we prepared this revision we thank the following professors: Lewis R. Husch, Rutgers University; Ken Seydell, Skyline College; Dalana L. Nugent, University of Arkansas; Dolores Stanton Smith, Coppin State College; Marjorie Senechal, Smith College; Suzanne Shelley, Sacramento Community College; and Jimmie A. Lakin, University of Wisconsin.

For their encouragement, support, and assistance we thank the staff at Prentice-Hall.

For her careful reading of the galleys, and for her thorough and careful writing of the Student Solutions Manual, we thank Margaret Hall Babcock of Chatam Township High School, New Jersey.

For his excellent work in writing the test bank, we thank David Pugh of the State University of New York at Cortland.

For his generosity in allowing us to use his art as our cover design, we thank Victor Vasarely of Aix-en-Provence, France.

Finally, we thank Betty Thomas for her excellent typing of the Student's Solution Manual, and Karin Lerner for her equally fine typing of the accompanying Instructor's Manual.

The authors sincerely hope that you will find this book teachable and enjoyable, and welcome your comments, criticisms, and suggestions.

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Suggestions for the Student

The most important suggestion we can give regarding your study habits for this mathematics course is that you make every possible effort to keep up to date. Set aside regular time periods for the work in this course and stick to this plan.

During your periods of study, read the text often. Few students do this because they find mathematics very difficult to read, and the frustrations they encounter do not seem to be worth the effort. We ask you to be patient; always try to dig things out on your own in spite of difficulties you may encounter. Even a modest effort along these lines will prove to be rewarding in the long run. We have tried to make this book readable and have included some special features (listed in the Preface) that are designed to help you during your periods of self-study.

How does one succeed in a mathematics course? Unfortunately, there is no universal prescription guaranteed to work. However, the experience that the authors have had with many students throughout their teaching careers provides some guidelines that seem to help. We suggest that you make an effort to follow these suggestions that have been useful to other students who have taken similar courses in the past:

1. Read the text! We recognize that mathematics is not always easy to read, but we have made every effort to make this book as readable as possible. Don't look upon the textbook merely as a source of exercises. Rather, read each section thoroughly to reinforce classroom instruction.
2. Try to complete the illustrative examples that appear within each section before studying the solutions provided. Mathematics is not a "spectator sport"; study the book with paper and pencil at hand.
3. Attempt each "Test Your Understanding" exercise and check your results with the answers given at the back of each chapter. Re-read the section if you have difficulty with these exercises.
4. Try to complete as many of the exercises as possible at the end of

each section. Complete the odd-numbered ones first and check your answers with those given at the back of the book. At times your answers may be in a different form than that given in the book; if so, try to show that the two results are equivalent. Considerable efforts have been made to assure that the answers are correct. However, if you happen to find an occasional incorrect answer, please write to let us know about it.

5. Prior to a test you should make use of the review exercises that appear at the end of each chapter. These are collections of representative examples from within each chapter. You can check your results by referring back to the designated section from which they are taken where you will find the worked-out solution for each one.
6. Each chapter ends with two sets of sample test questions that will help tell you whether or not you have understood the work of that chapter. Check your work with the answers that are given at the back of the book.
7. If you find yourself making a careless error in completing a problem, it would be wise to attempt another similar problem. If you make the same mistake again, it is best to go back and review, since a serious misunderstanding of basic concepts may be involved.
8. If convenient, find time to solve problems with a classmate. Such cooperative efforts can be quite beneficial as you attempt to explain ideas to each other.

We hope that you will have a profitable semester studying from this book. Good luck!

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1 Sets of Numbers, Equations, and Inequalities

1.1 Sets of numbers

Throughout this course we will use various sets of numbers, many of which you have already encountered in your past work in mathematics. The most fundamental set of numbers that we begin with is the set **N** of **natural numbers,** also referred to as the set of **counting numbers:**

$$N = \{1, 2, 3, . . .\}$$

WATCH THE MARGINS! We will use these for special notes, explanations, challenges, and hints.

Note that we named the set with a capital letter, N, and included the *elements* or *members* of the set within a pair of braces. This is an example of an **infinite set;** there is no last member of the set. We use the three dots to indicate that the set goes on without end.

By contrast, some of the sets of numbers that we will use will be **finite sets.** The members of a finite set can be listed and counted, and there is an end to this counting. For example, the set of natural numbers that are less than 5 is an example of a finite set:

$$\{1, 2, 3, 4\}$$

How long do you think it would take you to count to 1,000,000 at the rate of one number per second? (Don't use 1 million seconds as your answer!) First guess. Then use a calculator to help find the answer.

At times a set can be classified as finite even though it has so many elements that no one would really want to list them all. Thus the set of counting numbers from 1 through 1,000,000 is a large set, but nevertheless finite inasmuch as it does eventually have a last member. We adopt the convention of using three dots to indicate that some members of a set are not listed and can write this as

$$\{1, 2, 3, \ldots, 1{,}000{,}000\}$$

We shall now begin to consider other sets of numbers. Our first extension is to add the number 0 to the set of natural numbers. This produces the infinite set **W** of **whole numbers.**

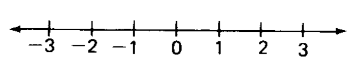
$$W = \{0, 1, 2, 3, \ldots\}$$

Note that every natural number is also a whole number, but not every whole number is a natural number.

Another set of numbers that we will refer to frequently is the set of **integers:**

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

We can represent the integers on a **number line** by locating the whole numbers and their opposites. For example, the opposite of 3 is located three units to the left of 0 on the number line and is named -3 (negative three). The opposite of 2 is -2, the opposite of 1 is -1, and the opposite of 0 is 0.



Observe that the *positive integers* are located to the right of 0 and that the *negative integers* are located to the left of 0. The number 0 is an integer, but is neither positive nor negative. We see that every integer is the **coordinate** of some point on the number line. Can every point on the line be named by an integer?

TEST YOUR UNDERSTANDING

Throughout this book we shall occasionally pause for you to test your understanding of the ideas just presented. If you have difficulty with these brief sets of exercises, you should reread the material of the section before going ahead. Answers are given at the end of the chapter.

List the elements in each of the following sets.

The word "list" here means that you are either to show all the members of a set or to indicate the unlisted members through the use of three dots.

1. The set of natural numbers less than 3.
2. The set of whole numbers less than 5.
3. The set of whole numbers greater than 10.
4. The set of whole numbers *between* 1 and 10. (*Note:* The word "between" means that 1 and 10 are not included.)
5. The set of integers between -3 and 4.
6. The set of negative integers greater than -5.
7. The set of integers less than 2.
8. The set of natural numbers between 5 and 6.

You may have been puzzled by Exercise 8 because there is no natural number between 5 and 6. Such a set is then said to be *empty*. Thus the **empty set,** or **null set,** is the set that contains no elements. Later we will have occasion to use this language to describe the solution of certain types of equations. For example, consider the set of numbers for which this equation is true:

$$x + 2 = x$$

Obviously there is no number than can be used in place of x to make this equation true. We then say that the solution is the empty set and denote this by the symbol $\varnothing$.

Although there are no natural numbers between 5 and 6, there are many other numbers between them, such as 5.2 and $\frac{43}{8}$. Some of these numbers can be identified by considering all of the numbers that can be written in the form $\frac{a}{b}$, where a and b are integers, with $b \neq 0$. The set of numbers that can be represented in this way is called the set of **rational numbers.**

A **rational number** is one that can be written in the form $\frac{a}{b}$, where a and b are integers, $b \neq 0$.

Recall that the representation of a rational number is not unique. For example, $\frac{1}{2} = \frac{5}{10} = \frac{-3}{-6} = \cdots$

Every integer is a rational number because it can be written as the quotient of integers. For example, $5 = \frac{5}{1}$. However, not every rational number is an integer. Thus $\frac{2}{3}$ and $-\frac{3}{4}$ are examples of rational numbers (*fractions*) that are not integers.

Every rational number can be written in decimal form. Sometimes the result will be a *terminating decimal*, as in the following examples:

$$\tfrac{3}{4} = 0.75 \qquad \tfrac{7}{8} = 0.875 \qquad \tfrac{23}{10} = 2.3$$

Other rational numbers will produce a *repeating decimal:*

$$\tfrac{2}{3} = 0.666\ldots \qquad \tfrac{19}{22} = 0.86363\ldots \qquad \tfrac{3}{7} = 0.428571428571\ldots$$

Usually, a bar is placed over the set of digits that repeat, so that the preceding illustrations can be written in this way:

A method for converting a repeating decimal into fraction form is considered in Exercises 45 and 46.

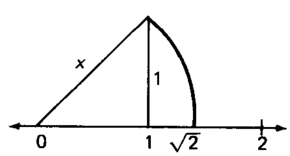
$$\tfrac{2}{3} = 0.\overline{6} \qquad \tfrac{19}{22} = 0.8\overline{63} \qquad \tfrac{3}{7} = 0.\overline{428571}$$

Some decimals neither terminate nor repeat. You are probably familiar with the number π from your earlier study of geometry. This number is *not* a rational number; it cannot be expressed as the quotient of two integers. The decimal representation for π goes on endlessly without repetition. In fact, one computer recently computed π as a decimal to 100,000 places. Here are the first 100 places:

$$\pi = 3.14159\ 26535\ 89793\ 23846\ 26433\ 83279\ 50288\ 41971\ 69399$$
$$37510\ 58209\ 74944\ 59230\ 78164\ 06286\ 20899\ 86280\ 34825$$
$$34211\ 70679\ \ldots$$

It is true that every rational number is the **coordinate** of some point on the number line. However, not every point on the number line can be labeled by a rational number. For example, here is a construction that can be used to locate a point on the number line whose coordinate is $\sqrt{2}$, the *square root* of 2.

At the point with coordinate 1 construct a one-unit segment perpendicular to the number line. Connect the endpoint of this segment to the point labeled 0. This becomes the hypotenuse of a right triangle, and by the Pythagorean theorem can be shown to be the square root of 2. Using a compass, this length can then be transferred to the number line, thus locating a point with coordinate $\sqrt{2}$.



$x^2 = 1^2 + 1^2$

$x^2 = 2$

$x = \sqrt{2}$

It can be proved that $\sqrt{2}$ cannot be expressed as the quotient $\frac{a}{b}$ of two integers, and thus is *not* a rational number. We call such a number an **irrational number.** Some other examples of irrational numbers are $\sqrt{5}$,