

FUNDAMENTALS
of
PLASMA PHYSICS

BY

J. A. BITTENCOURT

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Preface

This text is intended as a general introduction to plasma physics and was designed with the main purpose of presenting a comprehensive, logical and unified treatment of the fundamentals of plasma physics based on the statistical kinetic theory. It should be useful primarily for advanced undergraduate and first year graduate students meeting the subject of plasma physics for the first time, and presupposes only a basic elementary knowledge of vector analysis, differential equations and complex variables, as well as courses on classical mechanics and electromagnetic theory beyond the sophomore level. Some effort has been made to make the book self-contained by including in the text developments of the fluid mechanics and kinetic theory that is needed.

Throughout the text the emphasis is on clarity rather than formality. The various derivations are explained in detail and, wherever possible, the physical interpretations are emphasized. The equations are presented in such a way that they connect together without requiring the reader to do extensive algebra to bridge the gap. The features of clarity and completeness make the book suitable for self-learning and for self-paced courses.

The structure of this book is as follows. The first chapter consists of a basic introduction to plasma physics, at a descriptive level, intended to give the reader an overall view of the subject. The motion of charged particles under the influence of specified electric and magnetic fields is treated in detail in Chapters 2, 3 and 4. In the next five chapters the basic equations necessary for an elementary description of plasma phenomena are developed. Chapter 5 introduces the concepts of phase space and distribution function, and derives the basic differential kinetic equation which governs the distribution function in phase space. The definitions of the macroscopic variables in terms of the phase space distribution function are presented in Chapter 6 and their physical interpretations are discussed. The Maxwell-Boltzmann distribution function is introduced in Chapter 7 as the equilibrium solution of the Boltzmann equation and its kinetic properties are analyzed in some detail. In Chapter 8 the macroscopic transport equations for a plasma considered as a mixture of various interpenetrating fluids are derived, whereas the macroscopic transport equations for the whole plasma as a single conducting fluid are developed in Chapter 9. The remaining of the book is

devoted to applications of these basic equations in the description of a variety of important phenomena in plasmas. The problems of electrical conductivity and diffusion in plasmas are analyzed in Chapter 10, and other basic plasma phenomena, such as electron plasma oscillations and Debye shielding, are treated in Chapter 11. Simple applications of the magnetohydrodynamic equations, such as in plasma confinement by magnetic fields and the pinch effect, are presented in Chapters 12 and 13. The subject of wave phenomena in plasmas is organized in the next six chapters. A review of the basic concepts related to the propagation of electromagnetic waves in free space is given in Chapter 14. The propagation of very low frequency waves in a highly conducting fluid is analyzed in Chapter 15, under the title of magnetohydrodynamic waves. The various modes of wave propagation in cold and warm plasmas are considered in Chapters 16 and 17, respectively. In Chapters 18 and 19 the problems of wave propagation in hot nonmagnetized plasmas and in hot magnetized plasmas, respectively, are analyzed. Collision phenomena in plasmas and the derivation of the Boltzmann collision integral and of the Fokker-Planck collision term, are presented in Chapters 20 and 21. Finally, in Chapter 22 some applications of the Boltzmann equation to the analysis of transport phenomena in plasmas are presented.

A number of problems is provided at the end of each chapter, which illustrate additional applications of the theory and supplement the textual material. Most of the problems are designed in such a way as to provide a guideline for the student, including intermediate steps and answers in their statements.

The numbering of the equations starts over again at each section. When reference is made to an equation, using three numbers, the first number indicates the chapter and the last two numbers indicate the section and the equation, respectively. Within the same chapter the first number is omitted. Vectors are represented by an arrow above the corresponding letter and unit vectors by a circumflex above the letter.

The book contains more material than can normally be covered in one semester. This permits some freedom in the selection of topics depending on the level and desired emphasis of the course, and on the interests of the students.

In this, as in any introductory book, the topics included clearly do not cover all areas of plasma physics. No attempt was made to present the experimental aspects of the subject. Moreover, there are some important theoretical topics which are covered only very briefly and some which have been left for more advanced courses on plasma physics, such as plasma instabilities, plasma radiation, nonlinear plasma theory and plasma turbulence.

The system of units used in this text is the rationalized MKSA.

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