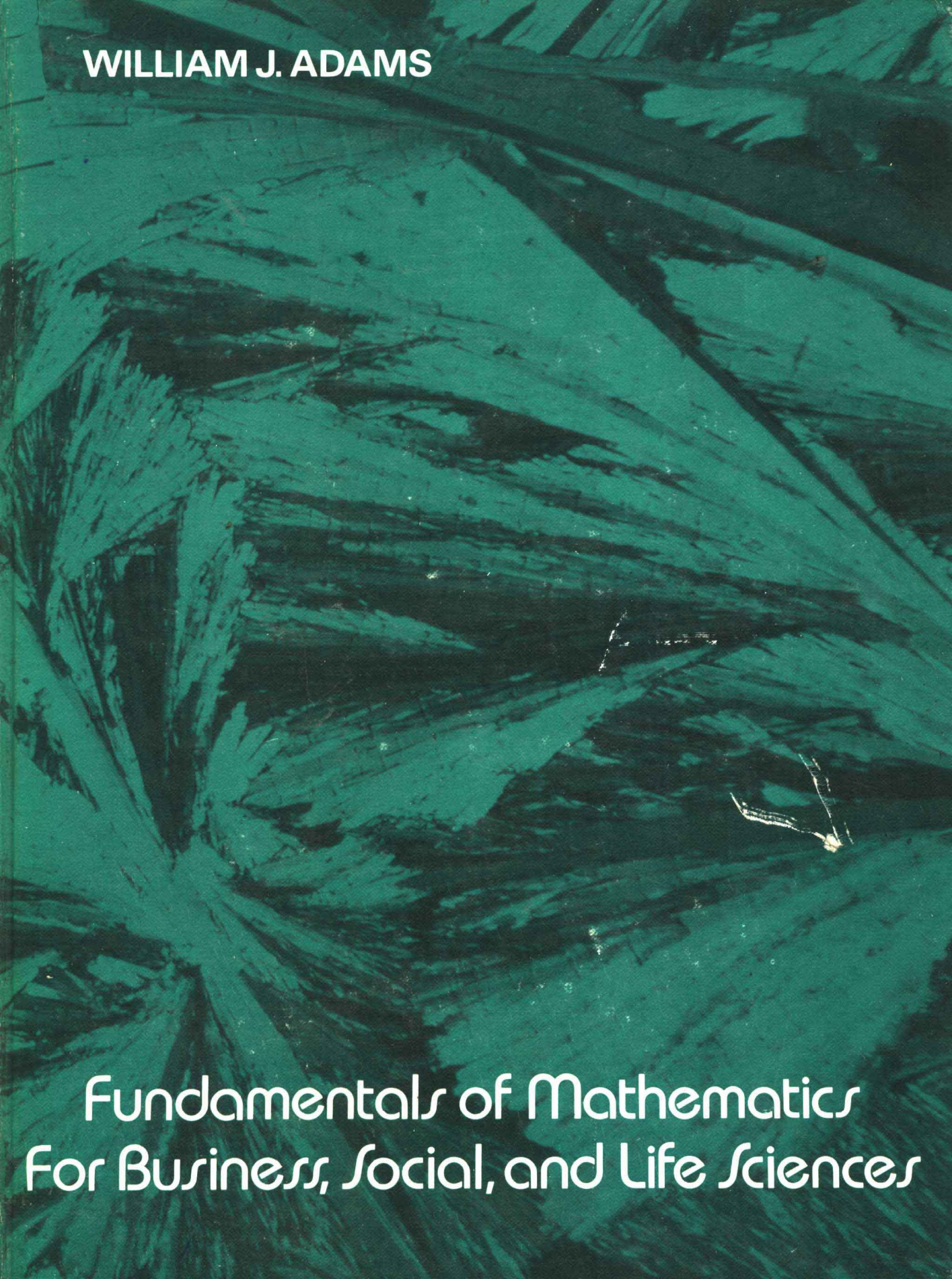
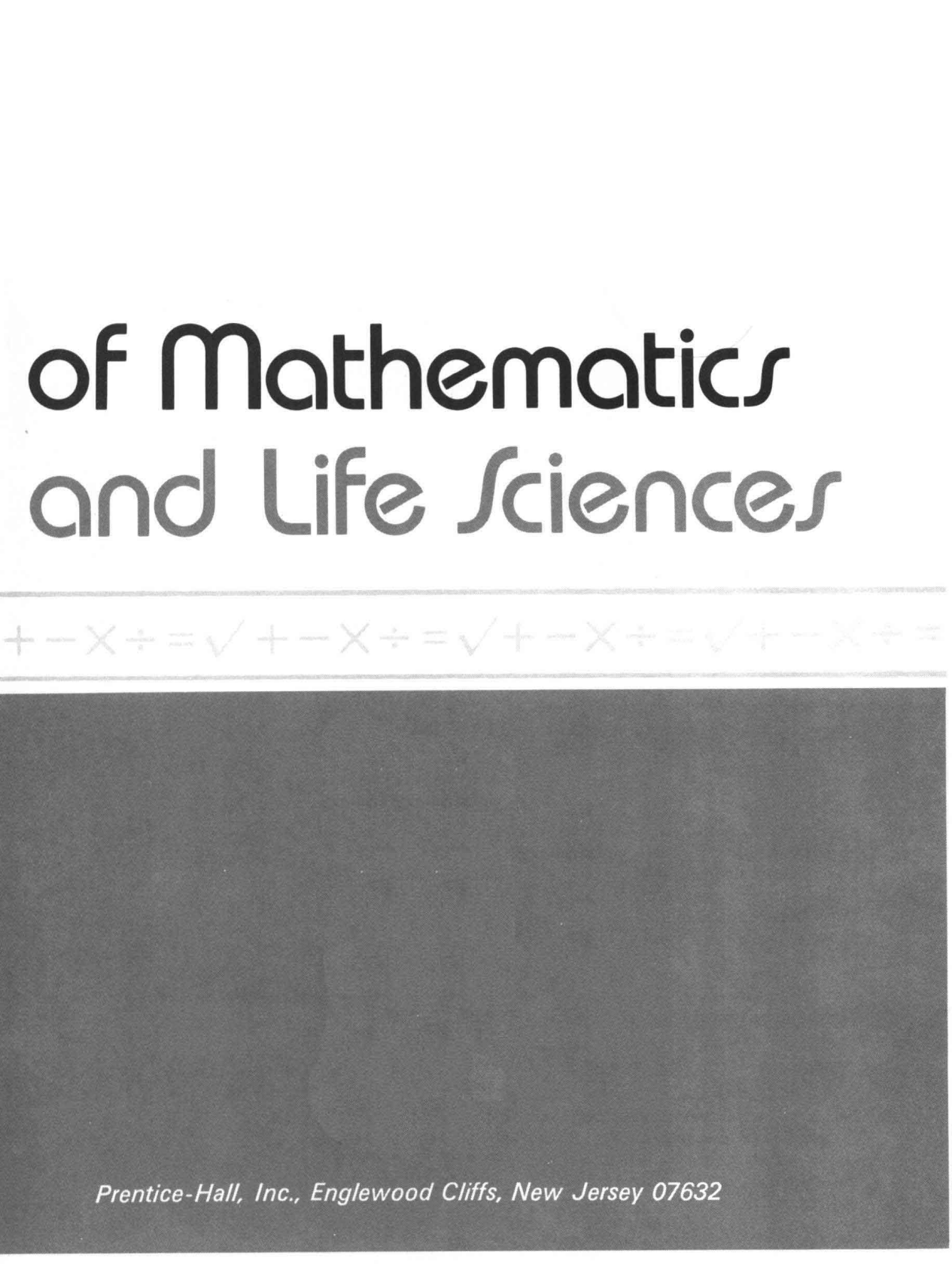
WILLIAM J. ADAMS



Fundamentals of Mathematics For Business, Social, and Life Sciences

of Mathematics and Life Sciences



Prentice-Hall, Inc., Englewood Cliffs, New Jersey 07632

Library of Congress Cataloging in Publication Data

ADAMS, WILLIAM J
Fundamentals of mathematics for business, social, and life sciences.

Bibliography: p.
Includes index.
1. Mathematics—1961- I. Title.
QA37.2.A3 510 78-13212
ISBN 0-13-341073-0

Fundamentals of Mathematics for Business, Social, and Life Sciences

William J. Adams

Editorial/production supervision by *Karen J. Clemments*
Cover/interior design by *Mark A. Binn*
Manufacturing buyer: *Phil Galea*
Cover photograph is entitled "*Crystallizations*," photographed by *Phillip Harrington*, supplied by *Peter Arnold*

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

PRENTICE-HALL INTERNATIONAL, INC., *London*
PRENTICE-HALL OF AUSTRALIA PTY. LIMITED, *Sydney*
PRENTICE-HALL OF CANADA, LTD., *Toronto*
PRENTICE-HALL OF INDIA PRIVATE LIMITED, *New Delhi*
PRENTICE-HALL OF JAPAN, INC., *Tokyo*
PRENTICE-HALL OF SOUTHEAST ASIA PTE. LTD., *Singapore*
WHITEHALL BOOKS LIMITED, *Wellington, New Zealand*

Preface

My objective in this book is to present a lucid exposition of topics in mathematics for students who plan to pursue studies in business, economics, psychology, the life and social sciences, and other disciplines that bring them into contact with mathematics. My other concern is with the student who, apart from professional interest, wishes to be a well-educated person. Mathematics, as one of the great triumphs of the human intellect, brings us together as human beings interested in partaking of the best of the human spirit.

This book is designed for multi-semester or quarter sequences in mathematics which presuppose a minimal mathematics background. It seeks to provide an introduction to basic concepts and methods of linear mathematics, probability and calculus along with the needed foundation in algebra. At the same time it seeks to provide a proper perspective on the scope and limitations of mathematical methods. In this connection, the distinction between the validity of a mathematical conclusion and its truth is pointed out on every suitable occasion. The presentation in this text is intuitive and informal, without sacrifice of correctness. Concepts are introduced through examples. Rigorous proofs above the expected level of mathematical maturity of the intended audience are avoided. To help stimulate student interest and appreciation for mathematical methods, emphasis has been placed on realistic applications that are appropriate at the freshman level.

To optimize flexibility, the book is organized into five parts that exhibit maximum independence. Part 1, Basic Concepts and Fundamental Algebra, develops and reviews

topics which serve as a foundation for the rest of the book and are in the mainstream regardless of one's subsequent direction. In response to such questions as, why should man go to the moon (or climb Mt. Everest), it was popular a few years ago to remark, "because it is there." This is certainly not true of Part 1; it should not be done in its entirety because it is there. Certain topics can be surveyed lightly or omitted entirely, depending on student needs and preparation.

Part 2, Linear Mathematics, begins with a discussion of linear equations, inequalities and functions (Chapter 4) which serves as a foundation for the discussion of matrix algebra (Chapter 5) and linear programming (Chapters 6–8). Chapter 5, Introduction to the World of Matrices, provides an introduction to basic matrix algebra and some of its applications, particularly to input-output analysis and a cost-allocation problem in accounting. Chapters 6 and 7 on linear programming apply linear methods to solve linear programs that are developed for a variety of situations. A good deal of attention is paid to the role of assumptions in the formulation of linear programming models so as to leave the student with an understanding of the scope and limitations of mathematical methods. A presentation of the simplex method which presupposes only a knowledge of arithmetic is given. Another dimension of linear programming that arises in connection with game theory is discussed in Chapter 8. Linear programming may be taken up before matrix algebra since Chapters 6, 7 and 8 are independent of Chapter 5.

Part 3, Probability, provides an introduction to basic probabilistic ideas and their interpretation. The distinction between probability as a mathematical structure and its relative-frequency and subjective interpretations is discussed and a good deal of attention is paid to the role of assumptions in the construction of probabilistic models for random processes. The only prerequisite for Part 3 in a formal sense is that matrix multiplication, discussed in Section 19, is needed for Markov chains, discussed in Section 44.

Part 4, Topics in Algebra, discusses a number of topics in algebra in appropriate depth. The purpose of Part 4 is to strengthen the student's background in algebraic techniques, especially in preparation for calculus. It can be treated in detail, surveyed quickly, assigned for independent review, referred to as needed, or omitted entirely, depending on class preparation and circumstances. Part 1 is prerequisite for Part 4.

Part 5, Calculus, provides an introduction to basic concepts of differential and integral calculus with applications. Part 1, or its equivalent, and some algebra treated in Part 4 are prerequisites for Part 5.

This text has been designed to accomodate course sequences which integrate algebra, finite mathematics, and calculus, as well as those courses in which these subjects are treated separately. In a course that focuses on algebra, one might, for example, take up Parts 1 and 4 along with Chapter 4 on linearity in Part 2, spiced with some discussion of matrix algebra or linear programming. At a glance, the general structural flexibility in the five parts is shown in the diagram of part dependence on the inside front cover.

In addition to the large number of exercises found at the end of each section, comprehensive review exercises are provided at the end of each part to help the stu-

dent obtain a better perspective on the parts as a whole. Answers to odd-numbered exercises are given at the end of the book, and a separate solutions guide with answers to even-numbered exercises and solutions to selected exercises is available for student use at the discretion of the instructor.

I should like to express my appreciation to Dr. Leo Vroman for the cartoon on calculator failure that appears on page 31, and to the following reviewers for their constructive criticism that helped make this a better book: Professors John W. Dettman, Oakland University; Claude Schochet, Wayne State University; Clinton W. Smullen III, University of Tennessee at Chattanooga; Lawrence E. Spence, Illinois State University; and Arnold Villone, San Diego State University.

W. J. A.

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Part One

+−×÷=√+−×÷=√+−×÷=√+−×÷=

Basic Concepts and Fundamental Algebra

Chapter One

The World of Real Numbers

1. THE REAL-NUMBER SYSTEM

Since algebraic operations and developments are dependent on the number system adopted as a foundation, let us begin our study of algebraic principles and applications by reviewing and summarizing basic data about numbers and their mathematical life. The real-number system, as it is called, underlies a wide range of developments and applications and will serve as the foundation for the topics considered in this book.

The real-number system is a collection of mathematical objects, called real numbers, which acquire mathematical life by virtue of certain fundamental principles, or rules, that we adopt. The situation is somewhat similar to a game, like chess, for example. The chess system, or game, is a collection of objects, called chess pieces, which acquire life by virtue of the rules of the game, that is, the principles that are adopted to define allowable moves for the pieces and the way in which they may interact. A word about representation by symbols is perhaps in order at this point. The number two, for example, is represented by such symbols as 2, $1 + 1$, $3 - 1$, $\frac{4}{2}$, and so on. We should be careful not to equate the symbol used to express a number and the number itself. They are not the same, just as the signature of a person is not the same as the person, although a person's signature can be used to identify or represent him under certain circumstances. When it is necessary to be ultracareful about this distinction, the symbol used to represent a number is called a **numeral**.