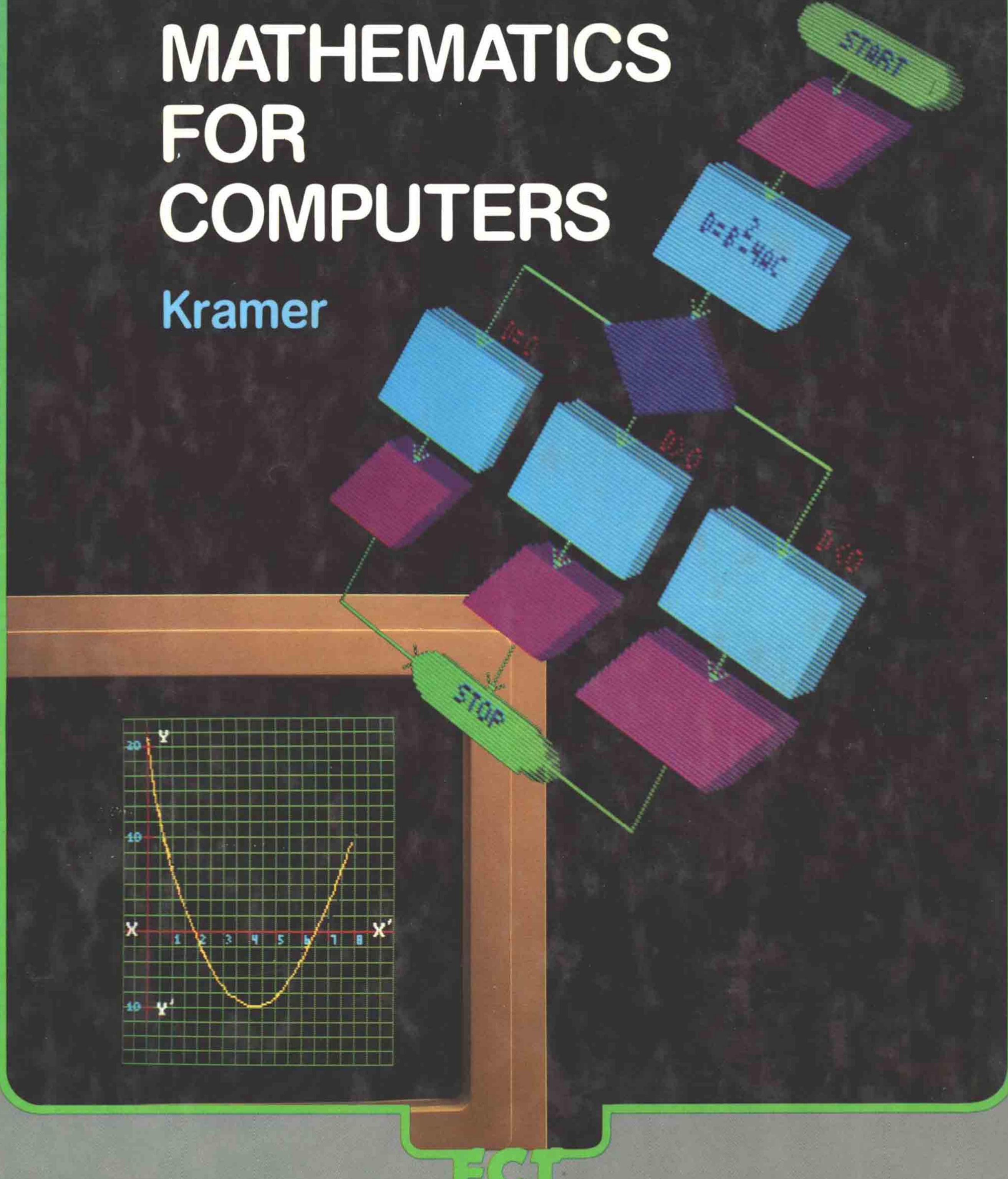
MATHEMATICS FOR COMPUTERS

Kramer



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ELECTRONIC
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MATHEMATICS FOR COMPUTERS

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Dedicated to my bride, Carol.

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EDITOR'S FOREWORD

The McGraw-Hill Electronic Computer Technology series is designed to prepare students for a variety of occupations in computer-based environments. The texts that constitute this series will not only help students attain technical competency, but will also serve as reference resources throughout their entire professional development.

The Electronic Computer Technology series has been developed according to a number of guiding principles. The series:

- Recognizes that employers need staff members who can communicate and interact knowledgeably with their colleagues and customers concerning computer-based technologies.
- Understands the backgrounds and goals of the prospective electronic computer technology graduate, and strives to concentrate on practical material without sacrificing the theory required to keep the graduate current with state-of-the-art computer technology.
- Provides the educational foundations necessary for a variety of specializations, training programs, local employment, and institutional needs.
- Encourages the recommendations of instructors and students, while analyzing and integrating into the series, as applicable, the reports of professional educational groups, technological societies, and government publications dealing with electronic computer technology curriculum and programs.

Because of the constant and rapid changes in the field of electronic computer technology, it is important that the authors in this series have relevant industrial and teaching experience. Their ongoing involvement in the field provides them with valuable insight into the needs and concerns of students, instructors, and practicing professionals.

The series components are coordinated around a core curriculum designed to present the latest technological material in a way that keeps both student and professional informed about recent advances in electronic computer technology. Concepts are explained in a clear and well-illustrated manner, and the links between theory and practical applications are explored.

The dynamic nature of technological developments in all facets of the computer industry emphasizes the importance of suggestions from instructors, students, and industry professionals to the success of a series like Electronic Computer Technology. These suggestions allow the series to evolve and develop in ways that satisfy the needs of programs that themselves are growing and changing to keep pace with the field of electronic computer technology.

Gordon Silverman
Project Editor

ELECTRONIC COMPUTER TECHNOLOGY
Gordon Silverman, Project Editor

Books in this series

Mathematics for Computers by Arthur D. Kramer
Integrated Circuits for Computers: Principles and Applications
by William L. Schweber
Computers and Computer Languages
by Gordon Silverman and David B. Turkiew *(in preparation)*

PREFACE

Mathematics for Computers is designed to be used by first-year students preparing for technical careers in computers, electronics, and electricity. Many of these students are enrolled in a two-year computer technology curriculum and need to take a one-year course in mathematics. These students are training for jobs that involve designing, installing, operating, maintaining, troubleshooting, and servicing electronic and computer systems. Although it is desirable to have some background in elementary algebra for the main body of the text, it is not essential. Chapters 1 and 2 provide a comprehensive review of basic arithmetic and algebra and can be used to introduce, or reinforce, material in these areas. The text stresses a working knowledge of mathematics and the application of mathematical ideas to solve realistic technical and practical problems.

Students master mathematical ideas best by doing many exercises and solving many problems. This text provides over 700 worked-out examples and over 3500 exercises and problems, almost all of which have meaningful applications to electronic and computer systems. Many of the examples and exercises are not found in existing texts, as computer technology is a new field of study. The applications do not necessarily require prior knowledge of computers and electronics. They enable the student to learn how mathematics is used in these areas, while introducing important technical concepts or reinforcing previously learned technical ideas.

Almost all the exercises and problems relate to worked-out examples in the text. This always provides the student with a model to help perform each exercise and solve each problem. The odd-numbered exercises are similar to the even-numbered exercises, which can be used for testing or review. The answers to the odd-numbered exercises are given in Appendix C, and answers to the even-numbered exercises are available in the *Instructor's Manual*. Exercises marked with a computer symbol in the left margin are those that can be solved using a calculator or personal computer. Every chapter is followed by a chapter summary and review exercises that help the student strengthen and reinforce the concepts. The review exercises link several of the ideas in the chapter and demonstrate their interrelationships. To illustrate important ideas, each chapter also contains one or more computer programs in BASIC. The programs have been tested on an Apple microcomputer but should work on most computers with little or no modification.

The chapters are written with an eye toward flexibility, and many can be taught independently. They are arranged in four basic groups. The first group is for review and reinforcement and consists of Chapters 1 and 2 on arithmetic and algebra. The second group is the most important. It consists of Chapters 3, 4, 5 and 6, and contains material directly related to computer and electronic systems. Chapter 3 introduces algorithms and flowcharts, which represent a recurring theme throughout the text. Flowcharts and algorithms are incorporated into almost every chapter in order to promote logical thinking and to help the student master new ideas. Chapters 4, 5, and 6, which cover number systems, computer arithmetic, and Boolean algebra are designed to be taught in sequence. However, Chapter 6 can be introduced earlier or later, if desired.

The third group contains algebraic material necessary to understand electronic and computer systems. It consists of Chapters 7, 8, 9, 13, and 14. Chapters 7 and 8 deal with linear equations and matrices, and they are designed to be taught in sequence, as are Chapters 9 and 14, which cover exponents and exponential functions. Chapter 13, on probability, can be taught independently at any time. The last group consists of Chapters 10, 11, 12, and 15. These chapters contain important geometric and trigonometric ideas. Chapter 11 reviews basic geometry and is designed so that, depending on the background of the students, certain topics can be taught and others used as reference material. Chapters 11, 12, and 15, are designed to be taught in sequence, and they cover trigonometry and complex numbers, which are important in electronics.

A possible course of study for a year might be as follows (exclusive of review material in Chapters 1, 2, and 10, which would depend upon the specific needs of the students): For a two-semester sequence, Chapters 3, 4, 5, 7, 8, and 13 would make up the first semester, and Chapters 6, 9, 11, 12, and 14 the second semester. Chapter 15 is more advanced and could be included if time and students' needs exist. For a three-quarter sequence, Chapters 3, 4 and 5 would make up the first quarter, 6, 7, 8, and 13 the second quarter, and 9, 11, 12, and 14 the last quarter. Students needs vary greatly, however, and to satisfy individual requirements the sequence of chapters to be taught may be arranged in many ways.

I extend my thanks to my colleagues for their reviews of the manuscript and helpful comments that have been incorporated into the text. Special thanks go to Bruce Broberg of Central Community College who, in addition to reviewing the manuscript, carefully checked all the examples and exercises, to John Cuniffe, James Bibeault, and Robert Campbell, for their in-depth review and helpful comments, and Jack Jacobs, who diligently typed the manuscript.

Arthur D. Kramer

CONTENTS

1 BASIC MATHEMATICS

The development of computers and their use have increased exponentially in recent years and continue to increase. Computers are becoming more common in almost every business, in industry, and in the home. The computer is a complex calculating machine that works not only with numbers but with *alphanumeric* data such as words, addresses, and equations. Mathematics is essential for a thorough understanding of computers and their operation. This chapter is designed to reinforce your understanding of basic mathematics. The first three sections review the four arithmetic operations (+, −, ×, ÷) and their application to fractions and decimals. Section 1-4 discusses powers and roots, and Sec. 1-5 introduces the metric system which is used in computers and electronics. The last section contains a summary of the important ideas in the chapter and review questions to test these ideas.

1-1 ARITHMETIC OPERATIONS

Two kinds of laws apply to the operations of addition and multiplication. The first kind states that it makes no difference in what order two numbers are added or multiplied. For example, $2 + 5 = 5 + 2$ and $3 \times 4 = 4 \times 3$. These are called the **commutative laws:**

$$a + b = b + a \quad \text{and} \quad a \times b = b \times a \tag{1-1}$$

The second kind states that if three numbers are to be added or multiplied together, it makes no difference whether the operations start with the first and second numbers or with the second and third. For example, in addition, $(2 + 3) + 5 = 2 + (3 + 5)$ or $5 + 5 = 2 + 8$. In multiplication, $(3 \times 4) \times 5 = 3 \times (4 \times 5)$ or

$12 \times 5 = 3 \times 20$. These are called the **associative laws:**

$$(a + b) + c = a + (b + c)$$

and
$$(a \times b) \times c = a \times (b \times c) \tag{1-2}$$

When the commutative and associative laws are applied together, it follows that three or more numbers can be added (or multiplied) in any order. For example, $2 + 3 + 4$ (or $2 \times 3 \times 4$) can be added (or multiplied) in any one of six different ways:

$$2 + 3 + 4,\ 2 + 4 + 3,\ 3 + 2 + 4,\ 3 + 4 + 2,\ 4 + 2 + 3,\ 4 + 3 + 2$$

Another important law of arithmetic which combines multiplication and addition is the **distributive law**. This law says that multiplication distributes over addition:

$$a \times (b + c) = a \times b + a \times c \tag{1-3}$$

The distributive law is important in algebra and is applied in Chap. 2.

The *order of operations* in arithmetic, when there are no parentheses, is: *multiplication or division first, addition or subtraction second.* Computers and most scientific calculators are programmed to perform the operations in this order. It is called *algebraic logic* or the *algebraic operating system*.

EXAMPLE 1-1

Calculate the following:

$$5 \times 21 - 36 + 4 \div 2$$

SOLUTION

Multiply and divide first:

$$105 - 36 + 2$$

Then subtract and add:

$$69 + 2 = 71$$

EXAMPLE 1-2

Calculate the following:

$$5 \times 21 - (36 + 4) \div 2$$

SOLUTION

Perform the operation in parentheses first:

$$5 \times 21 - (40) \div 2 = 105 - 20 = 85$$

EXAMPLE 1-3

Calculate the following:

$$\frac{5 \times 21 \times 12}{15 \times 7 \times 3}$$

SOLUTION

Multiplication and division can be done in any order. One way is to multiply across the top and bottom, and then divide:

$$\frac{5 \times 21 \times 12}{15 \times 7 \times 3} = \frac{1260}{315} = 4$$

An easier way is to divide common factors in the top and bottom first and then multiply:

$$\frac{\overset{1}{\cancel{5}} \times \overset{\overset{1}{\cancel{3}}}{\cancel{21}} \times \overset{4}{\cancel{12}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{15}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{3}}} = \frac{4}{1} = 4$$

EXERCISE 1-1

In problems 1–16, test your understanding of arithmetic by mentally calculating the result. Check by doing the problem by hand.

1. $6 + 5 + 7 + 3 + 5 + 4$
2. $8 + 2 - 3 + 9 - 1$
3. $5 \times 2 \times 3 \times 4$
4. $12 \div 3 \div 2 \div 2$
5. $(800 + 20) \div 20$
6. $10 \div (5 + 40) \times 9$
7. $8 + 13 \times 2 - 4$
8. $7 - 6 \div 3 + 8 \div 4$
9. $5 + (8 - 1) \times 6 \div 2$
10. $(5 - 1) \div 2 + 3 \times 4$
11. $\dfrac{9 \times 4}{3 \times 6} + \dfrac{18}{6}$
12. $\dfrac{8 \times 9}{4} - \dfrac{15}{3}$

13. $\dfrac{12 \times 15}{5 \times 3 \times 2}$

14. $\dfrac{8 \times 7 \times 6}{4 \times 28}$

15. $\dfrac{(3 + 5) \times 2}{13 - 11}$

16. $\dfrac{6 + 8 \times (4 - 1)}{4 - 1 \times 2}$

In problems 17–22, solve each applied problem by hand. (Check for accuracy with the calculator.)

17. One car travels 228 miles (mi) on 12 gallons (gal) of gasoline, and a second car travels 336 mi on 16 gal. How many more miles per gallon does the car with the better gas mileage get?

18. A computer technician earns $330 for a 40-hour week, and a civil engineer earns $287 for a 35-hour week. Who earns more per hour, and how much more?

19. A Mariner space probe traveling at an average speed of 6000 miles per hour (mi/h) takes 400 days to reach Mars. What is the total distance traveled by the space probe?

20. A bus route is 22 kilometers (km) long. It takes the bus 50 minutes (min) to complete the route in one direction and 70 min to complete it in the other direction. What is the average rate of speed of the bus in kilometers per hour for the total trip back and forth? (Average rate = total distance/total time.)

21. In the BASIC computer language the following symbols are used for the arithmetic operations:

Addition +
Subtraction −
Multiplication *
Division /

Applying the order of operations, calculate the following example written in BASIC:

```
2+1*8/4-3
```

22. The formula $N(N + 1)/2$ can be used to calculate the sum of the first N numbers. Check the formula for the first 12 numbers by (*a*) adding 1 through 12 directly; (*b*) letting $N = 12$ in the formula and calculating the result.

1-2 FRACTIONS

Calculations with fractions, decimals, and percents lead to mistakes because of a misunderstanding of the concepts involved. The calculator can prevent some of these mistakes, but it is not a substitute for clear understanding. The following examples review the basic arithmetic of fractions. Each example is designed to be done by hand.

EXAMPLE 1-4

Simplify (reduce to lowest terms):

$$\frac{28}{42}$$

SOLUTION

Divide out any common factors (divisors) in the top and bottom:

$$\frac{28}{42} = \frac{\overset{1}{\cancel{2}} \times 2 \times \overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}} \times 3 \times \underset{1}{\cancel{7}}} = \frac{2}{3}$$

It is not necessary to show all the factors. This is done to clearly illustrate the procedure. Simply divide top and bottom by 14. The numbers 28/42 and 2/3 are called *equivalent fractions*. A fraction can be changed to an equivalent fraction by dividing out common factors, or multiplying the top and the bottom by the same factor. For example:

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} \quad \text{and so on}$$

EXAMPLE 1-5

Calculate each of the following:

1. $\frac{3}{8} \times \frac{2}{9}$

2. $\frac{5}{12} \div \frac{15}{16}$

3. $4 \times \frac{3}{14} \times \frac{5}{9}$

4. $\frac{5}{4} \times 8 \div \frac{1}{4}$

SOLUTION

1. $\frac{3}{8} \times \frac{2}{9}$

To *multiply fractions,* first divide out common factors that occur in any numerator and denominator. Then multiply across the top and the bottom:

$$\frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{8}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{9}}} = \frac{1}{12}$$

2. $\frac{5}{12} \div \frac{15}{16}$

To *divide fractions,* invert the divisor, that is, the fraction after the division sign, and multiply:

$$\frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{12}}} \times \frac{\overset{4}{\cancel{16}}}{\underset{3}{\cancel{15}}} = \frac{4}{9}$$

3. $4 \times \frac{3}{14} \times \frac{5}{9} = \frac{\overset{2}{\cancel{4}}}{1} \times \frac{\overset{1}{\cancel{3}}}{\underset{7}{\cancel{14}}} \times \frac{5}{\underset{3}{\cancel{9}}} = \frac{10}{21}$

4. $\frac{5}{4} \times 8 \div \frac{1}{4} = \frac{5}{\underset{1}{\cancel{4}}} \times \frac{8}{1} \times \frac{\overset{1}{\cancel{4}}}{1} = 40$

In problems 3 and 4, note that a whole number can be written with a denominator of 1. To multiply a fraction by a whole number, multiply the numerator by the whole number: $3 \times \frac{3}{4} = \frac{9}{4}$.

EXAMPLE 1-6

Combine:

$$\frac{2}{3} + \frac{5}{6}$$

SOLUTION

To *combine fractions,* that is, add or subtract, first change each fraction to an equivalent fraction so that the denominators are the same. The best denominator to use is the lowest common denominator (LCD) which is the smallest number that each denominator divides into. Then combine the numerators over the LCD:

$$\frac{2(2)}{3(2)} + \frac{5}{6} = \frac{4}{6} + \frac{5}{6} = \frac{4+5}{6} = \frac{9}{6} = \frac{3}{2}$$

The LCD = 6, and only the first fraction needs to be changed by multiplying top and bottom by 2. Note that the result 9/6 is reduced to 3/2.

EXAMPLE 1-7

Combine:

$$\frac{7}{15} + \frac{5}{12} - \frac{1}{6}$$

SOLUTION

The LCD is 60. This can be found by taking multiples of the largest denominator—15, 30, and so on—until each denominator divides into a multiple. Another way is to factor each denominator:

$$\frac{7}{(3)(5)} + \frac{5}{(2)(2)(3)} - \frac{1}{(2)(3)}$$

and make up the LCD so that it contains all the factors that appear in each denominator: (2)(2)(3)(5) = 60. The solution is then

$$\frac{7(4)}{15(4)} + \frac{5(5)}{12(5)} - \frac{1(10)}{6(10)} = \frac{28 + 25 - 10}{60} = \frac{43}{60}$$

Note that parentheses ( ) or a dot · also indicate multiplication. They are used in algebra to avoid confusion of the multiplication sign × with the letter x.

EXAMPLE 1-8

Calculate:

$$\frac{13}{8} - \frac{7}{5} \times \frac{15}{14} + 2 \div \frac{8}{15}$$

SOLUTION

Invert the last fraction and change the operation of division to multiplication. Then divide common factors and multiply:

$$\frac{13}{8} - \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{14}}} + \overset{1}{\cancel{2}} \times \frac{15}{\underset{4}{\cancel{8}}} = \frac{13}{8} - \frac{3}{2} + \frac{15}{4}$$

Now combine over the LCD, 8:

$$\frac{13}{8} - \frac{3(4)}{2(4)} + \frac{15(2)}{4(2)} = \frac{13 - 12 + 30}{8} = \frac{31}{8}$$

EXERCISE 1-2

In problems 1–6, simplify each fraction (reduce to lowest terms).

1. $\frac{6}{10}$

2. $\frac{12}{36}$

3. $\frac{28}{35}$

4. $\frac{27}{54}$

5. $\frac{39}{52}$

6. $\frac{34}{51}$

In problems 7–30, calculate each exercise by hand.

7. $\frac{5}{9} \times \frac{6}{25}$

8. $\frac{2}{21} \times \frac{7}{16}$

9. $\frac{8}{9} \div \frac{2}{3}$

10. $\frac{3}{11} \div \frac{1}{22}$

11. $\frac{3}{5} \times \frac{15}{7} \times \frac{14}{9}$

12. $6 \times \frac{4}{5} \div \frac{8}{15}$

13. $\frac{3}{17} \div \left(\frac{1}{34} \times \frac{1}{2}\right)$

14. $\left(\frac{9}{8} \div \frac{3}{4}\right) \div \frac{3}{2}$

15. $\frac{3}{8} + \frac{1}{4}$

16. $\frac{4}{15} + \frac{5}{6}$

17. $\frac{3}{4} - \frac{1}{2} + \frac{7}{10}$

18. $\frac{1}{6} + \frac{11}{20} - \frac{2}{3}$

19. $2 + \frac{7}{8} + \frac{2}{3}$

20. $\frac{5}{2} + \frac{5}{3} + \frac{5}{6}$

21. $\frac{16}{9} \times \frac{1}{2} + \frac{1}{4}$

22. $\frac{1}{6} + \frac{3}{8} \div \frac{1}{4}$

23. $3 \times \frac{1}{6} + \frac{7}{2} - \frac{4}{5} \div 8$

24. $\frac{3}{100} + \frac{7}{10} \times \frac{2}{35} - \frac{1}{50}$

25. $\left(\frac{1}{2} + \frac{1}{3}\right) \times \left(8 \div \frac{4}{3}\right)$

26. $\left(1 + \frac{3}{8}\right) \div \left(1 - \frac{3}{8}\right)$

27. A $30,000 inheritance is distributed as follows: half to the spouse, two-thirds of what is left to the children, and the remainder to charity. How much money is given to charity?

28. A bookcase is to be 8-foot $3\frac{1}{2}$-inches (8-ft $3\frac{1}{2}$-in) high and to contain six equally spaced shelves and a top, each $\frac{1}{2}$-in thick. How many feet and inches apart should each shelf be?

29. Calculate the resistance of a series-parallel circuit given by

$$R = \frac{1}{1/12 + 1/4} + 3$$

30. In the BASIC computer language, operations in parentheses are performed first. Calculate the following example written in BASIC:

```
(1/10)*3/(3/2-1)*(20/3)
```

See Exercise 1-1, problem 21.

1-3 DECIMALS AND PERCENT

Decimals

Decimals represent fractions whose denominators are powers of 10: 10, 100, 1000, and so on. The number of decimal places equals the number of zeros in the denominator:

$$0.3 = \frac{3}{10}, \qquad 0.21 = \frac{21}{100}, \qquad 0.067 = \frac{67}{1000}$$

and so on.

EXAMPLE 1-9

Calculate each of the following:

1. $6.23 + 17.87 + 0.15$

2. 1.3×0.05

3. $\frac{13.2}{0.12}$

4. $\frac{0.5 \times 0.02}{0.06 - 0.01}$

SOLUTION

1. $6.23 + 17.87 + 0.15$

Add and subtract decimals in the same way as whole numbers, lining up the decimal point and the columns:

$$\begin{array}{r} 6.23 \\ 17.87 \\ 0.15 \\ \hline 24.25 \end{array}$$

2. 1.3×0.05

To *multiply decimals,* multiply the numbers and add the decimal places in each number to determine the number of decimal places in the answer:

$$1.3 \times 0.05 = 0.065$$

Decimal places: one + two = three

3. $\dfrac{13.2}{0.12}$

To *divide decimals,* first move the decimal point in the numerator and the denominator to the right as many places as there are in the *denominator*. Then divide the numbers:

$$\frac{13.2}{0.12} = \frac{1320.}{12.} = 110$$

Note that movement of the decimal point in the numerator and denominator to the right is the same as multiplication of the top and bottom by a power of 10 which does not change the value of the fraction. In problem 3 the numerator and the denominator are multiplied by 100.

4. $\dfrac{0.5 \times 0.02}{0.06 - 0.01} = \dfrac{0.010}{0.05} = \dfrac{1.0}{5} = 0.2$

Study problem 4, which combines subtraction, multiplication, and division of decimals. Observe that the decimal point in the numerator is carried along to the same place in the answer.

Percent

Percents are fractions with denominators of 100. To *change from a percent to a decimal,* move the decimal point two places to the left and vice versa.

EXAMPLE 1-10

Express each number as a fraction, decimal, and percent.

1. $\dfrac{53}{100} = 0.53 = 53\%$

2. $\dfrac{1}{10} = 0.10 = 10\%$