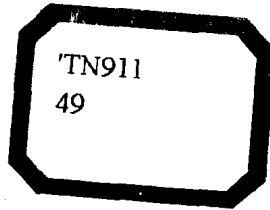


# Signals and Systems

Simon Haykin  
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# **Signals and Systems**

# Preface

The study of signals and systems is basic to the discipline of electrical engineering at all levels. It is an extraordinarily rich subject with diverse applications. Indeed, a thorough understanding of signals and systems is essential for a proper appreciation and application of other parts of electrical engineering, such as *signal processing*, *communication systems*, and *control systems*.

This book is intended to provide a modern treatment of signals and systems at an introductory level. As such, it is intended for use in electrical engineering curricula in the sophomore or junior years and is designed to prepare students for upper-level courses in communication systems, control systems, and digital signal processing.

The book provides a balanced and integrated treatment of continuous-time and discrete-time forms of signals and systems intended to reflect their roles in engineering practice. Specifically, these two forms of signals and systems are treated side by side. This approach has the pedagogical advantage of helping the student see the fundamental similarities and differences between discrete-time and continuous-time representations. Real-world problems often involve mixtures of continuous-time and discrete-time forms, so the integrated treatment also prepares the student for practical usage of these concepts. This integrated philosophy is carried over to the chapters of the book that deal with applications of signals and systems in modulation, filtering, and feedback systems.

Abundant use is made of examples and drill problems with answers throughout the book. All of these are designed to help the student understand and master the issues under consideration. The last chapter is the only one without drill problems. Each chapter, except for the last chapter, includes a large number of end-of-chapter problems designed to test the student on the material covered in the chapter. Each chapter also includes a list of references for further reading and a collection of historical remarks.

Another feature of the book is the emphasis given to design. In particular, the chapters dealing with applications include illustrative design examples.

MATLAB, acronym for MATrix LABoratory and product of The Math Works, Inc., has emerged as a powerful environment for the experimental study of signals and systems. We have chosen to integrate MATLAB in the text by including a section entitled “Exploring Concepts with MATLAB” in every chapter, except for the concluding chapter. In making this choice, we have been guided by the conviction that MATLAB provides a computationally efficient basis for a “Software Laboratory,” where concepts are explored and system designs are tested. Accordingly, we have placed the section on MATLAB before the “Summary” section, thereby relating to and building on the entire body of material discussed in the preceding sections of the pertinent chapter. This approach also offers the instructor flexibility to either formally incorporate MATLAB exploration into the classroom or leave it for the students to pursue on their own.

Each “Exploring Concepts with MATLAB” section is designed to instruct the student on the proper application of the relevant MATLAB commands and develop additional insight into the concepts introduced in the chapter. Minimal previous exposure to MATLAB is assumed. The MATLAB code for all the computations performed in the book, including the last chapter, are available on the Wiley Web Site: <http://www.mathworks.com/books>.

There are 10 chapters in the book, organized as follows:

- ▶ Chapter 1 begins by motivating the reader as to what signals and systems are and how they arise in communication systems, control systems, remote sensing, biomedical signal processing, and the auditory system. It then describes the different classes of signals, defines certain elementary signals, and introduces the basic notions involved in the characterization of systems.
- ▶ Chapter 2 presents a detailed treatment of time-domain representations of linear time-invariant (LTI) systems. It develops convolution from the representation of an input signal as a superposition of impulses. The notions of causality, memory, stability, and invertibility that were briefly introduced in Chapter 1 are then revisited in terms of the impulse response description for LTI systems. The steady-state response of a LTI system to a sinusoidal input is used to introduce the concept of frequency response. Differential- and difference-equation representations for linear time-invariant systems are also presented. Next, block diagram representations for LTI systems are introduced. The chapter finishes with a discussion of the state-variable description of LTI systems.
- ▶ Chapter 3 deals with the Fourier representation of signals. In particular, the Fourier representations of four fundamental classes of signals are thoroughly discussed in a unified manner:
  - ▶ Discrete-time periodic signals: the discrete-time Fourier series
  - ▶ Continuous-time periodic signals: the Fourier series
  - ▶ Discrete-time nonperiodic signals: the discrete-time Fourier transform
  - ▶ Continuous-time nonperiodic signals: the Fourier transform

A novel feature of the chapter is the way in which similarities between these four representations are exploited and the differences between them are highlighted. The fact that complex sinusoids are eigenfunctions of LTI systems is used to motivate the representation of signals in terms of complex sinusoids. The basic form of the Fourier representation for each signal class is introduced and the four representations are developed in sequence. Next, the properties of all four representations are studied side by side. A strict separation between signal classes and the corresponding Fourier representations is maintained throughout the chapter. It is our conviction that a parallel, yet separate, treatment minimizes confusion between representations and aids later mastery of proper application for each. Mixing of Fourier representations occurs naturally in the context of analysis and computational applications and is thus deferred to Chapter 4.

- ▶ Chapter 4 presents a thorough treatment of the applications of Fourier representations to the study of signals and LTI systems. Links between the frequency-domain and time-domain system representations presented in Chapter 2 are established. Both analysis and computational applications are then used to motivate derivation of the relationships between the four Fourier representations and develop the student’s skill in applying these tools. The continuous-time and discrete-time Fourier transform representations of periodic signals are introduced for analyzing problems in which there is a mixture of periodic and nonperiodic signals, such as application of a periodic input to a LTI system. The Fourier transform representation for discrete-time

signals is then developed as a tool for analyzing situations in which there is a mixture of continuous-time and discrete-time signals. The sampling process and continuous-time signal reconstruction from samples are studied in detail within this context. Systems for discrete-time processing of continuous-time signals are also discussed, including the issues of oversampling, decimation, and interpolation. The chapter concludes by developing relationships between the discrete-time Fourier series and the discrete-time and continuous-time Fourier transforms in order to introduce the computational aspects of the Fourier analysis of signals.

- ▶ Chapter 5 presents an introductory treatment of linear modulation systems applied to communication systems. Practical reasons for using modulation are described. Amplitude modulation and its variants, namely, double sideband-suppressed carrier modulation, single sideband modulation, and vestigial sideband modulation, are discussed. The chapter also includes a discussion of pulse-amplitude modulation and its role in digital communications to again highlight a natural interaction between continuous-time and discrete-time signals. The chapter includes a discussion of frequency-division and time-division multiplexing techniques. It finishes with a treatment of phase and group delays that arise when a modulated signal is transmitted through a linear channel.
- ▶ Chapter 6 discusses the Laplace transform and its use for the complex exponential representations of continuous-time signals and the characterization of systems. The eigenfunction property of LTI systems and the existence of complex exponential representations for signals that have no Fourier representation are used to motivate the study of Laplace transforms. The unilateral Laplace transform is studied first and applied to the solution of differential equations with initial conditions to reflect the dominant role of the Laplace transform in engineering applications. The bilateral Laplace transform is introduced next and is used to study issues of causality, stability, invertibility, and the relationship between poles and zeros and frequency response. The relationships between the transfer function description of LTI systems and the time-domain descriptions introduced in Chapter 2 are developed.
- ▶ Chapter 7 is devoted to the  $z$ -transform and its use in the complex exponential representation of discrete-time signals and the characterization of systems. As in Chapter 6, the  $z$ -transform is motivated as a more general representation than that of the discrete-time Fourier transform. Consistent with its primary role as an analysis tool, we begin with the bilateral  $z$ -transform. The properties of the  $z$ -transform and techniques for inversion are introduced. Next, the  $z$ -transform is used for transform analysis of systems. Relationships between the transfer function and time-domain descriptions introduced in Chapter 2 are developed. Issues of invertibility, stability, causality, and the relationship between the frequency response and poles and zeros are revisited. The use of the  $z$ -transform for deriving computational structures for implementing discrete-time systems on computers is introduced. Lastly, use of the unilateral  $z$ -transform for solving difference equations is presented.
- ▶ Chapter 8 discusses the characterization and design of linear filters and equalizers. The approximation problem, with emphasis on Butterworth functions and brief mention of Chebyshev functions, is introduced. Direct and indirect methods for the design of analog (i.e., continuous-time) and digital (i.e., discrete-time) types of filters are presented. The window method for the design of finite-duration impulse response digital filters and the bilateral transform method for the design of infinite-duration impulse response digital filters are treated in detail. Filter design offers another opportunity to reinforce the links between continuous-time and discrete-time systems. The chapter builds on material presented in Chapter 4 in developing a method for the

equalization of a linear channel using a discrete-time filter of finite impulse response. Filters and equalizers provide a natural vehicle for developing an appreciation for how to design systems required to meet prescribed frequency-domain specifications.

- ▶ Chapter 9 presents an introductory treatment of the many facets of linear feedback systems. The various practical advantages of feedback and the cost of its application are emphasized. The applications of feedback in the design of operational amplifiers and feedback control systems are discussed in detail. The stability problem, basic to the study of feedback systems, is treated in detail by considering the following methods:
  - ▶ The root-locus method, related to the closed-loop transient response of the system
  - ▶ Nyquist stability criterion, related to the open-loop frequency response of the system

The Nyquist stability criterion is studied using both the Nyquist locus and Bode diagram. The chapter also includes a discussion of sampled data systems to illustrate the natural interaction between continuous-time and discrete-time signals that occurs in control applications.

- ▶ Chapter 10, the final chapter in the book, takes a critical look at limitations of the representations of signals and systems presented in the previous chapters of the book. It highlights other advanced tools, namely, time-frequency analysis (the short-time Fourier transform and wavelets) and chaos, for the characterization of signals. It also highlights the notions of nonlinearity and adaptivity in the study of systems. In so doing, the student is made aware of the very broad nature of the subject of signals and systems and reminded of the limitations of the linear, time-invariance assumption.

In organizing the material as described, we have tried to follow theoretical material by appropriate applications drawn from the fields of communication systems, design of filters, and control systems. This has been done in order to provide a source of motivation for the reader.

The material in this book can be used for either a one- or two-semester course sequence on signals and systems. A two-semester course sequence would cover most, if not all, of the topics in the book. The material for a one-semester course can be arranged in a variety of ways, depending on the preference of the instructor. We have attempted to maintain maximum teaching flexibility in the selection and order of topics, subject to our philosophy of truly integrating continuous-time and discrete-time concepts. Some sections of the book include material that is considered to be of an advanced nature; these sections are marked with an asterisk. The material covered in these sections can be omitted without disrupting the continuity of the subject matter presented in the pertinent chapter.

The book finishes with the following appendices:

- ▶ Selected mathematical identities
- ▶ Partial fraction expansions
- ▶ Tables of Fourier representations and properties
- ▶ Tables of Laplace transforms and properties
- ▶ Tables of z-transforms and properties

A consistent set of notations is used throughout the book. Except for a few places, the derivations of all the formulas are integrated into the text.

The book is accompanied by a detailed *Solutions Manual* for all the end-of-chapter problems in the book. A copy of the *Manual* is only available to instructors adopting this book for use in classrooms and may be obtained by writing to the publisher.



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Simon Haykin  
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## Notation

- [ $\cdot$ ] indicates discrete-valued independent variable, for example,  $x[n]$
- ( $\cdot$ ) indicates continuous-valued independent variable, for example,  $x(t)$
- ▶ Lowercase functions denote time-domain quantities, for example,  $x(t)$ ,  $w[n]$
- ▶ Uppercase functions denote frequency- or transform-domain quantities
  - $X[k]$  discrete-time Fourier series coefficients for  $x[n]$
  - $X[k]$  Fourier series coefficients for  $x(t)$
  - $X(e^{j\Omega})$  discrete-time Fourier transform of  $x[n]$
  - $X(j\omega)$  Fourier transform of  $x(t)$
  - $X(s)$  Laplace transform of  $x(t)$
  - $X(z)$   $z$ -transform of  $x[n]$
- ▶ Boldface lowercase symbols denote vector quantities, for example,  $\mathbf{q}$
- ▶ Boldface uppercase symbols denote matrix quantities, for example,  $\mathbf{A}$
- ▶ Subscript  $\delta$  indicates continuous-time representation for a discrete-time signal
  - $x_\delta(t)$  continuous-time representation for  $x[n]$
  - $X_\delta(j\omega)$  Fourier transform of  $x_\delta(t)$
- ▶ Sans serif type indicates MATLAB variables or commands, for example,  
`X = fft(x, n)`
- $0^\circ$  is defined as 1 for convenience
- arctan refers to the four-quadrant function and produces a value between  $-\pi$  to  $\pi$  radians.

## Symbols

$ c $	magnitude of complex quantity $c$
$\arg\{c\}$	phase angle of complex quantity $c$
$\text{Re}\{c\}$	real part of $c$
$\text{Im}\{c\}$	imaginary part of $c$
$c^*$	complex conjugate of $c$
$j$	square root of $-1$
$i$	square root of $-1$ used by MATLAB
$\mathcal{T}$	sampling interval in seconds
$T$	fundamental period for continuous-time signal in seconds
$N$	fundamental period for discrete-time signal in samples
$\omega$	(angular) frequency for continuous-time signal in radians/second

$\Omega$	(angular) frequency for discrete-time signal in radians
$\omega_o$	fundamental (angular) frequency for continuous-time periodic signal in radians/second
$\Omega_o$	fundamental (angular) frequency for discrete-time periodic signal in radians
$u(t), u[n]$	step function of unit amplitude
$\delta[n], \delta(t)$	impulse function of unit strength
$H\{\cdot\}$	representation of a system as an operator $H$
$S^\tau\{\cdot\}$	time shift of $\tau$ units
$H^{-1}, h^{-1}$	superscript $-1$ denotes inverse system
*	denotes convolution operation
$H(e^{j\Omega})$	discrete-time system frequency response
$H(j\omega)$	continuous-time system frequency response
$h[n]$	discrete-time system impulse response
$h(t)$	continuous-time system impulse response
$y^{(n)}$	superscript $(n)$ denotes natural response
$y^{(f)}$	superscript $(f)$ denotes forced response
$y^{(p)}$	superscript $(p)$ denotes particular solution
$\longleftrightarrow_{DTFS; \Omega_o}$	discrete-time Fourier series pair with fundamental frequency $\Omega_o$
$\longleftrightarrow_{FS; \omega_o}$	Fourier series pair with fundamental frequency $\omega_o$
$\longleftrightarrow_{DTFT}$	discrete-time Fourier transform pair
$\longleftrightarrow_{FT}$	Fourier transform pair
$\longleftrightarrow_{\mathcal{L}}$	Laplace transform pair
$\longleftrightarrow_{\mathcal{L}_u}$	unilateral Laplace transform pair
$\longleftrightarrow_z$	$z$ -transform pair
$\longleftrightarrow_{z_u}$	unilateral $z$ -transform pair
$\text{sinc}(u)$	$\sin(\pi u)/\pi u$
$\otimes$	periodic convolution of two periodic signals
$\cap$	intersection
$T(s)$	closed-loop transfer function
$F(s)$	return difference
$L(s)$	loop transfer function
$\epsilon_{ss}$	steady-state error
$K_p$	position error constant
$K_v$	velocity error constant
$K_a$	acceleration error constant
P.O.	percentage overshoot
$T_p$	peak time
$T_r$	rise time



$T_s$	settling time
$X(\tau, j\omega)$	short-time Fourier transform of $x(t)$
$W_x(\tau, a)$	wavelet transform of $x(t)$

## Abbreviations

A/D	analog-to-digital (converter)
AM	amplitude modulation
BIBO	bounded input bounded output
CW	continuous wave
D/A	digital-to-analog (converter)
dB	decibel
DOF	degree of freedom
DSB-SC	double sideband-suppressed carrier
DTFS	discrete-time Fourier series
DTFT	discrete-time Fourier transform
FDM	frequency-division multiplexing
FFT	fast Fourier transform
FIR	finite-duration impulse response
FM	frequency modulation
FS	Fourier series
FT	Fourier transform
Hz	hertz
IIR	infinite-duration impulse response
LTI	linear time-invariant (system)
MRI	magnetic resonance image
MSE	mean squared error
PAM	pulse-amplitude modulation
PCM	pulse-code modulation
PM	phase modulation
QAM	quadrature-amplitude modulation
ROC	region of convergence
rad	radian(s)
s	second
SSB	single sideband modulation
STFT	short-time Fourier transform
TDM	time-division multiplexing
VSB	vestigial sideband modulation
WT	wavelet transform