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Edited by A. Dold and B. Eckmann

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Analytical Methods in Probability Theory

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Editors

Daniel Dugué

Institut de Statistique, Université de Paris VI
4, Place Jussieu, 75230 Paris Cedex 05, France

Eugene Lukacs

The Catholic University of America
Washington, DC 20064, USA

Vijay K. Rohatgi

Department of Mathematics and Statistics
Bowling Green State University
Bowling Green, OH 43403, USA

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LIST OF PARTICIPANTS

Artzner, Ph., France
Bergström, H., Sweden
Blum, J. R., USA
Bützer, P. L., Germany
Chevalier, J., France
Csörgö, M., Canada
Cuppens, R., France
Davies, P. L., Germany
Deheuvels, P. France
Dugué, D., France
Geffroy, J., France
Gyires, B., Hungary
de Haan, L., The Netherlands
van Harn, K., The Netherlands
Heyer, H., Germany
Jacob, P., France
Jacobs, K., Germany
Laha, R. G., USA
Letac, G., France
Lukacs, E., USA
Roeckerath, M. Th., Germany
Rohatgi, V. K., USA
Steutel, F. W., The Netherlands
Teghem, J., Belgium
Teicher, H., USA
Theodorescu, R., Canada
Thompson, J. W., Great Britain
Vincze, I., Hungary
Wang, Y. H., Canada
Wolfe, St. J., USA

RECORD OF MEETINGS

Monday, June 9.

Morning Session, Chair: E. Lukacs

Opening of the meeting

Geffroy, J. Approximate empirical distributions of random measures

Jacob, P. Convergence stochastique des processus ponctuels à signe

Cuppens, R. Decomposition of probabilities

Afternoon Session, Chair: D. Dugué

Letac, G. Problèmes classiques de probabilité sur un couple de Gelfand

Steutel, F. W. Divisibility of Lebesgue measure

Tuesday, June 10.

Morning Session, Chair: H. Heyer

Blum, J. R. Random sampling from a continuous time stochastic process

Csörgö, M. On a test of goodness of fit based on the empirical probability measure of Foutz and testing for exponentiality

Bergström, H. Reduction of weak limit problems by transformations

Roeckerath, M. Th. Central limit theorem with large 0-rates for martingales in Banach space

Afternoon Session, Chair: H. Bergström

Davies, P. L. A theorem of Dény with applications to characterization problems

Rohatgi, V. K. On the rate of convergence in the central limit theorem

Teicher, H. Almost certain behaviour of row sums of double arrays

Wednesday, June 11.

Morning Session, Chair: M. Csörgö

Lukacs, E. Construction of characterization theorems

Dugué, D. A nonparametric test of multivariate normality

Vincze, I. On a joint characterization of the Poisson and Gamma distributions

Afternoon Excursion

Thursday, June 12.

Morning Session, Chair: P. Révész

Chevalier, J. Estimation du support d'une loi de probabilité lorsque le support est une variété

Gyires, B. New characterizations of the normal distributions

Laha, R. G., & Rohatgi, V. K. Decomposition of probability measures on locally compact Abelian groups (presented by R. G. Laha)

Thursday, June 12 (cont.).

de Haan, L.	Local limit theorems for sample extremes
<u>Afternoon Session</u> , Wolfe, St. J.	Chair: V. K. Rohatgi On the unimodality of infinitely divisible distribution functions
van Harn, K., Steutel, F. W., & Verwaart, W.	Self decomposable discrete distributions and branching processes (presented by K. van Harn)
Thompson, J. W.	Dispersive distributions and strong unimodality

Friday, June 13.

<u>Morning Session</u> , Heyer, H.	Chair: R. G. Laha An application of the method of moments to the central limit theorem on hyperbolic spaces
Bertin, M. J., & Theodorescu, R.	Some characterizations of unimodal distribution functions (presented by R. Theodorescu)
Deheuvels, P.	Multivariate tests for independence
<u>Afternoon Session</u> , Révész, P.	Chair: R. Cuppens Local time and invariance
Wang, Y. H.	Extension of Lukacs' characterization of the Gamma distribution

PAPERS PRESENTED AT THE CONFERENCE,
TO BE PUBLISHED ELSEWHERE

BUTZER, P.L. and ROECKERATH, M.-Th. Central limit theorem with
large θ -rates for martingales in Banach spaces

CUPPENS, R. Decomposition of probabilities on R and R^n

DUGUE, D. A nonparametric test of multivariate normality

GEFFROY, J. Approximate empirical distributions of a random measure

GYIRES, B. New characterizations of the normal distributions

O'BRIAN, G.L. and STEUTEL, F.W. Divisibility of Lebesgue measure

THOMPSON, J.W. Dispersive distributions and strong unimodality

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REDUCTION OF WEAK LIMIT PROBLEMS BY TRANSFORMATIONS

Harald Bergström

Department of Mathematics

Chalmers University and the University of Göteborg

Göteborg

Abstract

Weak limit problems for sequences of measures on normal, particularly metric, spaces are reduced by transformations to limit problems on simpler spaces than the original one's. Subject classification: 60 F 17, 46 G 99.

1. Alexandrov's Theorems. In earlier papers [2] and [3] I have used fundamental theorems of A.D. Alexandrov in order to prove weak convergence of sequences of measures. Then I presented a certain reduction procedure. However, some of the conclusions I made there cannot be drawn so directly from Alexandrov's theorems as I asserted. This paper will give a more complete presentation of the reduction procedure and thus is also a complement to [2] and [3].

The following concepts and notations will be used (compare [1]): S is a topological or a σ -topological space. The latter is called an Alexandrov space (shorter A-space). Any topological space is an A-space, but the converse is only true if S is separable. A Stone vector lattice of realvalued functions from S is a linear space over the real numberfield, such that it contains $\max(f,g)$ if it contains f and g . If it also contains $f \cdot g$ at the same time as f and g , it is called a Stone vector lattice ring. We shall only consider Stone vector lattices which are bounded, $\|f\| = \sup |f(x)| < \infty$ for any f in the lattice. The class of realvalued bounded continuous functions from an A-space is a Stone vector lattice ring. It will be denoted by Ψ .

An A-space is said to be normal if there to any disjoint closed

sets F_1 and F_2 exists $f \in \Psi$, $0 \leq f(x) \leq 1$ for $x \in S$, such that $f(x) = 0$ on F_1 and $f(x) = 1$ on F_2 . The function f having this property is said to connect F_1 with F_2 . A normal A -space S is called completely normal if any closed set F in S has a representation $F = \{x : f(x) = 0\}$ with $f \in \Psi$, $0 \leq f(x) \leq 1$ on S . A metric space is completely normal. A linear functional L on a Stone vector lattice $\mathcal{C}$ is a mapping $f \in \mathcal{C}$ from $\mathcal{C}$ into R such that

$$L(\alpha f + \beta g) = \alpha L(f) + \beta L(g)$$

for any real numbers α and β and $f, g \in \mathcal{C}$. We say that L is non-negative if $f(x) \geq 0$ for all x implies $L(f) \geq 0$, bounded if $|L(f)| < \infty$ for $\|f\| < \infty$.

A finitely additive non-negative set function μ on an algebra of subsets on S is called a measure if $\mu(S) < \infty$. If μ is σ -additive we call it a σ -smooth measure. When we say that μ is a measure on A -spaces we require that it is a measure on the algebra generated by the closed sets, but on the σ -algebra generated by these sets if μ is σ -smooth. The abstract integral of any realvalued function f from S into R with respect to a measure μ on S is denoted by $\mu(f)$. Note that $\mu(f)$ determines a non-negative, bounded functional on Ψ .

A sequence $\{\mu_n\}$ of measures on an A -space S is said to converge weakly to a measure μ on S if $\mu_n(f) \rightarrow \mu(f)$ ($n \rightarrow \infty$) for any $f \in \Psi$.

Alexandrov's theory contains theorems which may be given in the following form

THEOREM 1. Let L be a non-negative, bounded linear functional on a Stone vector lattice ring Ψ_0 of realvalued bounded continuous functions from a normal σ -topological space S , where Ψ_0 contains the function $x \rightarrow 1$ from S and, to any two disjoint closed disjoint sets, a function connecting these sets. Then L determines uniquely a regular measure μ on S such that $\mu(f) = L(f)$ on Ψ_0 .

THEOREM 2. Let μ be a regular measure on a normal space S and Ψ_0 a Stone vector lattice of realvalued, bounded, continuous functions where Ψ_0 contains the function $x \rightarrow 1$ from S . Further suppose that there, to any closed set F and any $\epsilon > 0$, exist an open set G_ϵ with $\mu(G_\epsilon \setminus F) < \epsilon$ and a function in Ψ_0 connecting F and $S \setminus G_\epsilon$.

If $\{\mu_n\}$ is a sequence of measures on S , then the following conditions imply each other:

- (i) $\{\mu_n\}$ converges weakly to μ
- (ii) $\mu_n(f) \rightarrow \mu(f) (n \rightarrow \infty)$ for $f \in \Psi_0$.
- (iii) $\limsup_{n \rightarrow \infty} \mu_n(F) \leq \mu(F)$ for any closed set F , and $\mu_n(S) \rightarrow \mu(S)$ $(n \rightarrow \infty)$.
- (iv) $\liminf_{n \rightarrow \infty} \mu_n(G) \geq \mu(G)$ for any open set G , and $\mu_n(S) \rightarrow \mu(S) (n \rightarrow \infty)$
- (v) $\lim_{n \rightarrow \infty} \mu_n(E) = \mu(E)$ for any continuity set in S with respect to μ , and $\mu_n(S) \rightarrow \mu(S) (n \rightarrow \infty)$.

THEOREM 3. If a sequence $\{\mu_n\}$ of σ -smooth measures on a completely normal space S converges weakly to a measure μ , then μ is σ -smooth.

2. The Reduction Procedure. We shall deal with mappings π of an A -space S onto or into an A -space S' . Then we consider the algebras $\mathcal{S}$ and $\mathcal{S}'$ generated by the closed sets in S and S' respectively. We say that π is measurable S/S' if $\pi^{-1} E' \in \mathcal{S}$ for any $E' \in \mathcal{S}'$. If π is continuous then, of course $\pi^{-1} F'$ is a closed set in S for any closed set F' in S' . The reduction procedure is given by

LEMMA 1. Let the following conditions hold for a normal A -space S :

- (i) S is mapped onto a normal A -space $\mathcal{S}^{(r)}$ by a measurable mapping π_r and $\mathcal{S}^{(r)}$ is mapped onto $S^{(r)} \subset S$ by a continuous mapping V_r for $r = 1, 2, \dots$, where $\tilde{S}^{(r)}$ is projection of $\tilde{S}^{(s)}$ for $r > s$.
- (ii) Ψ_0 is a Stone vector lattice ring of realvalued, bounded, continuous functions from S , such that Ψ_0 contains the function $x \rightarrow 1$, and, to any disjoint closed sets F_1 and F_2 , a function which connects F_1 and F_2 .
- (iii) $\{\mu_n\}$ is a sequence of measures on S , such that $\sup \mu_n(S) < \infty$, and $\{\mu_n(\pi_r^{-1})\}$ converges weakly to a measure $\tilde{\mu}^{(r)}$ on $\mathcal{S}^{(r)}$ for $r = 1, 2, \dots$.
- (iv) $\lim_{r \rightarrow \infty} \limsup_{n \rightarrow \infty} \mu_n\{|f(x) - f(V_r \pi_r x)| \geq \varepsilon\} = 0$ for any $\varepsilon > 0$, $f \in \Psi_0$

Then, 1^o $\{\mu_n\}$ converges weakly to a measure μ on S as $n \rightarrow \infty$, 2^o if the μ_n are σ -smooth, then μ is σ -smooth, 3^o if the π_r are continuous, we have $\tilde{\mu}^{(r)} = \mu(\pi_r^{-1})$. Conversely, if the π_r are continuous, and if

$\{\mu_n\}$ is a sequence of σ -smooth measures, converging weakly to a measure μ , then (iii) necessarily holds; if furthermore

$$f(x) - f(V_r \pi_r x) \rightarrow 0 (r \rightarrow \infty) \quad \text{a.s.} \mu$$

for $f \in \Psi_0$, also (iv) is satisfied.

PROOF. For $f \in \Psi_0$ put

$$(1) \quad L_n(f) = \int_S f(x) \mu_n(dx)$$

$$(2) \quad L_n^{(r)}(f) = \int_{\tilde{S}(r)} f(V_r \tilde{x}^{(r)}) \mu_n(\pi_r^{-1} d\tilde{x}^{(r)}).$$

By transformation of the integral we can write (2)

$$(3) \quad L_n^{(r)}(f) = \int_S f(V_r \pi_r x) \mu_n(dx)$$

Regarding (iv) and the inequality in (iii), we get from (1) and (3)

$$(4) \quad \lim_{r \rightarrow \infty} \limsup_{n \rightarrow \infty} |L_n^{(r)}(f) - L_n(f)| = 0.$$

Further (iii) implies

$$(5) \quad \lim_{n \rightarrow \infty} L_n^{(r)}(f) = L^{(r)}(f) = \int_{\tilde{S}(r)} f(V_r \tilde{x}^{(r)}) \tilde{\mu}^{(r)}(d\tilde{x}^{(r)}),$$

since $f(V_r \cdot)$ is a bounded, continuous function from $S^{(r)}$ into $\mathbb{R}$. By (5) we obtain for positive integers r_1 and r_2 , $f \in \Psi_0$,

$$|(L^{(r_1)}(f) - L^{(r_2)}(f))| = \lim_{n \rightarrow \infty} |L_n^{(r_1)}(f) - L_n^{(r_2)}(f)|$$

$$\leq \limsup_{n \rightarrow \infty} |L_n^{(r_1)}(f) - L_n(f)| + \limsup_{n \rightarrow \infty} |L_n(f) - L_n^{(r_2)}(f)|$$

As $r_1 \rightarrow \infty$, $r_2 \rightarrow \infty$, the right hand side of the inequality tends to 0 according to (4). Hence $\{L^{(r)}(f)\}$ is Cauchy convergent and thus convergent.

$$(6) \quad \lim_{r \rightarrow \infty} L^{(r)}(f) = L(f)$$

It follows by the properties of limits, that L is a non-negative,