PRINCIPLES OF PLASMA PHYSICS

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PREFACE

This textbook was developed to provide graduate students in physics and astronomy at the University of Maryland with a comprehensive background in plasma physics. It was designed for a two-semester course, normally taken in the second year of graduate study by students who have completed the usual sequence of graduate courses in classical and statistical mechanics, electromagnetism, and mathematical methods. It is a basic introduction to the field and assumes no background in plasma physics. Although it was intended as a graduate text, it could be used for an advanced undergraduate course by proper selection of material.

The basic plan of the book is to begin with the exact statistical description of a many-body system and then to obtain various reduced descriptions of the plasma state. The most reduced description, called fluid theory, or macroscopic theory, is developed first and is used to explore a wide range of plasma phenomena and problems involving equilibrium, waves, and instabilities. Macroscopic theory, together with transport phenomena in plasmas (Chapter 6), are typically covered in the first semester of the course.

Next, the book takes up a less reduced description, the Vlasov or microscopic theory, based on a continuous distribution function of velocity and configuration space, and again considers problems of plasma equilibrium, waves, and instabilities, in addition to nonlinear interactions of waves and the plasma distribution. This "Vlasov-Maxwell" description of the plasma state predicts phenomena that depend on the details of the velocity-space distribution function, including, for example, Landau damping, velocity-space instabilities, velocity-space diffusion, etc. The Vlasov results for these plasma properties are compared with the results of the somewhat simpler fluid treatment of the earlier chapters.

Correlations, fluctuations, and radiation, being phenomena that depend on discreteness, are not included in either of the above reduced descriptions. A test-particle approach is used to calculate these quantities. Further discrete properties of plasmas are obtained from higher-order reduced descriptions of the exact statistical equations, leading to the Fokker-Planck or Balescu-Lenard equations and calculations of transport coefficients for plasmas.

The theory of the orbits of single-charged particles in static and time-varying electric and magnetic fields is included as an appendix, consistent with our objectives of trying to be complete, and at the same time not devote extensive text space to treatment of topics that are simple and adequately treated in many readily available reference sources.

One unique feature of this book is an extensive introductory chapter intended to eliminate the "jargon gap" that we found to be a major stumbling block for students first encountering plasma physics. Most of our students have an adequate background in mathematics and classical physics, but many of the concepts and the terminology associated with plasma physics are unfamiliar to them, creating an artificial difficulty.

This introductory chapter is really a short course (three to four lectures) in the concepts and terminology of plasma physics. As such it is useful to anyone who desires a qualitative introduction to the field. Another unique feature is a chapter on the equilibrium statistical mechanics of plasmas. The purpose of this chapter is to provide the student with the proper perspective of plasma physics as a many-body problem and the plasma state as a statistical system. This chapter calculates the thermodynamic properties of the equilibrium plasma, and sets the stage for the rest of the book, which deals with the many realistic situations in which the plasma is not in a state of thermodynamic equilibrium.

With regard to references, we selected primarily books and review papers that we found of particular value to students. It is our opinion that a proper grasp of the physics is the overriding concern, to which the historical aspects of the field are subordinate.

Recent advances in the use of digital computers to simulate a plasma have provided considerable insight into plasma processes. Some simulation results are included where they amplify or extend a particular analytical result, or contribute to a better understanding of plasma properties.

There are many problems in each chapter, varying in difficulty from routine algebraic manipulations leading to well-known results to challenging problems that will help the student interested in research in plasma physics obtain a better feeling for this diverse and intriguing field.

Gaussian-cgs units are used throughout this book, since their use is widespread in the plasma physics research literature. A conversion table from Gaussian-cgs units to standard practical units for quantities such as resistivity, capacitance, electric field strength, etc., is included in Appendix III, along with a list of the most frequently occurring symbols (such as plasma frequency, cyclotron frequency, etc.).

A project as extensive as this text naturally involves the contribution of a large number of people.

We thank our friends and colleagues in plasma physics with whom we have worked and learned about this subject. We thank the students who took the plasma physics course at Maryland during the period this book was being written. Their conscientious help and advice in the development of the course and the textbook was very valuable to us. We are particularly grateful to Ted Northrop, Derek Tidman, and Allan Kaufman for their early suggestions and advice on how to develop and teach a graduate course in plasma physics. We benefited greatly from their previous experience in teaching plasma physics. We appreciate the advice, encouragement, and support of our colleagues at Maryland, particularly Hans Griem, Ron Davidson, and Bob Pechacek. We are grateful to our Department Chairman, Howard Laster, for allowing us to concentrate our efforts on teaching and developing the plasma physics course on which this book is based. We appreciate the careful reading of one or more chapters of the manuscript by Ron Davidson, Alan DeSilva, Seishi Hamasaki, John Hey, Walt Jones, Chris Kapetanakos, Hank Klein, Paulett Liewer, Marvin Schwartz, Don Spero, Derek Tidman, and Maria Zales-Caponi. Our special thanks go to Mrs. Mary Ann Ferg and Mrs. Clara Rodriguez for their excellent work in typing the various drafts of the manuscript, and to Mrs. Barbara Hornady for preparing the Indexes.

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INTRODUCTION TO PLASMA PHYSICS

Plasma physics is the study of charged particles collected in sufficient number so that the long-range Coulomb force is a factor in determining their statistical properties, yet low enough in density so that the force due to a near-neighbor particle is much less than the long-range Coulomb force exerted by the many distant particles. It is the study of low-density ionized gases. The term "plasma" was first used to describe a collection of charged particles by Tonks and Langmuir,[1] in 1929, in their studies of oscillations in electric discharges. However, the most characteristic aspect of the plasma state, the fact that because of the long range of the Coulomb force the charged particles exhibit a collective behavior, was known much earlier, and was probably first described by Lord Rayleigh,[2] in 1906, in his analysis of electron oscillations in the Thomson model of the atom.

The term "fourth state of matter," often used to describe the plasma state, was coined by W. Crookes[3] in 1879 to describe the ionized medium created in a gas discharge. The term fourth state of matter follows from the idea that as heat is added to a solid, it undergoes a phase transition to a new state, usually liquid.

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[1] L. Tonks and I. Langmuir, Oscillations in Ionized Gases, *Phys. Rev.*, **33**:195 (1929).

[2] Lord Rayleigh, *Phil. Mag.*, **11**:117 (1906).

[3] W. Crookes, *Phil. Trans.*, **1**:135 (1879).

If heat is added to a liquid, it undergoes a phase transition to the gaseous state. The addition of still more energy to the gas results in the ionization of some of the atoms. At a temperature above 100,000°K most matter exists in an ionized state; this ionized state of matter is called the *fourth state.* A plasma state can exist at temperatures lower than 100,000°K provided there is a mechanism for ionizing the gas, and if the density is low enough so that recombination is not rapid.

Although 99.9 percent of the apparent universe exists in a plasma state, there is very little in the way of natural plasma here on earth because the low temperature and high density of the earth and its near atmosphere preclude the existence of plasma. This means that plasma must be created by experimental means to study its properties. However, in the upper atmosphere (ionosphere), plasma does exist, created by photoionization of the tenuous atmosphere. Farther out from the earth, plasma is trapped in the earth's magnetic field in the near vacuum of space. Plasma streams toward the earth from the sun (the solar wind), and fills many regions of interstellar space, forming the medium through which outer space is viewed.

Plasma physics generally involves the well-known physics of classical mechanics, electromagnetism, and nonrelativistic statistical mechanics. The challenge of plasma physics comes from the fact that many plasma properties result from the long-range Coulomb interaction, and therefore are collective properties that involve many particles interacting simultaneously.

In its simplest form, a plasma is a collection of protons and electrons at sufficiently low density so that binary (short-range) interactions are negligible. Many-body theory, or the many-body problem, is the study of the properties of such a medium. When a collection of protons and electrons coexist in an equilibrium state, the properties of this state are described by equilibrium statistical mechanics with the appropriate Gibbs ensemble. However, most of the interesting features of plasmas occur for nonequilibrium situations.

Revived interest in plasma physics in the United States began in 1952 with the attempts of a program, then classified, known as Project Sherwood,[1] to develop a controlled thermonuclear fusion reactor. Similar programs were started in England, France, and the U.S.S.R. at about the same time. These programs have grown substantially since that time, and now there are many nations with major research programs in this field. Although the development of a controlled fusion reactor is one of the more challenging practical applications of plasma physics, it is only one of the many areas in which plasma physics plays a role. Plasma physics has played a major role in the development of much

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[1] A. S. Bishop, "Project Sherwood," Addison-Wesley, Reading, Mass., 1958.

of contemporary physics, and it is important in the study of problems in such areas as astrophysics, atomic physics, chemistry, life sciences, molecular physics, magnetohydrodynamic power generation, and atmospheric physics.

Plasma physics has its own vocabulary and set of ideas. The main purpose of this chapter is to review plasma physics on an elementary level, provide a background sketch of the familiar concepts of the field, identify many of the terms used repeatedly in discussing the plasma state, review some of the schemes by which plasma is produced in the laboratory, and review some of the methods by which plasma properties are measured.

PART ONE: PLASMA CONCEPTS AND TERMINOLOGY

1.1 EQUILIBRIUM AND METAEQUILIBRIUM

The term "equilibrium" is often loosely used in plasma physics to describe a quasi-steady-state condition that persists only until the plasma particles collide with each other. Frequently, plasma studies are made by investigating small perturbations about such a *metaequilibrium* state.

Thermodynamic equilibrium means that the ions and the electrons are each described by a maxwellian distribution characterized by the same single parameter, the temperature. In this situation, the medium is in equilibrium with its surroundings, and it radiates and absorbs energy at the same rate. The spectrum of emitted radiation is *blackbody*.

In many of the theoretical and experimental situations of interest in plasma physics, the ions and electrons are neither at the same temperature nor in thermodynamic equilibrium with their surroundings. The term metaequilibrium is used to describe situations that eventually will be altered by binary collisions.

1.2 DEBYE LENGTH

The electrostatic potential of an isolated particle of charge q is

$$\phi = \frac{q}{r} \qquad (1.2.1)$$

In a plasma, electrons are attracted to the vicinity of an ion and shield its electrostatic field from the rest of the plasma. Similarly, an electron at rest repels other electrons and attracts ions. This effect alters the potential in the vicinity of a charged particle. The potential of a charge at rest in a plasma is given by

$$\phi = \frac{q}{r} e^{-r/\lambda_D} \qquad (1.2.2)$$

where λ_D is the Debye length originally defined in the Debye-Hückel theory of electrolytes. For an electron-proton plasma

$$\lambda_D = \left(\frac{\kappa T}{8\pi n e^2}\right)^{1/2} = 4.9\left(\frac{T}{n}\right)^{1/2} \qquad (1.2.3)$$

where n = density of electrons (or ions), cm^{-3}
T = temperature, °K
κ = Boltzmann's constant ($= 1.38 \times 10^{-16}$ ergs/°K)

The Debye length is a measure of the sphere of influence of a given test charge in a plasma. In general, the Debye length depends on the speed of the test charge with respect to the plasma.

1.3 PLASMA PARAMETER

The plasma parameter g indicates the number of plasma particles in a Debye sphere, and is defined by

$$g = \frac{1}{n{\lambda_D}^3} \qquad (1.3.1)$$

For Debye shielding to occur, and for the description of a plasma to be statistically meaningful, the number of particles in a Debye sphere must be large; that is, $g \ll 1$. The assumption $g \ll 1$ is called the *plasma approximation*. The plasma parameter is also a measure of the ratio of the mean interparticle potential energy to the mean plasma kinetic energy. An ideal gas corresponds to *zero* potential energy between the particles. In many situations the plasma parameter is small and the plasma is treated as an ideal gas of charged particles, that is, a gas that can have a charge density and electric field but in which no two *discrete* particles interact.

To ensure that $n{\lambda_D}^3$ be large, the density must be *low*, since

$$g = \frac{1}{n{\lambda_D}^3} \propto \frac{n^{1/2}}{T^{3/2}}$$

Because the collision frequency decreases with density n, and also decreases with increasing temperature T, the condition $g \to 0$ corresponds to a decreasing collision frequency.

Problem 1.3.1 Show that, if ratio of mean kinetic energy to mean interparticle potential energy is much greater than 1, that is, if

$$\frac{\langle KE \rangle}{\langle PE \rangle} \gg 1$$

the number of particles per Debye sphere $n\lambda_D{}^3$ must also be much larger than 1. ////

The plasma parameter g is one of the more important dimensionless parameters associated with a plasma, and may be interpreted as a measure of the degree to which plasma or collective effects dominate over single-particle behavior. The plasma state is described by equations obtained from an expansion of the exact many-body equations in powers of g.

A distinctive constrast between the statistical mechanics of a plasma and that of a neutral gas is that in a plasma the expansion parameter g is small when *many* particles interact at the same time, since $\lambda_D{}^3$ is essentially the volume of the interaction region; for a neutral gas, the atomic radius R is a measure of the interaction region and nR^3 ($\ll 1$) is the expansion parameter. The plasma behaves as a nearly ideal gas *in spite of* the presence of many interacting particles; the reason is that the strength of the interaction between individual particles is so weak, as shown in Prob. 1.3.1.

1.4 DISTRIBUTION FUNCTION

The most detailed description of a plasma gives the location and velocity of each plasma particle as a function of time. It is impossible to obtain such a description of a real plasma, except in some recent "experiments" which involve the use of digital computers to follow the position and velocity of a large number of ions and electrons. Therefore it is customary to use the distribution function f to describe a plasma. The distribution function is the number of particles per unit volume in six-dimensional velocity-configuration phase space. From the Boltzmann H theorem[1] it is known that under the action of binary collisions an ideal gas relaxes to a maxwellian distribution of velocities.

$$\bar{n}f(v) = \bar{n}\left(\frac{m}{2\pi\kappa T}\right)^{3/2} e^{-mv^2/2\kappa T} \qquad (1.4.1)$$

where $\bar{n} = N/V$, with N the number of particles of a certain type (e.g., ions or electrons) in the system and V the volume of the system. Although laboratory plasmas probably never achieve exactly a maxwellian distribution, they may approach it closely, and it is useful in many theoretical treatments to assume that the plasma is described by a maxwellian velocity distribution.

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[1] S. Chapman and T. G. Cowling, "The Mathematical Theory of Non-uniform Gases," Cambridge, London, 1952.