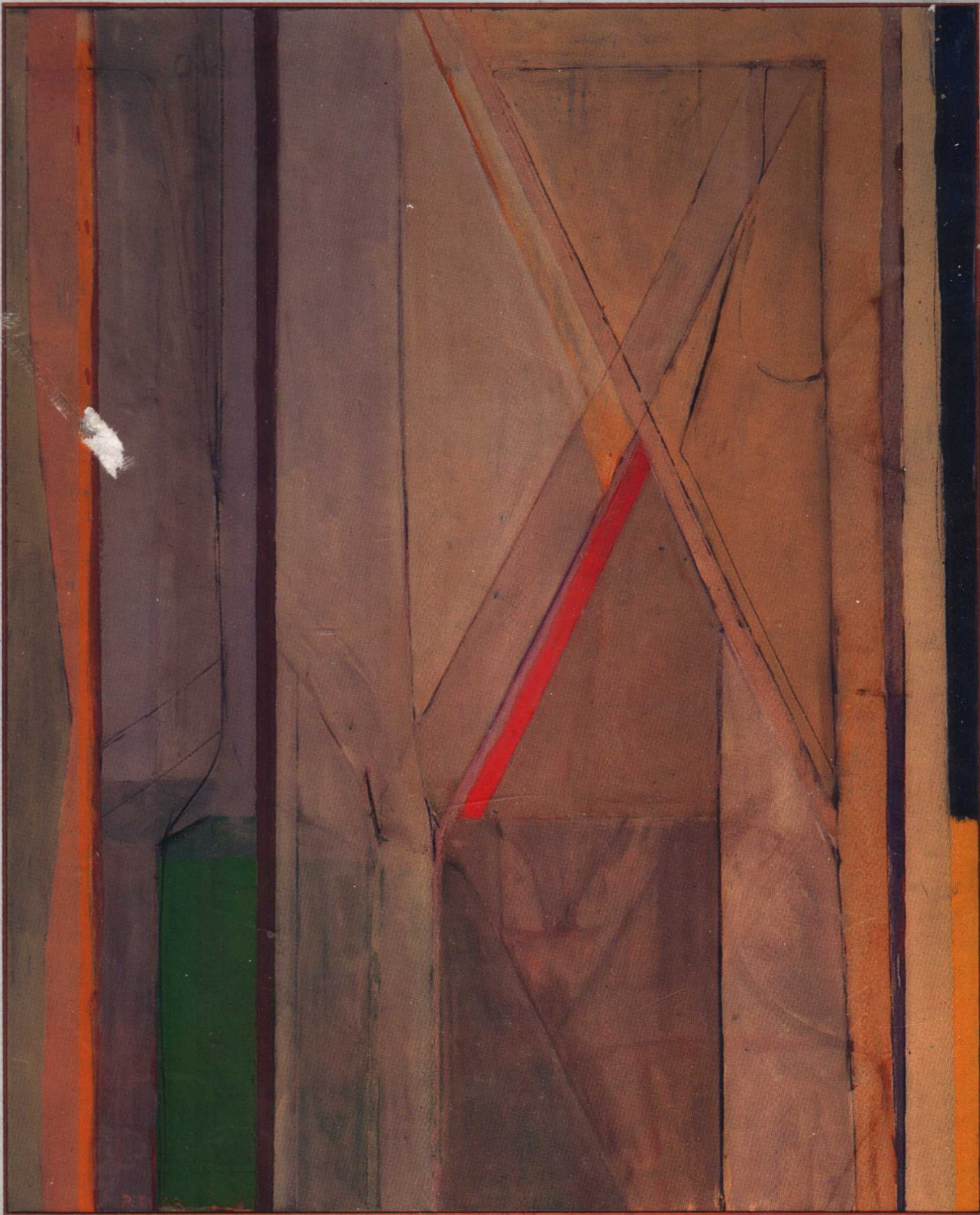
INTERMEDIATE ALGEBRA



Gene Sellers

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Preface

The mathematical concepts and principles presented in *Intermediate Algebra* can be mastered by students in a one-semester or two-quarter course. The topics bridge the gap between elementary algebra and courses that are generally considered pre-calculus. Students who successfully complete a course using this text will be prepared to continue their mathematical studies: in a college algebra course; in a trigonometry course; in many applied mathematics courses, such as statistics; or in a science course that requires mathematical competence, such as physics.

Problem-solving orientation

This text has a problem-solving orientation that leads students through the material in an organized fashion and leads them step by step through the many examples and solutions. The following are principle features of this design.

First, each section begins with a list of key topics covered in the section. Within the section, sub-headings clearly indicate where the topic is presented; each section is thus fragmented into smaller portions that are easy for students to assimilate.

Second, most sections begin with a short introductory paragraph linking the material in the section to previously learned skills, or to a skill that will be introduced later in the text. This feature provides answers to the questions, "What have we learned before?", "What are we going to learn now?", and "What will we learn later?"

Third, most general mathematical statements are followed by explicit examples labeled **a., b., c.,** and so on, that illustrate what the symbols in the general statement may represent in a specific example.

Fourth, every section contains numerous worked-out examples, each showing all the steps leading to a detailed solution. Marginal comments on many solutions provide additional insight into how the problem was solved.

Fifth, every section includes a large number of exercises, giving students ample opportunity to develop their skills through practice. Marginal references indicate which examples in the section illustrate the method of solution for a given block of exercises. Should students have difficulty working a problem, the reference guides them to the example with the detailed solution that will assist them.

Sixth, in the exercise sets of many sections are one or two additional examples that illustrate a concept related to others in that section, or that solve *more difficult* problems. These examples and related exercises should be treated as optional material that the instructor may either ignore or use to challenge students. The exercises can be assigned as extra credit problems or worked in class to increase the mathematical scope of all the students.

Graded exercise sets

The problems in most exercise sets accomplish several goals. First, they survey numerous variations of a particular type of problem. Second, the level of difficulty of each variation increases within the exercise set, from easy to moderately difficult to most difficult. Third, many sets contain problems that can have different kinds of answers, such as the solutions to equations. Here, the answers represent each level of difficulty. Fourth, the exercise sets frequently include applied problems, in sufficient quantity for each topic. Finally, each exercise set is intended to challenge the students but not to overwhelm them. Therefore, the problems can be worked by applying the techniques illustrated in the example, and do not require any skill beyond the abilities of the average student at this level.

Concise and accurate

In this text words are kept to a minimum. Each mathematical principle is stated as concisely and accurately as possible. Although explanations are provided when necessary, words are replaced with graphic illustrations whenever they can be. Such illustrations give a structure to the material that helps students learn. The lists of key points, and the step-by-step procedures for problem solving, train students to organize similar material on their own.

Chapter reviews

A review at the end of each chapter consists of moderately difficult exercises representing each section of the chapter, with each exercise keyed to the section from which it was taken. These exercises can be used in class to review for chapter tests, or selections of them might be used for take-home tests.

Cumulative reviews

Beginning with Chapter 2, each chapter review is followed by a cumulative review containing about thirty problems from the chapters covered so far. These reviews are a constant reminder to the student that previously learned skills must not be forgotten. Studying the cumulative reviews can also reduce the anxiety associated with a comprehensive final examination.

Extended coverage

At the end of the text are five appendices, "Sets," "Synthetic Division," "Significant Digits," "Rounding a Number," and "Interpolation." The appendices are followed by three tables: "Powers and Roots," "Logarithms," and "Exponential Functions." Following the tables are answers to all the odd-numbered exercises, and selected answers to the chapter reviews and cumulative reviews. An index is also included.

Instructional aids

In addition to the text itself, supplementary materials are available. The *Study Guide*, for students, has eleven chapters corresponding to the chapters in the main text, each consisting of three parts. Part A summarizes the material in the text chapter, Part B offers completely worked solutions to 25% of the problems in a chapter exercise set, and Part C contains practice tests with answers. The *Instructor's Manual* offers seven tests with answers for each chapter in *Intermediate Algebra*, including cumulative review tests. Also included in this manual are the answers to the even-numbered problems in the text.

Acknowledgments

There are several people that I would like to acknowledge for their help in this project. First, I want to thank Craig Barth, Mathematics Editor at Brooks/Cole, for his help in developing this text. He provided much valuable assistance throughout the project. Second, I am deeply indebted to Production Editor Penelope Sky. The patience that she graciously extended to me was greatly appreciated. Furthermore, her intense desire to produce an accurate and beautiful book resulted in a product that truly reflects this attitude. Third, I wish to thank Jamie Sue Brooks for a design and cover that enhance the teaching aspects of the book.

Finally, I want to thank David Conroy, Northern Virginia Community College; Steven Hatfield, Marshall University; Herbert Kasube, Bradley University; Edwin Schulz, Elgin Community College; and Jerry Wilkerson, Missouri Western State College for their constructive reviews of the manuscript. Many of the positive suggestions they shared with me have made this book a better text.

I wish to dedicate this book to my wife, Linda, whose love and encouragement are a constant source of strength and happiness.

Gene Sellers

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CHAPTER

1

THE REAL NUMBERS

An intermediate algebra course builds on many of the mathematical concepts and skills learned in a beginning algebra course. However, many basic concepts from elementary algebra will be reviewed in Chapter 1. Specifically, the sets of numbers that comprise the real numbers will be identified.* Then the relations used to compare these numbers, and the number line used to graph real numbers, will be studied. The properties of the real numbers will also be reviewed in this chapter. Finally, the operations of addition, subtraction, multiplication, and division of real numbers will be reviewed.

Section 1-1 The Subsets of the Real Numbers

KEY TOPICS

1. Whole numbers
2. Multiple and factor
3. Integers
4. Rational numbers
5. Irrational numbers
6. Real numbers
7. Number line

For many reasons mathematicians have found it useful to separate numbers into sets. The numbers that are members of a given set are identified by some clearly defined characteristic that each number in the set must possess. In this section the characteristics that identify the subsets of the real numbers will be studied.

Whole numbers

The number zero, together with the numbers used in counting, comprise the set of **whole numbers**.

> The set of whole numbers is **W**, and
>
> $\mathbf{W} = \{0, 1, 2, 3, 4, 5, \ldots\}$

*Appendix A contains a review of those terms from set theory that will be used in this text.

A subset of **W** is the set of **natural numbers**, which are also called counting numbers. If **N** is the set of natural numbers, then

$$\mathbf{N} = \{1, 2, 3, 4, 5, \ldots\}$$

Multiple and factor

Two words frequently used in mathematics are **multiple** and **factor**. The following description explains these words by giving the multiples of a whole number m. Later in this section is a description of the factors of a whole number m.

> If m is a whole number, then the products
>
> $1 \cdot m, 2 \cdot m, 3 \cdot m, 4 \cdot m, \ldots$ are multiples of m.

When a letter is used to represent more than one number in a set, the letter can be called a **variable**. For example, in the statement on multiples, the letter m is a variable, since it represents more than one whole number. If a letter is used to represent exactly one number in a set of numbers, then the letter can be called a **constant**.

Example 1 List the first five multiples of 8.

Solution The first five multiples of 8 are the products of 8 and 1, 2, 3, 4, and 5. Thus,

$$1 \cdot 8 = 8 \qquad 2 \cdot 8 = 16 \qquad 3 \cdot 8 = 24$$
$$4 \cdot 8 = 32 \qquad 5 \cdot 8 = 40$$

and 8, 16, 24, 32, 40 are the first five multiples of 8. ∎

If m and n are whole numbers, then there are whole numbers that are multiples of both m and n. Such whole numbers are called **common multiples** of m and n. For example, 72 is a common multiple of 8 and 12, since $8 \cdot 9 = 72$ and $12 \cdot 6 = 72$.

For any two whole numbers m and n, there are infinitely many common multiples. The smallest (or least) of these numbers is called the **least common multiple**, written LCM.

Example 2 Find **a.** the common multiples of 10 and 15 and **b.** the LCM of 10 and 15.

Solution **a.** First list the multiples of 10 and 15. The common multiples are circled.

10, 20, (30) 40, 50, (60) 70, 80, (90) . . .
15, (30) 45, (60) 75, (90) 105, 120, . . .

b. Since 30 is the first number in the list of common multiples, the LCM of 10 and 15 is 30. ∎

> If m is a whole number and n is a natural number that divides m with a remainder of 0, then n is a **factor** of m.

Example 3 List all the factors of the given numbers.

a. 24 **b.** 29

Solution **a.** The factors of 24 are any whole numbers greater than or equal to 1 that evenly divide 24.

$1 \cdot 24 = 24$	therefore	1 and 24 are factors of 24
$2 \cdot 12 = 24$	therefore	2 and 12 are factors of 24
$3 \cdot 8 = 24$	therefore	3 and 8 are factors of 24
$4 \cdot 6 = 24$	therefore	4 and 6 are factors of 24

Thus 1, 2, 3, 4, 6, 8, 12, and 24 are the factors of 24.

b. $1 \cdot 29 = 29$

Thus 1 and 29 are factors of 29. ■

There are some whole numbers greater than 1 that have only two distinct factors, namely, the number itself and 1. Such numbers are called **prime numbers**. The first twenty prime numbers are

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, and 71

Integers

Numbers with values less than zero are called **negative numbers**. A minus bar ($-$) is used to indicate a negative number. To illustrate, -5 is read "negative 5," and -5 is less than zero. The union of the whole numbers and the negative natural numbers is the set of numbers called integers.

> The set of **integers** is denoted by **I**, where
>
> $\mathbf{I} = \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$

Note 1 All whole numbers are also integers.

Rational numbers

A comparison of two numbers by division can be called a *ratio*. When a ratio of two integers is formed and the second one is not zero, the number so formed is called a rational number.

> If m and n are integers and n does not equal zero ($n \neq 0$), then m/n is a **rational number**.

Any number that can be written in the form of an integer m divided by a nonzero integer n is a rational number.

Example 4 Write the integer 10 in the m/n form of a rational number.

Solution Since 10 can be written as $\frac{10}{1}$ or $\frac{20}{2}$ or $\frac{30}{3}$ and so on, the integer 10 is a rational number. ■

Note 2 All integers are also rational numbers.

Example 5 Write the *mixed number* $2\frac{1}{3}$ in the m/n form of a rational number.

Solution $2\frac{1}{3} = 2 + \frac{1}{3} = \frac{6}{3} + \frac{1}{3} = \frac{6+1}{3} = \frac{7}{3}$

Since 7 and 3 are integers, $\frac{7}{3}$ is a rational number. ■

Note 3 All mixed numbers are rational numbers.

Example 6 Write the decimal fraction 0.37 in the m/n form of a rational number.

Solution $0.37 = \frac{37}{100}$

Since 37 and 100 are integers, 0.37 is a rational number. ■

Note 4 All decimal fractions (also called **terminating decimals**) are rational numbers.

The decimal representations of the rational numbers $\frac{1}{3}$, $\frac{5}{11}$, and $\frac{4}{7}$ do not terminate. Such a rational number has a decimal representation with a pattern of digits that repeats at some point. To illustrate,

$\frac{1}{3} = 0.3333\ldots$	the digit 3 repeats
$\frac{2}{11} = 0.181818\ldots$	the digits 18 repeat
$\frac{4}{7} = 0.571428571428\ldots$	the digits 571428 repeat

It can be shown that if a number has a decimal representation with a repeating pattern of digits (called the *period* of the number), then that number can be written in the m/n form of a rational number. Such numbers are called **periodic decimals** (or *repeating decimals*).

Note 5 All periodic decimals are rational numbers.

Irrational numbers

There are infinitely many numbers whose decimal representations do not terminate or repeat a period of digits. Such numbers cannot be written in the m/n form of a rational number. These numbers are called **irrational**.

Examples **a–e** illustrate some irrational numbers.

a.	$0.12112111211112\ldots$	**b.**	$-1.57557555755557\ldots$
c.	$\sqrt{2} = 1.414213562\ldots$	**d.**	$-\sqrt{3} = -1.7320508\ldots$
e.	$\pi = 3.14159265\ldots$		

Real numbers

If a number is rational, then it cannot be irrational. If a number is irrational, then it cannot be rational. The union of the sets of rational and irrational numbers is the set of **real numbers**. In other words, any real number is either a rational number or an irrational number.

The diagram in Figure 1-1 shows the several subsets of numbers that comprise the set of real numbers.

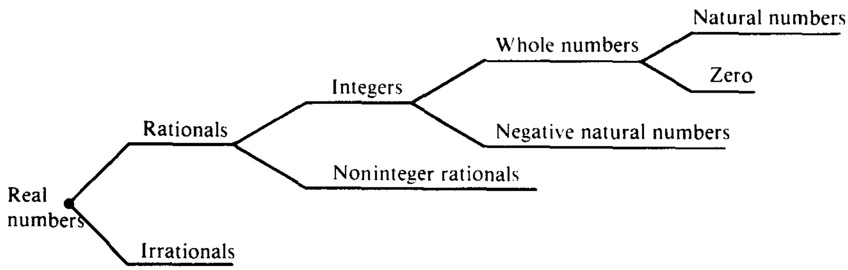

Figure 1-1 The subsets of the real numbers

Number line

A geometric line can be used as a visual model of the set of real numbers. To each point of the line is assigned a number, called the **coordinate** of the point. For each real number there is a point, called the **graph** of the number. The set of points of a line and the set of real numbers form a **one-to-one correspondence**, that is, for each point there is a number coordinate and for each number there is a graph.

Usually a few integers are shown on a number line to indicate the **unit length** of the line, that is, the distance between any two consecutive integers. An arrow on the end of a number line shows the direction of the increasing values of numbers on the line. A number line is shown in Figure 1-2.

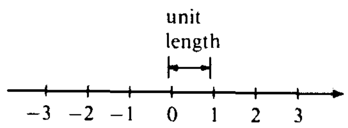

Figure 1-2 A number line

Example 7 Graph the following numbers on a number line.

a. 4 **b.** $\sqrt{2}$ **c.** $-\pi$

Solution

a. The number 4 is four units to the right of 0. Therefore point A in Figure 1-3 is the graph of 4.

b. Since $\sqrt{2} = 1.414213\ldots$, the graph is between 1 and 2. Point B in Figure 1-3 is the approximate location of the graph of $\sqrt{2}$.

c. Since $-\pi = -3.14159\ldots$, the graph is between -3 and -4. Point C in Figure 1-3 is the approximate location of the graph of $-\pi$.

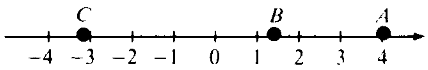

Figure 1-3 Graphs of 4, $\sqrt{2}$, and $-\pi$

Exercise Set 1-1

In exercises 1–20, place an X in a column if the number belongs to the set identified at the top of the column. A given number may belong to more than one set.

	Whole numbers	Integers	Rational numbers	Irrational numbers	Real numbers
1. 89					
2. -25					
3. $\frac{-3}{7}$					
4. $\frac{5}{8}$					
5. 0					
6. 1					
7. 0.21836 . . .					
8. $-2.05719\ldots$					
9. $10\frac{2}{3}$					
10. $-9\frac{1}{2}$					
11. $-0.999\ldots$					
12. 8.252525 . . .					
13. $\sqrt{5}$					
14. $-\sqrt{7}$					
15. 0.3					
16. -9.25					
17. $\frac{20}{4}$					
18. $\frac{-6}{3}$					
19. $\sqrt{9}$					
20. $\sqrt{12}$					

In exercises 21–30, find the first five multiples of each number.

[Example 1]

21. 5 **22.** 9 **23.** 13 **24.** 10

25. 19 **26.** 21 **27.** 25 **28.** 33

29. 39 **30.** 89

In exercises 31–40, find the LCM.

[Example 2]

31. 6 and 15 **32.** 8 and 12

33. 12 and 15 **34.** 10 and 14

35. 14 and 42 **36.** 35 and 210

37. 10, 15, and 25 **38.** 12, 18, and 30

39. 14, 21, and 35 **40.** 16, 24, and 40

In exercises 41–50, list all the whole number factors of each number.

[Example 3]

41. 12 **42.** 18 **43.** 21 **44.** 23

45. 35 **46.** 42 **47.** 43 **48.** 78

49. 96 **50.** 100

In exercises 51–70, write each number in the m/n form of a rational number. Answers can vary for each number.

[Examples 4–6]

51. 39 **52.** 17 **53.** $4\frac{3}{5}$ **54.** $5\frac{2}{3}$

55. 0.25 **56.** 0.6 **57.** -8 **58.** -15

59. $-6\frac{1}{2}$ **60.** $-10\frac{3}{4}$ **61.** -3.75 **62.** -7.8

63. 101 **64.** 260 **65.** $18\frac{3}{8}$ **66.** $21\frac{2}{7}$

67. 0.005 **68.** 0.0045 **69.** 0 **70.** $-30\frac{7}{10}$

In exercises 71–80, **a.** *write the decimal representation of each rational number and* **b.** *state the period.*

71. $\frac{4}{9}$ **72.** $\frac{5}{11}$ **73.** $\frac{5}{6}$ **74.** $\frac{11}{12}$

75. $\frac{7}{15}$ **76.** $\frac{13}{18}$ **77.** $\frac{40}{41}$ **78.** $\frac{17}{22}$

79. $\frac{5}{7}$ **80.** $\frac{54}{55}$

In exercises 81–90, graph each number on the given number line.

[Example 7]

81. $\frac{7}{8}$ **82.** $\frac{-1}{3}$ **83.** $4\frac{1}{2}$ **84.** $-5\frac{1}{5}$