

# PRELUDES IN THEORETICAL PHYSICS

IN HONOR OF V. F. WEISSKOPF

*edited by*

A. DE-SHALIT

*Department of Nuclear Physics,  
The Weizmann Institute of Science, Rehovoth, Israel*

H. FESHBACH

*Department of Physics and Laboratory for Nuclear Science,  
MIT, Cambridge, Mass., USA*

L. VAN HOVE

*Theoretical Study Division, CERN, Geneva, Switzerland*

NORTH-HOLLAND PUBLISHING COMPANY - AMSTERDAM

*No part of this book may be reproduced in any form  
by print, photoprint, microfilm or any other means  
without written permission from the publisher*

PUBLISHERS:

NORTH-HOLLAND PUBLISHING CO. - AMSTERDAM

SOLE DISTRIBUTORS IN THE U.S.A. AND CANADA :

INTERSCIENCE PUBLISHERS, a division of  
JOHN WILEY & SONS, INC. - NEW YORK

PRINTED IN THE NETHERLANDS

## EDITORS' FOREWORD

*Towards the end of 1964 it became known that Viki Weisskopf had decided to go back to MIT after having served as Director General of CERN for five years. Hopes were expressed that Viki might still change his mind, but it became clear that this time his decision was definite.*

*Many of us who have been visiting CERN, and working there, for shorter or longer periods, felt an urge to express our gratitude to Viki for everything he has done during these five years to make CERN such a pleasant and stimulating place. Suggestions of various sorts were brought up, but it seemed to us that this purpose could be best served by collecting together remarks and studies of a special character and dedicating this collection to Viki Weisskopf. Viki has won a special reputation for this insistence on looking at any given problem in physics from a variety of angles, and for his attempts to reduce to bare minimum formal derivations. His "intuitive" way of looking at things has been a source of aesthetic pleasure to everyone who has had the good fortune of working with him. As a matter of fact it is this "philosophy" of his that has given rise to some of the most exciting seminars at CERN and has guided the thinking of many of its scientists. The "preludes" collected in this volume are intended to illustrate some such approaches to a variety of physical problems.*

*The list of Viki's close friends is too long to have been covered in a volume like the present one. We have, therefore, limited our invitations only to those theoretical physicists who visited CERN and spent some time there. We tried to make sure that the list was as complete as possible, but there might have been some omissions, and we express our apologies to them.*

*Finally, Viki's work at CERN would have been impossible without the understanding, encouragement and help of Ellen Weisskopf; we*

*wish to take this opportunity to thank her, too, for the warm atmosphere all of us have always found at their home, and for complementing so harmoniously with Viki's contributions to CERN.*

*A. de-Shalit*

*H. Feshbach*

*L. Van Hove*

## INTRODUCTION

*In 1960 C. J. Bakker was killed in an airplane accident and a new director general of CERN had to be appointed. But also in other respects CERN was then in a state of transition. The construction of the synchrocyclotron and of the big accelerator had been successfully completed; physicists were gradually taking over from engineers and beginning to obtain interesting experimental results. It was important that the original fervour and spirit of cooperation, that had led to the creation of a European centre of high energy physics should be maintained, now that the first building period was over. Important not only to those working at CERN but in a broader sense to all physicists. It has always been the claim of scientists, that they have little difficulty to arrive at international understanding, as long as they are not hampered by the dullness of commercial acumen or the insipidity of diplomatic adroitness. Through CERN they had to prove their point, for this was an organization created by physicists for the pursuit of physics, not by governments for economic purposes or for some vague reasons of prestige.*

*When Viki Weisskopf accepted the appointment this was a great relief even to those who were only indirectly involved, but who knew the man and his background.*

*A few words about this background. Although the years from 1924 to 1935 with their grievous economic depression and the threat and finally the arrival of nazism were in many ways alarming, they will be remembered by theoretical physicists as a happy era. There was the feeling of a great spiritual breakthrough, followed by a surprisingly rich harvest, there was a feeling of belonging to a small and select inner circle headed by a few really outstanding men.*

*Weisskopf, who had worked at Göttingen, Zürich and Copenhagen before moving to the United States, was one of the prominent younger*

## VIII

*members of this group. He worked with Wigner and with Pauli and their power of mathematical penetration left their mark upon him. He knew Ehrenfest well and felt akin to him because of his preference for simple, clear and beautiful formulations. And above all he underwent the influence of Bohr's depth and wisdom.*

*But while others may wistfully remember those days, it is Weisskopf's unique achievement that he has carried over the devoted idealism and the enthusiasm of his early days into a new world of organized research and large scale experimentation.*

*Through the work he did at CERN, through the impact of his mature personality, he has had a profound influence on modern physics in Europe.*

*The present essays, in which we try to capture something of his spirit, is offered to him as a small token of gratitude.*

*H. B. G. Casimir*

## CONTENTS

Editors' foreword . . . . .	v
Introduction. . . . .	vii
M. FIERZ, Die unitären Darstellungen der homogenen Lorentzgruppe . . .	1
T. D. LEE, An elementary discussion of possible non-invariance under $T$ , $CP$ and $CPT$ in hyperon decays . . . . .	5
A. MARTIN, Born approximation and dispersion relations for singular poten- tials. . . . .	17
O. KLEIN, Boundary conditions and general relativity . . . . .	23
H. J. LIPKIN, Parity and momentum, a prelude to the use of group theory in physics . . . . .	27
A. DE-SHALIT, Polarization and zeros of the scattering amplitude . . . . .	35
L. VAN HOVE, Strongly interacting particles and the triplet hypothesis . . .	44
S. OKUBO AND R. E. MARSHAK, The charge conjugation operation and mixed space-time-internal symmetry groups . . . . .	51
J. D. WALECKA, Giant resonances in nuclei . . . . .	59
R. OPPENHEIMER, The symmetries of forces and states . . . . .	70
Y. YAMAGUCHI, The group $S_3$ and strong interactions . . . . .	78
E. M. HENLEY, Diffraction models for direct nuclear and high energy processes . . . . .	89
G. KÄLLEN, Intuitive analyticity . . . . .	100
B. T. FELD, A note on baryon masses, mass differences and magnetic moments, according to various symmetry schemes . . . . .	110
T. KINOSHITA AND N. N. KHURI, Some theoretical considerations on the real part of the forward scattering amplitude . . . . .	120
Y. NAMBU, A systematics of hadrons in subnuclear physics . . . . .	133
R. OEHME, A Lorentz covariant supermultiplet scheme for strong interactions	143
R. HAGEDORN, Causality and dispersion relations . . . . .	154
W. HEISENBERG, Die Rolle der phänomenologischen Theorien im System der theoretischen Physik . . . . .	166
L. WOLFENSTEIN, The concept of maximal $CP$ violation . . . . .	170
K. HUANG, The $SU_3$ mass formula . . . . .	177
F. E. LOW, Are wave functions finite? . . . . .	183
B. D'ESPAGNAT, An elementary note about 'mixtures' . . . . .	185
D. C. PEASLEE, Boson beta decay . . . . .	192
G. WENTZEL, On the localization in classical fields of energy, momentum, and charge. . . . .	199
L. L. FOLDY, Bottles for neutrons . . . . .	205
K. GOTTFRIED, Multipole radiation . . . . .	210
D. R. INGLIS, Inelastic scattering and associated gamma radiation . . . .	218

G. C. WICK, On symmetry transformations . . . . .	231
H. A. BETHE, Shadow scattering by atoms . . . . .	240
J. PRENTKI AND M. VELTMAN, <i>C</i> violation in strong interactions . . . . .	250
H. FESHBACH AND A. K. KERMAN, Studies of hypernuclei with K meson beams. . . . .	260
W. THIRRING, On the quantum theory of electric conductivity . . . . .	266
J. S. BELL AND M. NAUENBERG, The moral aspect of quantum mechanics . . .	279
H. B. G. CASIMIR, Energies and Hamiltonians in magnetic fields . . . . .	287
S. D. DRELL, D. R. SPEISER AND J. WEYERS, Test of role of statistical model at high energies . . . . .	294
A. PAIS, Vertices with partial SU(6,6) structure . . . . .	302
A. S. GOLDBABER AND M. GOLDBABER, Coherent high energy reactions with nuclei. . . . .	313
T. E. O. ERICSON, Diffraction scattering of strongly absorbed particles . . .	321
M. CINI, Pion-nucleon scattering and SU(4) spin-isospin symmetry . . . . .	330
D. AMATI, A semiclassical approach to the peripheral model . . . . .	339
P. MORRISON, Time's arrow and external perturbations . . . . .	347
AUTHOR INDEX. . . . .	



# DIE UNITÄREN DARSTELLUNGEN DER HOMOGENEN LORENTZGRUPPE

MARKUS FIERZ

*E. T. H. Zürich*

*(Received January 17, 1965)*

Wer kann was Kluges, wer was  
Dummes denken, Das nicht die  
Vorwelt schon gedacht?

*(Mephisto)*

Der Gegenstand der folgenden Betrachtungen gehört heute zum klassischen Bestand der mathematischen Physik. Niemand soll darum erwarten, daß ich etwas bieten kann, was nicht andere im wesentlichen schon gesagt hätten [1]. Der Sinn dieser Mitteilung ist darum ein pädagogischer.

Ich möchte eine anschauliche Methode vorführen, die zu den irreduziblen unitären Darstellungen der homogenen Lorentzgruppe führt.

Als Objekt, das wir Lorentztransformationen unterwerfen, wählen wir die Feldstärken  $\mathbf{E}$ ,  $\mathbf{B}$  und den Ausbreitungsvektor  $\mathbf{p}$  einer ebenen, elektromagnetischen Welle in einem festen Punkt des Raumes und der Zeit.

Die Maßeinheiten können stets so gewählt werden, daß

$$|\mathbf{E}| = |\mathbf{B}| = |\mathbf{p}| = p. \quad (1)$$

Diese Normierung ist lorentzinvariant, wie die vierdimensionale Gleichung

$$F_{ik}F^{k:l} = p_i p_l$$

zeigt. Hier entspricht  $F_{ik} = -F_{ki}$  den Feldstärken  $\mathbf{E}$ ,  $\mathbf{B}$  und  $p_k$  ist der zu  $\mathbf{p}$  gehörige lichtartige Vierervektor:

$$p_k p^k = 0.$$

Die drei Vektoren  $\mathbf{E}$ ,  $\mathbf{B}$  und  $\mathbf{p}$  bilden ein orthogonales Tripel im Raum der dreidimensionalen Vektoren. Jede Lorentztransformation läßt ein gegebenes Tripel entweder unverändert, oder sie führt es in

ein anderes über. Und jedes Tripel kann in jedes andere übergeführt werden, da man durch Dopplereffekt  $p$  beliebig ändern kann.

Wir beschreiben die Tripel durch  $p$  und drei Euler'sche Winkel  $\vartheta, \varphi, \psi$ . Dabei soll  $\psi$  die Drehung um die Richtung von  $\mathbf{p}$  beschreiben, die von  $\mathbf{E}$  nach  $\mathbf{B}$  führt.  $\mathbf{p}$  spielt also die Rolle der Figurenachse im symmetrischen Kreisel. Somit ist

$$\mathbf{F} = \mathbf{E} + i\mathbf{B} = e^{i\psi} \mathbf{a}(p, \vartheta, \varphi); \quad (2)$$

denn eine Drehung um  $\mathbf{p}$  ist eine solche der  $(\mathbf{E}, \mathbf{B})$ -Ebene in sich.

Bei Lorentztransformationen werden  $\mathbf{p}$  und  $\mathbf{F}$  je unter sich und linear-homogen transformiert.

Im Raume der Tripel ist

$$d\Omega = \frac{d^3 p}{p} d\psi \quad (3)$$

ein invariantes Volumenelement.

Wir betrachten in diesem Raume skalare, in bezug auf  $d\Omega$  quadratintegrierbare Funktionen  $\Phi(p, \vartheta, \varphi, \psi)$ :

$$\int |\Phi|^2 d\Omega = J. \quad (4)$$

Das Integral  $J$  ist invariant, da Lorentztransformationen lediglich eine Substitution der Integrationsvariablen erzeugen. Wir haben somit eine unitäre Darstellung der homogenen Lorentzgruppe im Hilbertraum der  $\Phi$  vor uns.

Setzt man nun

$$\log p = r, \quad \Phi = 1/p \cdot F$$

dann kann man, weil  $\Phi$  quadratintegrierbar ist,  $F$  wie folgt entwickeln:

$$F = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} d\mu e^{i\mu r} \sum_{j=0}^{\infty} \sum_{l=-j}^{+j} \sum_{m=-j}^{+j} e^{i(l\psi + m\varphi)} \cdot P_{lm}^j(\vartheta) C_{lm}^j(\mu). \quad (5)$$

Die  $P_{lm}^j(\vartheta)$  sind hier die Eigenfunktionen des symmetrischen Kreisels mit dem Impulsmoment  $j$ .  $l$  ist das Impulsmoment um die Figuren-achse,  $m$  dasjenige um die  $z$ -Achse.

Das invariante Integral wird jetzt

$$J = \int_{-\infty}^{+\infty} d\mu \sum_j \sum_l \sum_m |C_{lm}^j(\mu)|^2. \quad (6)$$

Hier ist aber schon

$$J_l(\mu) = \sum_{j \geq |l|} \sum_{|m| \leq j} |C_{lm}^j(\mu)|^2 \quad (7)$$

invariant. Denn zu festem  $\mu$  und  $l$  entsprechen die  $C_{lm}^j(\mu)$  Funktionen  $\Phi$ , die homogen sind in  $p$  vom Grad  $i\mu - 1$ , und homogen sind in  $e^{i\psi}$  vom Grad  $l$ . Da nun  $p$  und  $F$  linear-homogen transformiert werden, bleiben die Homogenitätsgrade  $i\mu - 1$  und  $l$  invariant. Zu festem  $\mu$  und  $l$  bilden die  $C_{lm}^j(\mu)$  also einen invarianten Raum und es ist leicht zu sehen, daß dieser auch irreduzibel ist. Wir haben also durch  $\mu$  und  $l$  charakterisierte, unitäre und irreduzible Darstellungen gewonnen.

Wenn man die Raumspiegelung als weiteres Element der Gruppe hinzunimmt, so hat man neben  $l$  auch  $-l$  zu betrachten, denn die Spiegelung führt  $l$  nach  $-l$ .

Die Darstellungen  $l = 0$  erhält man schon, wenn man  $p$  allein betrachtet.  $\Phi(p)$  ist alsdann eine Funktion auf dem Lichtkegel.

Man kann unsere Tripel als „anschauliche“ Darstellung von Spinoren  $a_\alpha$  ansehen. Es existiert nämlich die folgende, eindeutige Zuordnung:

$$a_\alpha \bar{a}_\beta \rightarrow p, \quad a_\alpha a_\beta \rightarrow E + iB. \quad (8)$$

Einem Tripel sind aber immer zwei Spinoren,  $a_\alpha$  und  $-a_\alpha$  zugeordnet.

Mit Hilfe der Spinoren erkennt man sogleich, daß es Lorentztransformationen gibt, welche ein gegebenes Tripel nicht ändern. Sei nämlich der zugehörige Spinor

$$a_1 = a, \quad a_2 = 0$$

so ändert die unimodulare Transformation

$$a_1 + Ca_2 = a'_1, \quad a_2 = a'_2 \quad (9)$$

den Spinor nicht. Dabei ist  $C$  eine beliebige komplexe Konstante. Im allgemeinen läßt freilich eine Lorentztransformation kein einziges Tripel invariant. Wenn sie jedoch eines invariant läßt, dann auch alle

diejenigen anderen, für welche  $p$  die gleiche Richtung besitzt.

Ist in der Transformation (9)  $C = 2 \operatorname{tg} \alpha$  reell, so kann man die Transformationsmatrix wie folgt aufspalten:

$$\begin{pmatrix} \cos \alpha, & \sin \alpha \\ -\sin \alpha, & \cos \alpha \end{pmatrix} \begin{pmatrix} \cos \alpha, & \sin \alpha \\ \sin \alpha, & (1 + \sin^2 \alpha)/\cos \alpha \end{pmatrix} = \begin{pmatrix} 1 & 2\operatorname{tg} \alpha \\ 0 & 1 \end{pmatrix}. \quad (10)$$

Es sei dem Leser überlassen, den Sinn dieser Darstellung zu ergründen.

#### REFERENZEN

- 1) V. Bargmann, Ann. of Mathem. **48** (1947) 568–640;  
Harish Chandra, Proc. Roy. Soc. **A189** (1947) 372–401;  
N. A. Neumark, Lineare Darstellungen der Lorentzgruppe (Berlin, 1963).

# AN ELEMENTARY DISCUSSION OF POSSIBLE NON-INVARIANCE UNDER $T$ , $CP$ AND $CPT$ IN HYPERON DECAYS \*

T. D. LEE

*Department of Physics, Columbia University, New York, N.Y.*

*(Received April 20, 1965)*

## 1. INTRODUCTION

The question whether the weak interactions are invariant under the time reversal operation  $T$ , or  $CP$  the product of the charge conjugation  $C$  and the space inversion  $P$ , has been raised [1] in the early days when the possibility of non-conservation of parity was being studied. After the discoveries [2, 3] that parity is not conserved, several experiments were performed to test the time reversal invariance in weak interactions. It was found that within the experimental accuracy [4], of about a few % in relative amplitudes, time reversal invariance holds in the  $\beta$ -decay, and to a much lesser accuracy, the same holds [5] for the  $\Lambda^0$  decay. If  $CPT$  invariance [6] is assumed, to the same degree of experimental accuracy  $CP$  is also conserved in these decays.

Recently, Christenson *et al.* observed [7] that the long-lived component of the neutral  $K^0$  meson can decay into  $(\pi^+ + \pi^-)$ , thus suggesting that  $CP$  invariance is violated in the  $K_{2\pi}^0$  decay. The observed non-invariant amplitude is quite small, being only  $\sim 2 \times 10^{-3}$  relative to the corresponding  $CP$  conserving amplitude. If  $CPT$  invariance is assumed, then the same experiment implies that time reversal invariance is also violated.

The experiments which established parity non-conservation usually consist of directly observing a right-left asymmetric effect from an, otherwise, initially right-left symmetric state. The conclusions that space inversion symmetry is violated in these experiments can be reached without any theoretical assumptions. The same is also true for the violation of charge conjugation symmetry. It is important to note

\* This research was supported in part by the U.S. Atomic Energy Commission.

that in all these weak interaction experiments, which pertain to testing the time reversal invariance or non-invariance, not a single one consists of comparing a reaction with its time reversal process. The relations between these experimental observations and time reversal symmetry are obtained through indirect theoretical reasoning, and some of these conclusions are valid only under additional assumptions such as *CPT* invariance. Similar criticism applies also to many of the existing tests of *CP* invariance.

It seems, therefore, desirable to review the underlying theoretical arguments of some of these tests, and to separate out the various implications of different symmetry requirements. With this motive, we will analyze in this note the simple example of the decay of a spin  $\frac{1}{2}$  hyperon, say,

$$\Lambda^0 \rightarrow N + \pi \quad (1)$$

where  $N$  stands for either  $p$  or  $n$  and  $\pi$  represents the corresponding  $\pi^-$  or  $\pi^0$ . The consequences of possible non-invariance under  $T$ ,  $CP$  and  $CPT$  are derived in Section II. As is well known, the time reversal invariance in the hyperon decay means [8] that the relative phase between the final  $s_{\frac{1}{2}}$  and  $p_{\frac{1}{2}}$  amplitudes is determined by the corresponding strong interaction phase shifts. In Section III, the same result is obtained by an alternative proof which is based only on the reciprocity relation between different reaction rates, without the explicit use of the time reversal operator [9]  $T$ . A simple example is given in Section IV which illustrates the difference between the consequences of time reversal invariance in quantum mechanics and that in classical mechanics, and which emphasizes again that the present tests of time reversal invariance concern only the reciprocity relations between various differential cross-sections, rather than the detailed time reversal operation  $T$ .

Throughout these discussions we assume that the amplitude of reaction (1) can be represented by the corresponding matrix element of a Hermitian operator [10]  $H_{\text{weak}}$ . The validity of the local field theory, or *CPT* invariance, is not assumed.

## 2. $\Lambda^0$ DECAY

In the decay of  $\Lambda^0$ , the final  $(N + \pi)$  system can be in either a  $s_{\frac{1}{2}}$  or a

$p_{\frac{1}{2}}$  spin-orbital state. Let these two transition amplitudes be denoted by  $A_s(I)$  and  $A_p(I)$  where  $I = \frac{1}{2}$ , or  $\frac{3}{2}$ , is the total iso-spin of the final state. The relative phase  $\phi(I)$  is given by

$$\frac{A_s(I)}{A_p(I)} = \left| \frac{A_s(I)}{A_p(I)} \right| e^{i\phi(I)}. \quad (2)$$

Similarly, in the decay of the anti-lambda,

$$\bar{\Lambda}^0 \rightarrow \bar{N} + \pi, \quad (3)$$

the corresponding  $s_{\frac{1}{2}}$  and  $p_{\frac{1}{2}}$  transition amplitudes are  $\bar{A}_s(I)$  and  $\bar{A}_p(I)$ , and their relative phase  $\bar{\phi}(I)$  is given by

$$\frac{\bar{A}_s(I)}{\bar{A}_p(I)} = \left| \frac{\bar{A}_s(I)}{\bar{A}_p(I)} \right| e^{i\bar{\phi}(I)}. \quad (4)$$

The following theorem states the separate consequences of the invariance requirements under  $T$ ,  $CP$  and  $CPT$  for the  $\Lambda^0$  and  $\bar{\Lambda}^0$  decays. Throughout the present paper, we neglect the effects of electromagnetic interactions and assume that the strong interaction is separately invariant under  $T$ ,  $C$  and  $P$ .

### Theorem

1. If  $T$  invariance holds then, independent of  $CP$  invariance,

$$\phi(I) = \begin{cases} \delta_s(I) - \delta_p(I), & \text{or} \\ \delta_s(I) - \delta_p(I) - \pi \end{cases} \quad (5)$$

and

$$\bar{\phi}(I) = \begin{cases} \delta_s(I) - \delta_p(I), & \text{or} \\ \delta_s(I) - \delta_p(I) - \pi \end{cases} \quad (6)$$

where  $\delta_s(I)$  and  $\delta_p(I)$  are, respectively, the  $s_{\frac{1}{2}}$  and  $p_{\frac{1}{2}}$  phase shifts due to the strong interactions of the  $(N + \pi)$  system with a total iso-spin  $I$ .

2. If  $CP$  invariance holds then, independent of  $T$  invariance,

$$A_s(I) = -\bar{A}_s(I) \quad (7)$$

$$A_p(I) = +\bar{A}_p(I) \quad (8)$$

and, consequently,

$$\phi(I) = \bar{\phi}(I) + \pi. \quad (9)$$

For convenience, we chose the anti-particle states  $\bar{A}^0$  and  $\bar{N}$  to be identically related to their respective particle states  $A^0$  and  $N$  through the  $CP$  operation.

3. If  $CPT$  invariance holds then, independent of either  $T$  invariance or  $CP$  invariance,

$$|A_s(I)| = |\bar{A}_s(I)|, \quad (10)$$

$$|A_p(I)| = |\bar{A}_p(I)| \quad (11)$$

and

$$\frac{1}{2}[\phi(I) + \bar{\phi}(I)] = \begin{cases} [\delta_s(I) - \delta_p(I)] + \frac{1}{2}\pi, & \text{or} \\ [\delta_s(I) - \delta_p(I)] - \frac{1}{2}\pi. \end{cases} \quad (12)$$

Some of these results, e.g. Eqs. (5) and (6), are well known and have already been proved in the literature [8]. For pedagogical reasons, a formal proof of this theorem is given below.

*Proof.* We consider the rest system of  $A^0$ . Let  $|(N\pi)_{I,s}^{\text{st}}\rangle$  and  $|(N\pi)_{I,p}^{\text{st}}\rangle$  be, respectively, the stationary wave eigen-states of the strongly interacting  $(N+\pi)$  system in the  $s_{\frac{1}{2}}$  and  $p_{\frac{1}{2}}$  orbits and with a total iso-spin  $I = \frac{1}{2}$  or  $\frac{3}{2}$ . The corresponding incoming wave states  $|(N\pi)_{I,s}^{\text{in}}\rangle$  and  $|(N\pi)_{I,p}^{\text{in}}\rangle$  are related to these stationary states by

$$|(N\pi)_{I,l}^{\text{in}}\rangle = e^{-i\delta_l(I)} |(N\pi)_{I,l}^{\text{st}}\rangle \quad (13)$$

where  $l = s$  or  $p$ . The transition amplitude  $A_l(I)$  is given by

$$A_l(I) = \langle (N\pi)_{I,l}^{\text{in}} | H_{\text{weak}} | A^0 \rangle \quad (14)$$

$$= e^{+i\delta_l(I)} \langle (N\pi)_{I,l}^{\text{st}} | H_{\text{weak}} | A^0 \rangle \quad (15)$$

where  $|A^0\rangle$  is the physical  $A^0$  state. The time reversal operator  $T$  is represented [11] by the joint operation of a complex conjugation times a unitary operator  $U_T$ . Since the strong interaction is invariant under  $T$ , all its stationary eigen-states  $|j, m\rangle$  which have zero total momentum can be chosen to transform under  $T$  as

$$T|j, m\rangle = U_T|j, m\rangle^* = (-1)^m |j, -m\rangle, \quad (16)$$

where  $j$  is the total angular momentum quantum number and  $m$  its  $z$ -component. Both  $|A^0\rangle$  and  $|(N\pi)_{I,l}^{\text{st}}\rangle$  satisfy Eq. (16). If  $H_{\text{weak}}$  is invariant under the time reversal operation, then

$$TH_{\text{weak}}T^{-1} = U_TH_{\text{weak}}^*U_T^\dagger = H_{\text{weak}}, \quad (17)$$



and, as a consequence,  $\langle (\bar{N}\pi)_{I,l}^{\text{st}} | H_{\text{weak}} | \Lambda^0 \rangle$  are real. Thus, Eq. (5) can be obtained by using Eq. (15).

For the decay of  $\bar{\Lambda}^0$ , we may denote the corresponding incoming wave eigen-state of the strongly interacting  $(\bar{N} + \pi)$  system by  $|(\bar{N}\pi)_{I,l}^{\text{in}}\rangle$ . Since the strong interaction is invariant under  $C$ , Eq. (13) implies that the  $|(\bar{N}\pi)_{I,l}^{\text{in}}\rangle$  state is also related to the stationary state  $|(\bar{N}\pi)_{I,l}^{\text{st}}\rangle$  by

$$|(\bar{N}\pi)_{I,l}^{\text{in}}\rangle = e^{-i\delta_l(I)} |(\bar{N}\pi)_{I,l}^{\text{st}}\rangle. \quad (18)$$

Equation (6) can be derived by using the relation

$$\bar{A}_l(I) = \langle (\bar{N}\pi)_{I,l}^{\text{in}} | H_{\text{weak}} | \bar{\Lambda}^0 \rangle. \quad (19)$$

To establish the consequences of  $CP$  invariance, we may choose

$$|\bar{\Lambda}^0\rangle = CP|\Lambda^0\rangle, \quad (20)$$

$$|(\bar{N}\pi)_{I,p}^\alpha\rangle = +CP|(N\pi)_{I,p}^\alpha\rangle \quad (21)$$

and

$$|(\bar{N}\pi)_{I,s}^\alpha\rangle = -CP|(N\pi)_{I,s}^\alpha\rangle \quad (22)$$

where  $\alpha$  = stationary or incoming. Equations (7)–(9) are the direct consequences of the assumption that  $H_{\text{weak}}$  is invariant under  $CP$ , i.e.

$$CPH_{\text{weak}}P^{-1}C^{-1} = H_{\text{weak}}. \quad (23)$$

If  $H_{\text{weak}}$  is invariant under  $CPT$ , then

$$CPTH_{\text{weak}}T^{-1}P^{-1}C^{-1} = H_{\text{weak}}. \quad (24)$$

Equations (10), (11) and (12) follow immediately by using Eqs (14)–(16) and (18)–(22). A special consequence of Eqs (10) and (11) is that  $CPT$  invariance implies [1] the equality of life time between  $\Lambda^0$  and  $\bar{\Lambda}^0$ .

We note that Eqs (5) and (6) are consequences of Eqs (7)–(12) [i.e.  $T$  invariance is a consequence of  $CP$  invariance and  $CPT$  invariance], Eqs (7)–(9) are consequences of Eqs (5), (6) and (10)–(12) [i.e.  $T$  invariance and  $CPT$  invariance imply  $CP$  invariance], and that Eqs (10)–(12) are consequences of Eqs (5)–(9) [i.e.  $T$  invariance and  $CP$  invariance imply  $CPT$  invariances].

The absolute magnitudes and the relative phases of these transition amplitudes can be directly measured by studying the decay rates and