MECHANICS OF ELASTIC-PLASTIC FRACTURE

Second Edition, Revised

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PREFACE TO THE SECOND EDITION

From ancient times, man has been concerned with determining both the strength of certain materials and the cause of their fracture. However, over the centuries, knowledge of the strength of materials and the nature of fracture was accumulated only sporadically, having been passed down from generation to generation in the framework of an art or skill rather than as a science.

Even today, the true nature of fracture is not completely understood. The increasing number of catastrophic failures of ships, aircraft, and rockets as caused by a sudden fracture or crack propagation has demonstrated the inadequacy of currently-used methods of analysis. Indeed, these dramatic failures have served to place the study of fracture in the forefront of advanced engineering research.

Today, scientists are studying the phenomenon of fracture from the perspective of solid body mechanics. The aim of these studies is to describe the major features of fracture in various groups of materials by means of rigorously-formulated models. The use of the fundamental principles and methods of solid body mechanics when studying the fracture process has led to the name of this new science, which is called *fracture mechanics.*

In a broad sense, fracture mechanics is that portion of the science of strength of materials which is concerned with analyzing the carrying capacity of a body either with or without existing cracks. Of necessity, fracture mechanics also considers the process of crack formation and growth.

Galileo must be given credit as a founder of fracture mechanics. He was the first to find that the breaking load of a bar in tension is proportional to its cross-sectional area, and is independent of its length. The work of Galileo and others—such as Hooke, Coulomb, Saint Venant, and Mohr—is generally considered the first step in the study of fracture mechanics. Their findings, characterized by both an extensive investigation of deformation and the development of various fracture criteria (having phenomenological implications), are referred to as strength theories. The essence of these theories is that a fracture occurs at the moment when, at some point in a body, such quantities as stress and strain (or their combination) reach a critical level. These approaches ignore the process of crack propagation in the body, and are justified only in cases where the development of cracks (and the attendant loss in carrying capacity) takes place in a relatively small zone of the critical region.

If a strength analysis were performed today, an appropriate strength theory would be used that takes into account criteria such as a maximum principal stress, shear stress, or octahedral stress, depending on the type of material and the operating conditions of the structure. Such an analysis would still be insufficient, and will continue to be so even if an allowance is made for future improvements in strength theories. For example, problems concerning the equilibrium of crack-containing elastic bodies are of particular importance. However, general solutions to these problems based on strength theory are associated with enormous mathematical difficulties. Moreover, these solutions contain much more information than is needed. What is really needed is to determine whether a body has sufficient capacity at a given load. That is, the major object of interest is not in a general solution to a complex problem of equilibrium of a cracked body, but rather the existence or nonexistence of a particular solution at a given load. In other words, from a mathematical point of view, fracture occurs with limit states that ensure the nonexistence of strength-theory solutions. These limit states are integral characteristics of the fracture process, and are in agreement with the general global concept of fracture of solid bodies.

When the phenomenological strength-theory approach is used to develop models for crack development in solids, it is most common to initially assume some disturbance in the form of a pre-existing (original) crack. This assumption is consistent with the observed presence of material imperfections, such as cracks that occur during the fabrication process of a particular structure. Hence, when deriving various strength criteria based on the process of fracturing, relations are obtained that are in formal agreement with regular strength criteria, differing only in that the derived relations involve constants that depend on the coordinates, lengths, and geometry of the pre-existing cracks.

Current research in the field of fracture mechanics is aimed at more than the carrying-capacity concept. In fact, the investigation of the fracture process now represents an independent field of interest. Monitoring the process of fracture, and learning the laws that govern it are of great practical importance. For example, in operating structures it is essential to suppress the process of fracture, whereas when cutting metals, it is desirable to facilitate rupture.

This book presents the fundamentals of the mechanics of crack development in solids, and some special problems of fracture mechanics involving higher levels of mathematics.

Part One is devoted to the fundamental concepts and methods concerning elastic and elastic-plastic fracture. Examples are given that demonstrate the results of using various fracture criteria to determine both the critical and allowable crack lengths at static and cyclic loadings. For the purposes of independent study or for delivering a short course, Sections 1 to 3, 12, 16, 17, 25, 30, 33, and 34 are recommended.

Part Two is a systematic presentation of several problems of fracture mechanics such as the effects of hydrogen-containing materials, stress corrosion, thermal and dynamic loadings, and electromagnetic interaction in piezoelectric materials. Solutions of such problems involve complex mathematics, in which readers should be adequately grounded.

Fracture mechanics problems are so extensive that some topics are not covered; these include the fracture of plates and shells, and methods of experimental determination of fracture toughness. It is felt that these topics are sufficiently represented in related publications.

This second edition of the book, which contains both revisions and additional material, can now be used as a text for graduate students in universities.

The authors express their deep gratitude to Dr. N. Robes for his thorough review and comments, which contributed to the further improvement of this edition.

FOREWORD

The term *fracture mechanics* is somewhat unsettling to many people. This is because, until recently, the major emphasis in mechanics was on the strength and resistance of materials. To speak of fracture is as uncomfortable for some as it is to speak of a fatal illness. However, just as in preventing a fatal disease, one must know its nature, symptoms, and behavior; to ensure the strength of a structure, one must be aware of the causes and nature of its potential failure.

The problem of fracture is vital in the science of strength of materials. However, not only has fracture mechanics, as an independent branch of the mechanics of deformable solids, originated quite recently, but its boundaries are not yet clearly defined. Therefore, it is of paramount interest to combine the efforts of representatives from many different branches of science and engineering for a complete study of the fracture concept. It is also important that differences in terminology (that are usual for different sciences), and the widespread conviction that the solution to everything lies in a particular portion of the general problem, do not lead to a situation in which disputes about the concepts are replaced by arguments about the words.

At present, routine fracture mechanics is the study of conditions under which a crack or a system of cracks undergoes propagation. However, cracks are of different natures, and are considered on different scale levels. The case on one extreme is the fracture of a crystal grain, which initiates with a submicroscopic crack when two atomic layers move apart by such a distance that the forces of interaction between the atoms may be neglected. An example of the other extreme is a crack occurring in a welded turbine rotor in a nuclear reactor, when the crack's length and width may amount to centimeters; this is referred to as a macroscopic fracture.

In the first case, the condition for crack propagation is defined by the configuration of atoms at the crack tip. Considered here is a discrete crystal lattice formed by atoms rather than a continuous medium; therefore, the very concept of the "crack tip" becomes uncertain. The study of this kind of submicroscopic crack and its behavior in interaction with other lattice defects is, essentially, in the province of solid-state physics rather than mechanics; however, the methods of classical theory of elasticity are fully applicable to problems of this nature. The line between modern physics and mechanics is not well defined; nevertheless, it must be drawn to avoid possible terminological confusion.

A macroscopic fracture has dimensions exceeding by several orders the size of the largest structural constituent of the material (the constituent must contain a sufficient number of crystal grains for its properties not to differ from those of any other element of similar size which may be isolated from the material). It is precisely this condition that makes it possible to solve such a crack problem within the framework of mechanics of a solid body. The formulated condition refers to an ideal situation in order to make the theory applicable; in real conditions one may depart from this stringent requirement, but this in no way makes the theory groundless. Assuming the material to be continuous, homogeneous, and elastic, and using the techniques of the classical theory of elasticity, we inevitably arrive at the paradoxical conclusion that the stresses grow infinitely near the crack tip. This paradox is a sort of penalty paid for the simplicity associated with using the linear theory of elasticity in a region where its application is know to be invalid.

So-called linear fracture mechanics assume that a physically impossible singularity is a reality. Such an approach is not new and not so unusual for continuum mechanics; recall, for example, the vortex filaments with zero cross section and finite circulation. It appears that the work of crack propagation, which is done either as a result of increase of external forces or reduction of the elastic energy of the body with the crack size increase, is expressed directly through the coefficient of the singular term in the formula for stress. This coefficient is referred to as the stress intensity factor, and is of fundamental importance for the entire theory. The work of crack propagation may be associated with overcoming the forces of surface tension (Griffith's concept), or the plastic deformation in the small region of the immediate neighborhood of the crack tip, or other physical causes. The factor to be emphasized is that the size of the region, where the laws of the linear theory of elasticity are in some way violated, must be very small. The ability of the crack to further propagate is then determined by the sole characteristic: the work per unit length of the propagation path, or the critical stress intensity factor.

If the size of the zone, where the relations of the linear theory of elasticity are violated, is large, one should consider the laws of nonlinear fracture mechanics. It appeared at the beginning that formal indifference of linear fracture mechanics to both the object and the scale, mathematical equivalence of problems associated with entirely different physical phenomena, would make it possible to establish nonlinear mechanics in a similar uniform manner. It was later found to be quite different.

The principal problem, on which the efforts of scientists have been focused in recent years, concerns the conditions of either equilibrium or the propagation of a large crack in a sufficiently plastic material. Scientists have been involved in the theory and practical applications of fracture mechanics for evaluating the strength of large-scale structural elements. They have shown that the plastic zone ahead of a crack is sufficiently extensive so that the macroscopic theory of plasticity, which assumes that the medium is continuous and homogeneous, holds good. For the plane state of stress, the Leonov-Panasyuk-Dugdale model, which substitutes the plastic zone by a no-thickness segment extending the crack, appears to be satisfactory. In particular, this book presents an analysis of the corresponding elastic-plastic problem that is solved numerically by using the finite element method (FEM). The presented FEM solution confirms the validity of the model used: as the number of elements in the model is increased, the plastic zone contracts; in the limit state, the plastic zone is expected to degenerate into a segment when, with the infinite refinement of the mesh, the solution approaches the exact result. Many authors, when considering submicroscopic cracks at the atomic scale level, assume the hypothesis that nonlinear behavior in interaction between atoms is significant only within a single interactomic layer (similar to the computation of the so-called Peierls dislocation). Again, as in the linear theory, the analogy is merely formal; in this case, it is of an artificial nature, and the judgments about the relative validity of the model in various cases are based on entirely different considerations; the persuasiveness of argumentation in favor of this model appears to vary in a wide range.

Unfortunately, a plane state of stress can never been realized in practice; at a distance from the crack tip of approximately the thickness of a sheet, the state of stress is essentially three-dimensional and far too complex for analysis. In the case of plane strain, the shape of the plastic zone appears to be different: it spreads transversely rather than longitudinally, and the model of the plastic segment assumed for plane stress in no way reflects the reality.

The situation faced by a design engineer is complicated. The size of the plastic zone in structural elements made of modern alloys is of the same order as the thickness of the element; consequently, the state of stress throughout the plastic region is essentially three-dimensional. Also, the most common structural materials, carbon and alloy steels, are quite ductile. Crack propagation begins when the plastic deformation encountered in the vicinity of its tip becomes extensive, amounting to the order of tens of percent. The tip of an originally sharp, say, fatigue, crack becomes blunt. Its flanks, which were initially closed, separate transversely in a parallel fashion by a finite distance, and further fracture takes place only when the opening reaches a certain critical value. Thus, the theory of crack propagation in ductile materials includes at least two elements: 1) the solution of an elastic-plastic problem by taking into consideration both the finiteness of the plastic strain and the boundary conditions over the deformed boundary, and 2) determination of the condition of macrocrack formation in material which has undergone a significant deformation accompanied by accumulation of microdefects.

The book by V. Z. Parton and E. M. Morozov is the first Russian monograph on the above-discussed subject. It is based mainly on the results obtained by the authors during their original research, and concerns the problems of nonlinear fracture mechanics. It presents some elastic-plastic problems for crack-containing bodies. The greater part of the book is devoted to linear fracture mechanics and also to some new developments in this field which lead to governing equations that may be nonlinear.

In spite of certain limits imposed by linear fracture mechanics, a wide variety of problems may be reliably solved using its methods. Development of this theory is focused on accumulating data from already solved elasticity problems concerning cracks of various shape in various bodies. The amount of such information continually grows both abroad and in the USSR. Many results obtained by foreign authors became available by means of numerous translations of books and published articles. In particular, a translation of the seven-volume advanced treatise "Fracture," edited by H. Liebowitz (USA) [247], is now being published.

The present book may be considered as a significant contribution to the database of fracture mechanics. Some features of the book deserve special mention. First of all, it is the new variational principle that makes it possible to approximately solve numerous problems; in particular, to find the trajectory of crack propagation in a nonuniform stress field. Also, a straightforward approach for an approximate determination of the stress intensity factor is included; it enables one to obtain a reasonable evaluation for those cases where an exact solution of the elasticity problem is impossible, and the numerical computation is extremely laborious. In addition, a series of newly solved dynamic problems for bodies subjected to cyclic (periodic) loading is provided.

Linear fracture mechanics has been developed by applying its concepts to the problems of crack-growth kinetics as a function of either time or, in the case of fatigue fracture, the number of cycles. It should be noted that the kinetics, both linear and nonlinear, are presumed to be essentially local: all fracture processes regardless of their nature are assumed to occur in an end region of very small size; outside this region the material is in an elastic state. Then, the stress intensity factor becomes the only representative of the state of stress in any kinetic equations. The chapters of books concerned with fatigue fracture are structured following this approach.

In conclusion we shall note that significant advancement in the field of the mechanics of crack propagation has led to such a wide-spread perception that it reflects the entire fracture mechanics. However, the subject matter of fracture mechanics should be understood in a much wider context. For instance, in metals loaded at high temperature, the fracture is of a scattered nature: micropores are accumulated at the grain boundaries over the entire volume of the body, followed by their merging and, finally, combining to form a macrocrack. The macrocrack is merely the final, visible result of the damage accumulation process that cannot be recognized by the naked eye, but can well be seen using appropriate optical devices. A similar character of fracture is apparently observed in some polymers, but in this case a more precise technique is needed to detect the microdamage.

The importance of statistics in evaluating the strength of structures is widely known. The statistical theory of fracture must also be considered as an inherent part of fracture mechanics. It should be mentioned, however, that the sophisticated and theoretical probabilistic analysis is often used with rather primitive mechanical modelling. This, no doubt, may be explained by the complexity of the studied subject.

The book elaborates on an extremely important problem. It contains a number of interesting remarks and considerations, sometimes of a tentative nature, that fuel thinking and stimulate further work.

Yu. N. Rabotnov, Academician
USSR Academy of Sciences

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PART

ONE

FUNDAMENTALS OF FRACTURE MECHANICS

CHAPTER

ONE

MECHANISMS OF CRACK FORMATION AND GROWTH

§1. FRACTURE MECHANICS AND STRENGTH OF SOLID BODIES

In recent years there has been dramatic progress in the ancient science of strength of deformable solids. This burst of knowledge is due primarily to a new approach to the problem of brittle fracture. When the preexisting cracks are taken into consideration, this new approach renders sufficiently reliable solutions.

Fracture mechanics, in a general sense, encompasses that part of strength of materials which is concerned with the carrying capacity of structural elements, both with and without account of preexisting cracks. Fracture mechanics also considers various histories of crack formation and development.

It is well known that fracture itself involves a complex and multistage process, which initiates well before the occurrence of visible cracks. Because a common theory of crack formation is lacking, the various recurring patterns of this phenomenon are being studied at various scale levels. The linear scales of fracturing are shown in Fig. 1.1. Within each scale range, the fracture must be studied in conjunction with the model corresponding to the material constitution, and boundary conditions on both left-hand and right-hand neighboring ranges (according to the adopted scale).

A. F. Ioffe and others [64] have conducted experimental studies on the strength of sodium salt crystals at various surface conditions of specimens. The strength of a