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7

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TRANSLATOR'S PREFACE

In the interest of speed and economy the notation of the original text has been retained so that the cross product of two vectors **A** and **B** is denoted by [AB], the dot product by (AB), the Laplacian operator by Δ, etc. It might also be worth pointing out that the temperature is frequently expressed in energy units in the Soviet literature so that the Boltzmann constant will be missing in various familiar expressions. In matters of terminology, whenever possible several forms are used when a term is first introduced, e.g., magnetoacoustic and magnetosonic waves, "probkotron" and mirror machine, etc. It is hoped in this way to help the reader to relate the terms used here with those in existing translations and with the conventional nomenclature. In general the system of literature citation used in the bibliographies follows that of the American Institute of Physics "Soviet Physics" series. Except for the correction of some obvious misprints the text is that of the original.

We wish to express our gratitude to Academician Leontovich for kindly providing the latest corrections and additions to the Russian text.

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A. A. Galeev and R. S. Sagdeev

INTRODUCTION

In linear plasma theory an arbitrary perturbation can be expressed as a superposition of characteristic eigenmodes, each of which is independent of the others. In the nonlinear theory the eigenmodes interact with each other as a result of the nonlinearity. This interaction is, in many respects, reminiscent of the interaction between motions on different scales in hydrodynamic turbulence. In a plasma, however, the pattern of this interaction can frequently be represented in the familiar form of a superposition of linear eigenmodes if account is taken of the fact that the nonlinearity only leads to a weak interaction between the modes. This means that the coefficients in an expansion in characteristic eigenmodes become slowly varying functions of time and ultimately take on values which are very different from the values given by the linear theory.

This approach is now generally called the theory of weak turbulence. The equations of this theory can be derived from first principles by means of an expansion of original equations for the plasma in powers of a small parameter, the ratio of energy in the oscillations to the total energy in the plasma. The energy source for the perturbations in this theory are usually the various plasma instabilities.

The theory of weak turbulence was developed at the beginning of the 1960's; at the present time, by means of this theory it has been possible to explain a number of important nonlinear effects: the interaction of a beam of charged particles with a plasma; turbulent heating of a plasma; the dissipation mechanism in collisionless shock waves; and anomalous resistivity. The methods of weak turbulence go beyond the framework of plasma physics and have been applied successfully in the analysis of nonlinear dispersive media in general, and, in particular, in the nonlinear dynamics of water waves. Thus, it has been possible to formulate a quantitative theory of water ripples, effects which were only amenable to a qualitative description for a long time.

The theory of weak turbulence has been the subject of a number of books and reviews in the last decade*; nevertheless, it is felt that a need has arisen to summarize the results of the theory of weak turbulence from a single point of view, including phenomenon that are not found in laboratory plasma physics.

It is useful to consider nonlinear plasma theory in terms of three basic interactions: nonlinear wave–wave interactions, wave–particle interactions, and finally, wave–particle–wave interactions (sometimes called the nonlinear wave–particle interactions).

The first interaction, the *wave – wave* interaction is frequently called *resonance wave – wave scattering*. The resonance conditions can be written

$$\sum_i \omega_i = 0,\ \sum_i \mathbf{k}_i = 0,\ i = 1, 2, \ldots,$$

where ω_i and $\mathbf{k}_i$ are the frequencies and wave vectors of the waves which participate in the interaction. The simplest interaction of this kind is the one in which three waves are involved. The coupling between the waves is especially strong if the resonance condition is satisfied. Since this interaction does not involve resonance particles, it can be described by means of the fluid equations (in other words, it is not necessary to use the kinetic equations). The wave–wave interaction lies at the basis of many effects in nonlinear wave dynamics: parametric wave instabilities (the case of small amplitudes corresponds to the well-known decay instability); the modulational instability of wave packets in a plasma and in nonlinear optics; and self-focusing of waves in nonlinear optics. If the quantities ω and $\mathbf{k}$ are interpreted as the energy and momentum of the photon associated with the ω, $\mathbf{k}$ wave, it will be evident that the resonance condition is simply a statement of the conser-

*B. B. Kadomtsev, Plasma Turbulence, Academic Press, New York, 1965.

vation of energy and momentum in the elementary process in which a single photon decays into two other photons or in the inverse process. Consequently, it is not surprising that the wave interaction conserves the total energy and momentum.

The second interaction can be pictured as being almost linear (or quasilinear). The wave – particle interaction is especially strong near resonance $\omega = \mathbf{k} \cdot \mathbf{v}$ ($\mathbf{v}$ is the velocity of the particle that participates in the interaction). If this so-called Landau resonance condition is satisfied, the particle maintains a constant phase with respect to the wave and is effectively accelerated (or retarded) by the electric field associated with the wave. An analogous resonance arises in a magnetic field when the following condition is satisfied:

$$\omega - l\omega_H = \mathbf{k}\cdot\mathbf{v}, \quad l = 0, \pm 1, \ldots,$$

where ω_H is the particle gyrofrequency. Since this interaction involves resonant particles it is necessary to make use of the kinetic equations. From the quantum-mechanical point of view the resonance condition for this interaction is a statement of the conservation of energy and momentum in the elementary process involving the emission or absorption of a photon with energy $\hbar\omega$ and momentum $\hbar\mathbf{k}$ by a particle moving with velocity $\mathbf{v}$. Thus it is not surprising that the wave–particle interaction conserves the total energy and momentum of the waves and particles (rather than the energy of the waves alone). The change in the wave amplitude associated with this interaction is called Landau damping (or inverse Landau damping) while the corresponding change in the velocity distribution of the particles is called quasilinear diffusion.

The third interaction, the wave–particle–wave interaction, is frequently called nonlinear Landau damping. The resonance condition for this interaction is written in the form $\omega_1 - \omega_2 = (\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{v}$, while the basic mechanism is reminiscent of the linear wave–particle interaction. In the present case, however, the particle maintains a constant phase with respect to the beat wave produced by two waves. This interaction also involves resonant particles and must be considered within the framework of kinetic theory. The resonance condition written above taken with the plus sign corresponds to the elementary process involving simultaneous emission or absorption of two photons by a particle.

With the minus sign the resonance condition refers to the elementary process involving emission of a single photon and absorption of another (in other words, the scattering process). In addition to the conservation of the total energy and the total momentum of the waves and particles in the scattering process it also turns out that the total number of photons is conserved. In the classical case the number of photons can be defined as the energy of the wave W_k divided by the frequency [that is to say, W_k/ω_k is the action of the $(\omega, \mathbf{k})$ wave].

It will be evident that in general all three interactions described above can occur in a plasma at the same time; the behavior of the plasma is then determined by the total effect of all three interactions. The problem of anomalous resistivity in the plasma is treated in a separate chapter as an example of this interaction.

Nonlinear phenomenon in a plasma cannot always be treated by means of the theory of weak turbulence. Many plasma effects are a result of strong turbulence, which is similar to the usual hydrodynamic turbulence. At the present time no reliable quantitative methods are available in the theory of strong turbulence. As a rule, one tries to obtain reasonable estimates as to the orders of magnitude involved. Certain examples of this kind are discussed in various sections of the present review.

The original step in the writing of this review was the presentation of lectures by the authors at the International Center for Theoretical Physics in Trieste in 1966. A report containing the lectures [1] was edited by Book and O'Neil and published in 1969.

In the ensuing period so many new results have been obtained in nonlinear plasma theory that it has become necessary to subject the original version to a thorough revision. In the process we have also added a new chapter on anomalous resistivity in plasmas. It is assumed that the reader of the present review is familiar with the linear theory of plasma waves and instabilities.*

*A. B. Mikhailovskii, Theory of Plasma Instabilities, Consultants Bureau, New York, 1974.

Chapter 1

WAVE–WAVE INTERACTION

§ 1.1. Resonance Interaction between Plasma Waves

Let us consider the nonlinear wave–wave interaction between three plasma waves. An example of this kind of interaction is the process in which a wave of finite amplitude decays into two daughter waves, which was first treated by Oraevskii and Sagdeev [1]. In order for this interaction to occur, the wave vectors and frequencies must satisfy a resonance condition, that is to say, $\mathbf{k}_0 = \mathbf{k}_1 \pm \mathbf{k}_2$ and $\omega_0 = \omega_1 \pm \omega_2$. It will be evident that the frequencies and wave vectors of each of the waves are coupled by the linear dispersion equation $\omega = \omega(\mathbf{k})$. The nature of the dispersion plays an important role in determining whether or not a resonance interaction is possible for a given set of waves. In order to illustrate this point we note the difference between nonlinear resonances which dominate the situation in ordinary gas dynamics and those which are important in plasma physics. In the case of a monochromatic acoustic wave of large amplitude, gas dynamic theory predicts that the primary mechanism responsible for the nonlinear distortion of the wave is the steepening of the wave front (Fig. 1). This steepening can be understood in terms of the resonance generation of higher harmonics. If the original large-amplitude wave is characterized by a frequency ω and wave vector $\mathbf{k}$, the nonlinear interaction of the wave leads to the appearance of a second harmonic $(2\omega, 2\mathbf{k})$. Since the dispersion relation for the acoustic wave is a linear relation $\omega = kc_s$, the harmonics, like the fundamental mode, are characteristic modes of the system and are thus at all times in resonance with the fundamental mode; thus,

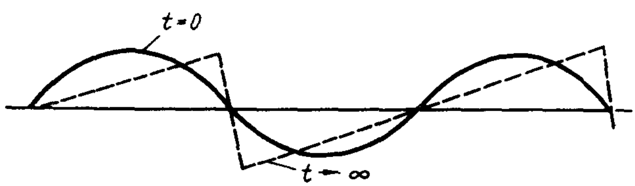

Fig. 1. Steepening of the profile of an acoustic wave of finite amplitude.

the harmonics grow in time. Higher harmonics grow in similar fashion and the appearance of higher and higher values of **k**, that is to say, shorter wavelengths, is responsible for the steepening of the front (Fig. 1).

On the other hand, as a rule plasma oscillations exhibit a strong dispersion (i.e., the frequency ω is usually a nonlinear function of **k**) so that the harmonics of the normal modes are themselves not normal modes of the system. Consequently, harmonics of a plasma wave of large amplitude are usually limited at a very low level and only lead to an insignificant distortion of the wave shape [2]. However, this result does not mean that plasma waves always propagate without change of shape. Even if a plasma wave does not interact with its own harmonics, it can still be in resonance with two other waves.

Resonance generation of harmonics can also be suppressed by a choice of wave polarization such that the matrix element of the interaction operator between the wave and its harmonics can vanish identically. An example of this kind is furnished by transverse Alfvén waves. Nonetheless, as we have seen earlier, this does not mean that these waves propagate in a plasma without change of shape.

For purposes of illustration we now consider a wave–wave interaction between an Alfvén wave of large amplitude, an Alfvén wave of low amplitude, and an acoustic wave. The magnetic field, the electric field, the velocity of the electron fluid, and its density are written in the form

$$\left.\begin{aligned} \mathbf{H} &= \mathbf{H}_0 + \delta\mathbf{H}_\perp(z,\ t) + \mathbf{h}_\perp(z,\ t); \\ \mathbf{E} &= \delta\mathbf{E}_\perp(z,\ t) + \mathbf{e}_\perp(z,\ t); \\ \mathbf{v} &= \delta\mathbf{V}_\perp(z,\ t) + \mathbf{v}_\perp(z,\ t) + \mathbf{v}_\parallel(z,\ t); \\ N &= N_0 + N(z,\ t), \end{aligned}\right\} \qquad (1.1)$$

where $\mathbf{H}_0$ is a fixed magnetic field in the z direction; the functions $\delta\mathbf{H}_\perp(z, t)$, $\delta\mathbf{E}_\perp(z, t)$, and $\delta\mathbf{V}_\perp(z, t)$ characterize the finite-amplitude Alfvén wave; $\mathbf{h}_\perp$, $\mathbf{e}_\perp$, and $\mathbf{v}_\perp$ are the perturbations of the fields and fluid velocity of the low-amplitude Alfvén wave; N and $\mathbf{v}_\parallel$ are the perturbations in the density and fluid velocity of the acoustic wave. It is assumed that all three waves propagate along the fixed magnetic field $\mathbf{H}_0$.

We now solve the two-fluid MHD equations by perturbation theory, taking the quantities $\mathbf{H}_0$, N_0, $\delta\mathbf{H}_\perp$, $\delta\mathbf{E}_\perp$ and $\delta\mathbf{V}_\perp$ to be the unperturbed quantities; the quantities $\mathbf{h}_\perp$, $\mathbf{e}_\perp$, $\mathbf{v}_\perp$, N, and $\mathbf{v}_\parallel$ are small perturbations.

The Alfvén wave of finite amplitude can be represented as a circularly polarized wave which is written in the form

$$\delta V_x - i\delta V_y = \delta V \exp(-i\,\omega_0 t + ik_0 z) + \text{c.c.} \tag{1.2}$$

The connection between the field amplitudes $\delta\mathcal{H}$ and $\delta\mathcal{E}$, which are defined in similar fashion, and the flow velocity δV is given by the equations of motion of electron fluid and Maxwell's equations

$$\left.\begin{aligned} -i(\omega_0+\omega_H)\delta V &= -(e/m)\delta\mathcal{E}; \\ k_0\delta\mathcal{H} &= -(4\pi e N_0/c)\delta V - (i\omega_0/c)\delta\mathcal{E}; \\ k_0\delta\mathcal{E} &= (i\omega_0/c)\delta\mathcal{H}; \\ \omega_{Hj} = e_j H_0/m_j c;\ \omega_{He} \equiv \omega_H,\ &\omega_{Hi} \equiv \Omega_H,\ m_e \equiv m,\ m_i \equiv M. \end{aligned}\right\} \tag{1.3}$$

The solution of these equations leads to the linear dispersion relation

$$\left.\begin{aligned} &k_0^2 c^2 = \omega_0^2\,\varepsilon(\omega_0); \quad \varepsilon(\omega) \equiv 1 - \left[\omega_p^2/\omega(\omega+\omega_H)\right]; \\ &\omega_{pj}^2 = 4\pi e_j^2 N_0/m_j; \quad \omega_{pe}^2 \equiv \omega_p^2; \quad \omega_{pi}^2 \equiv \Omega_p^2 . \end{aligned}\right\} \tag{1.4}$$

The first-order perturbation equations are of the form

$$\partial v_\parallel/\partial t + (c_s^2/N_0)(\partial N/\partial z) = -(\partial/\partial z)(\mathbf{h}_\perp\cdot\delta\mathbf{H}_\perp/4\pi N_0 M); \tag{1.5}$$

$$\partial N/\partial t + N_0\,(\partial v_\parallel/\partial z) = 0; \tag{1.6}$$

$$m\,(\partial\mathbf{v}_\perp/\partial t) + e\{\mathbf{e}_\perp + (1/c)[\mathbf{v}_\perp\times\mathbf{H}_0]\} = -m v_\parallel\,(\partial/\partial z)\delta\mathbf{V}_\perp - -(e/c)[\mathbf{v}_\parallel\times\delta\mathbf{H}_\perp]; \tag{1.7}$$

$$\operatorname{curl}\mathbf{h}_\perp - (1/c)\,(\partial\mathbf{e}_\perp/\partial t) + (4\pi e/c)N_0\mathbf{v}_\perp = (-4\pi e/c)\,N\delta\mathbf{V}_\perp; \tag{1.8}$$

$$\operatorname{curl}\mathbf{e}_\perp + (1/c)(\partial\mathbf{h}_\perp/\partial t) = 0. \tag{1.9}$$

The left-hand sides of Eqs. (1.5) and (1.6) describe the acoustic wave while the right-hand sides of these equations represent the coupling between the acoustic wave and the two Alfvén waves. Similarly, the left-hand side of Eqs. (1.7)-(1.9) describe the low-amplitude Alfvén wave while the right-hand sides indicate the coupl-