PASSIVE AND ACTIVE FILTERS

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John Wiley & Sons
New York
Chichester
Brisbane
Toronto
Singapore

Library of Congress Cataloging in Publication Data:

Chen, Wai-Kai, 1936-
Passive and active filters.

Includes index.
1. Electric filters, Active. 2. Electric filters, Passive. 3. Analog electronic systems. I. Title.
TK7872.F5C469 1986 621.3815'324 85-9497

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

PREFACE

Electrical filters permeate modern technology so much that it is difficult to find any electronic system that does not employ a filter in one form or another. Thus, the topic of filters is important for study by electrical engineering students preparing to enter the profession and by practicing engineers wishing to extend their skills. The purpose of the book is to fill such needs.

The book is designed to provide the basic material for an introductory senior or first-year graduate course in the theory and design of modern analog filters, as opposed to digital filters. It is intended for use as a text for a two-quarter or one-semester course. It should be preceded by a course on linear networks and systems analysis in which Laplace transform and state-variable techniques are introduced. Some familiarity of the theory of a complex variable and matrix algebra is necessary for the last three chapters.

With the rapid advance of solid-state technology and the wide use of digital computers, few curricula in electrical engineering can afford more than a one-semester or two-quarter course in analog filter design. The trend is toward shortening the passive filter synthesis and expanding the active design. This is to recognize the tremendous advances made in integrated-circuit processing techniques, which frequently render active elements much cheaper to produce than some of the passive ones. As a result, modern synthesis techniques lean heavily toward the use of active elements, which, together with resistors and capacitors, virtually eliminate the need to use inductors in many frequency ranges. Such active *RC* or inductorless filters are attractive in that they usually weigh less and require less space than their passive counterparts, and they can be fabricated in microminiature form using integrated-circuit technology, thereby making inexpensive mass production possible. Thus, they are taking over an ever-increasing share of the total filter production and applications. However, active *RC* filters have the finite bandwidth of the active devices, which places a limit on the high-frequency performance. Most of the active *RC* filters are used up to approximately 30 kHz. This is quite adequate for use in voice and data communication systems. The passive filters, on the other hand, do not have such a limitation, and they can be employed up to approximately 500 MHz, the limitation being the parasitics associated with the passive elements. Furthermore, passive filters generally have a lower sensitivity than the active ones.

Based on these observations, the decision was made to concentrate on inductorless filters in which the active element is the operational amplifier. The passive filter synthesis is included to provide background material for other topics. For example, active filters are designed from passive resistively terminated *LC* ladder prototypes. In addition, other basic passive synthesis techniques are included to provide the students with a solid background in analog filters. By putting together theories, techniques, and procedures that can be used to analyze, design, and implement analog filters, it is hoped that after the completion of this book the students will be able to do some simple filter design work and will possess enough background for advanced study.

A by-product of this choice of topics is the stress of the importance of the operational amplifier. Two developments that have profoundly affected the modern practice of electrical engineering are the microprocessor for digital systems and the operational amplifier for the analog systems. It is imperative that the students have experience with both. This book stresses the usefulness of the operational amplifier.

The scope of this book should be quite clear from a glance at the table of contents. Chapter 1 introduces the fundamentals of analog filter design by covering the basic active building blocks and properties of network functions. Chapter 2 gives a fairly complete exposition of the subject of approximation, which includes the usual Butterworth, Chebyshev, inverse Chebyshev, and Bessel–Thomson responses. Frequency transformation is also covered. With the concept of positive-real functions established in Chapter 1, the properties of the driving-point immittance functions of the *LC* and *RC* one-port networks and their realization techniques are examined in Chapter 3. These techniques are extended to realize various classes of transfer functions in the form of *LC* or *RC* ladders or parallel ladders. Chapter 4 discusses the synthesis of resistively terminated lossless two-port networks and presents explicit design formulas for the doubly terminated lossless Butterworth and Chebyshev ladder networks. These ladder structures also serve as the prototypes for the synthesis of the coupled active filters described in Chapter 8.

The next four chapters deal with active filter synthesis. Chapter 5 introduces two basic approaches and two network configurations. The advantages and disadvantages of each approach are examined. The subject of sensitivity is covered in Chapter 6. In addition to discussing the various types of sensitivity and their interrelations, general relations of network function sensitivities are given. Chapter 7 begins the discussion of active filters, which include the single-amplifier general biquads and the multiple-amplifier general biquads. The study of simulated ladder networks is undertaken in Chapter 8. Simulation of the passive ladder is accomplished in three ways: the use of simulated inductors, the use of frequency-dependent negative resistors, and the simulation of the block-diagram representation of the ladder.

The last three chapters contain material on advanced passive filter synthesis. Chapter 9 is concerned with the design of broadband matching networks, which are used to equalize a resistive generator to a frequency-dependent load and to achieve a preassigned transducer power-gain characteristic. The theory of passive cascade synthesis is presented in Chapter 10. The students will soon discover that active synthesis is much simpler than its passive counterpart, because with the use of the operational amplifier as a buffer, individual sections can be designed independently.

Finally, the general problem of compatibility of two frequency-dependent impedances is taken up in Chapter 11. It is shown that the problem essentially reduces to that of the existence of a certain type of all-pass function.

The intent of the book is to provide a unified and modern treatment of filter design techniques. The arrangement of topics is such that they reinforce one another. The book stresses basic concepts, modern design techniques, and implementation procedures. The theory is supplemented and illustrated by numerous practical examples. The prerequisite knowledge is a typical undergraduate mathematics background of calculus, complex variables, and simple matrix algebra plus a working knowledge in Laplace transform technique. When the level of the material is beyond that assumed, references are given to the relevant literature. As a result, the book can also be used as a guide on filters to the practicing engineers who desire a good solid introduction to the field.

The material presented in the book has been classroom-tested at the University of Illinois at Chicago for the past several years. There is little difficulty in fitting the book into a one-semester or two-quarter course on analog filter design. For example, the first eight chapters contain material suitable for a one-semester course. The entire book is designed for a two-quarter course. For this reason, the material of the last three chapters on passive synthesis, which could have been made to follow Chapter 4, is relegated to the end, so that the first eight chapters would fit nicely into a one-semester course.

A special feature of the book is that it contains material on the design of broadband matching networks and on compatible impedances. These topics are normally excluded from undergraduate curricula, but recent advances makes these important filters available to undergraduates. The serious students will find the perusal of these chapters to be a gratifying and stimulating experience.

A variety of problems are included at the end of each chapter to enhance and extend the presentation. Most of these problems have been class-tested to ensure that their levels of difficulty and their degrees of complexity are consistent with the intent of the book. Also, the triangle symbol is used to represent the operational amplifier, because it appears to be the most commonly accepted usage in the literature.

I am indebted to many of my students over the years who participated in testing the material of this book, and to my colleagues at the University of Illinois at Chicago for providing a stimulating milieu for discussions. A personal note of appreciation goes to Puszka Łuszka for inspirational discussions. Special thanks are due to Yi-Sheng Zhu, Joseph Chiang and Eishi Yasui, who gave the complete manuscript a careful and critical reading and assisted me in preparing many of the tables, and to Jing-Liang Wan of Dalian Institute of Technology for proofreading the complete manuscript. Yi-Sheng Zhu and Eishi Yasui assisted me in preparing the index. Dr. Carlos Lisboa also proofread the first three chapters with some detailed comments. Finally, I express my appreciation to my wife, Shiao-Ling, and children, Jerome and Melissa, for their patience and understanding during the preparation of the book.

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Chicago, Illinois

CONTENTS

chapter 1

FUNDAMENTALS OF NETWORK SYNTHESIS

Design is the primary concern for the engineer as opposed to analysis for the scientist. It has traditionally implied the use of cut-and-try methods, the know-how from experience, and the use of handbooks and charts. It was not until after the Second World War that a scientific basis for design fully emerged, distinguished by the name *synthesis*. Synthesis has since become one of the most important subjects in modern electrical engineering. The contrast between analysis and synthesis is illustrated schematically in Figure 1.1. If the network and the excitation are given and the response is to be determined, the solution process is called *analysis*. When the excitation and the response are given and it is required to determine a network with no trial and error, it is known as *synthesis*. In this sense, analysis and synthesis are opposites. The word analysis comes from the Greek *lysis* and *ana*. It means the loosening up of a complex system. Synthesis, on the other hand, means putting together a complex system from its parts or components. In network synthesis, we are primarily concerned with the design of networks to meet prescribed excitation–response characteristics.

An electrical *filter* is a device designed to separate, pass, or suppress a group of signals from a mixture of signals. On a larger scale, televisions and radios are typical examples of electrical filters. When a television is tuned to a particular channel, it will only pass those signals transmitted by that channel and will block all other signals.

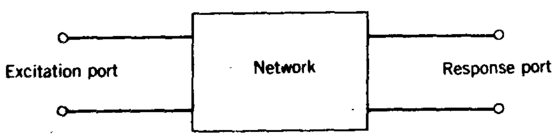

Figure 1.1

On a smaller scale, filters are basic electronic components used in the design of communication systems such as telephone, television, radio, radar, and computer. In fact, electrical filters permeate modern technology so much that it is difficult to find any electronic system that does not employ a filter in one form or another. The purpose of this book is to introduce some fundamental concepts and methods of modern filter design, and no electrical engineering graduate is equipped to understand the literature or advanced practice without a knowledge of these fundamentals.

1.1 NETWORK SYNTHESIS PROBLEM

There are important differences between analysis and synthesis. First of all, in analysis there is normally a unique solution. By contrast, in synthesis solutions are not unique and there may exist no solution at all. For example, in Figure 1.2, the generator with internal resistance $R_1 = 10\ \Omega$ is capable of supplying a maximum power of $|V_g|^2/4R_1 = 100/40 = 2.5$ W; yet the required output power is $|V_2|^2/R_2 = 36/10 = 3.6$ W. It is impossible to design a passive network to achieve this. As another example, consider the impedance function

$$Z(s) = \frac{4(s^2+7s+10)}{s^2+5s+4} \tag{1.1}$$

This impedance can be realized as the driving-point impedance of at least four different RC networks shown in Figure 1.3. Thus, by specifying the driving-point impedance there are many networks that will yield the same input impedance.

Second, network analysis uses a few basic methods such as nodal, loop, or state-variable techniques. Synthesis, on the other hand, makes use of a variety of methods and involves approximation. Figure 1.4 is the ideal low-pass filter transmission characteristic, where the amplitude of the desired transfer function is constant from $\omega = 0$ to $\omega = \omega_c$ and zero for all ω greater than ω_c. Such niceties cannot be achieved with a finite number of network elements. What then can be done to obtain a desired transmission characteristic? Instead of seeking overly idealistic performance criteria, we specify the maximum permissible loss or attenuation over a given frequency band and the minimum allowable loss over another frequency band. The shaded area of Figure 1.5 represents such a compromise, where maximum deviations are specified from 0 to ω_c and from ω_s to ∞. This is known as the *approximation problem*, and will be discussed in greater detail in Chapter 2.

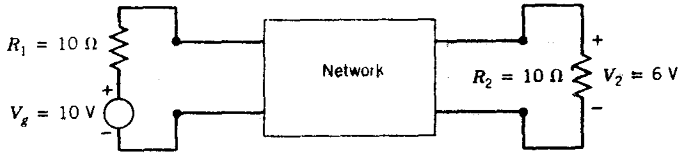

Figure 1.2

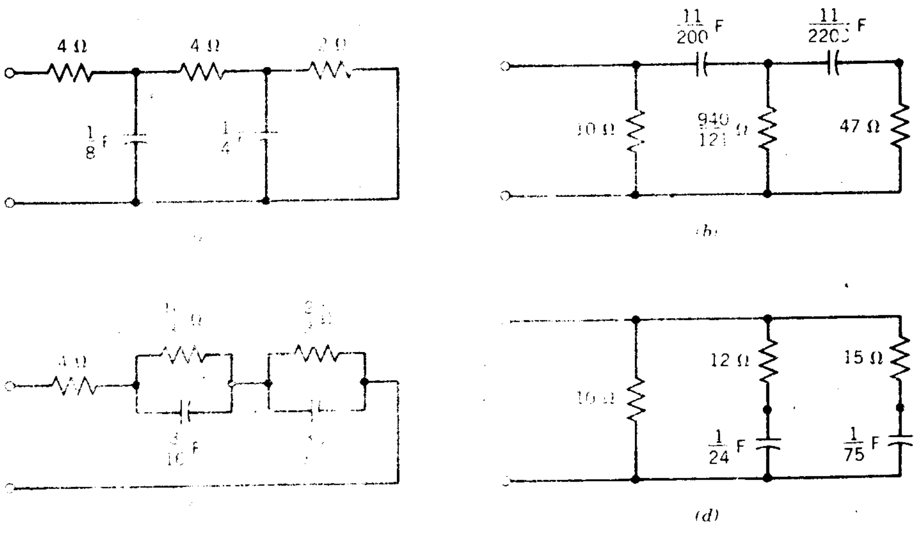

Figure 1.3

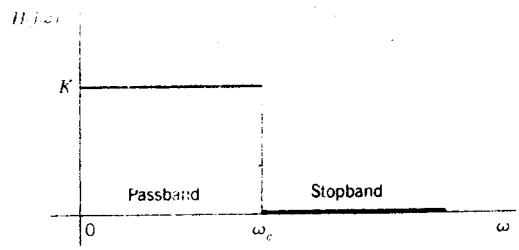

Figure 1.4

How is the synthesis of a network accomplished? The steps we follow are usually as follows: The first step is the determination of a suitable model to represent the system and the setting of specifications. For example, if one wishes to design a network to couple a voltage generator to the input of an operational amplifier, and to achieve the ideal low-pass transmission characteristic of Figure 1.4, the network model of Figure 1.6 and the transfer voltage-ratio function

$$|H(j\omega)| = \frac{|V_2(j\omega)|}{|V_g(j\omega)|} \tag{1.2}$$

are most appropriate. As mentioned above, with a finite number of elements, the characteristic of Figure 1.4 is not realizable. To circumvent this difficulty, we must alter our requirements. The second step is, therefore, to determine the allowable tolerances from the specifications. One such compromise is shown in Figure 1.5. The next

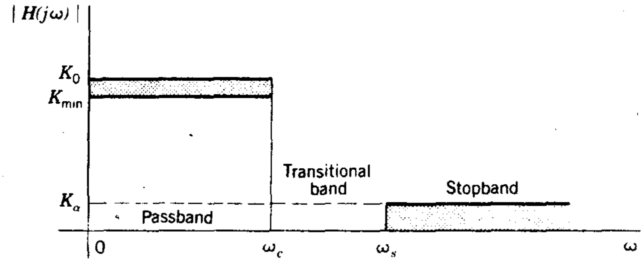

Figure 1.5

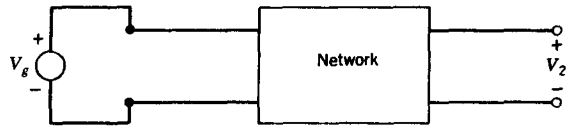

Figure 1.6

step is the determination of methods for expressing or approximating the filter characteristic so as to facilitate its realization. Once this is done, we proceed to the synthesis of physical networks that meet our specifications. In general, there are many possible solutions with different network configurations and different element values. The final step is then to select what appears to be the best solution based on criteria involving costs, performance, sensitivity, convenience, and engineering judgment. The selection of a solution from a number of alternatives is an integral part of engineering that manifests itself in synthesis much more than in analysis.

1.2 CLASSIFICATION OF FILTERS

Electrical filters may be classified in a number of ways. An *analog filter* is a filter used to process analog or continuous-time signals, whereas a *digital filter* is used to process discrete-time or digital signals. Analog filters may further be divided into *passive* or *active* filters, depending on the type of elements used in their realizations.

Filters are also classified according to the functions they perform. A *passband* is a frequency band in which the attenuation of the filter transmission characteristic is small, whereas in the *stopband* the opposite is true. The patterns of passband and stopband give rise to the four most common filter names whose ideal characteristics are depicted in Figure 1.7. An *ideal low-pass* characteristic is shown in (*a*) with passband extending from $\omega = 0$ to $\omega = \omega_c$ and stopband from ω_c to infinity, where ω_c is called the *angular* or *radian cutoff frequency*, or simply *cutoff frequency*. An *ideal high-pass* characteristic is the one shown in (*b*) with passband extending from ω_c to infinity and stopband from 0 to ω_c. Figure 1.7(*c*) is the characteristic for an *ideal*

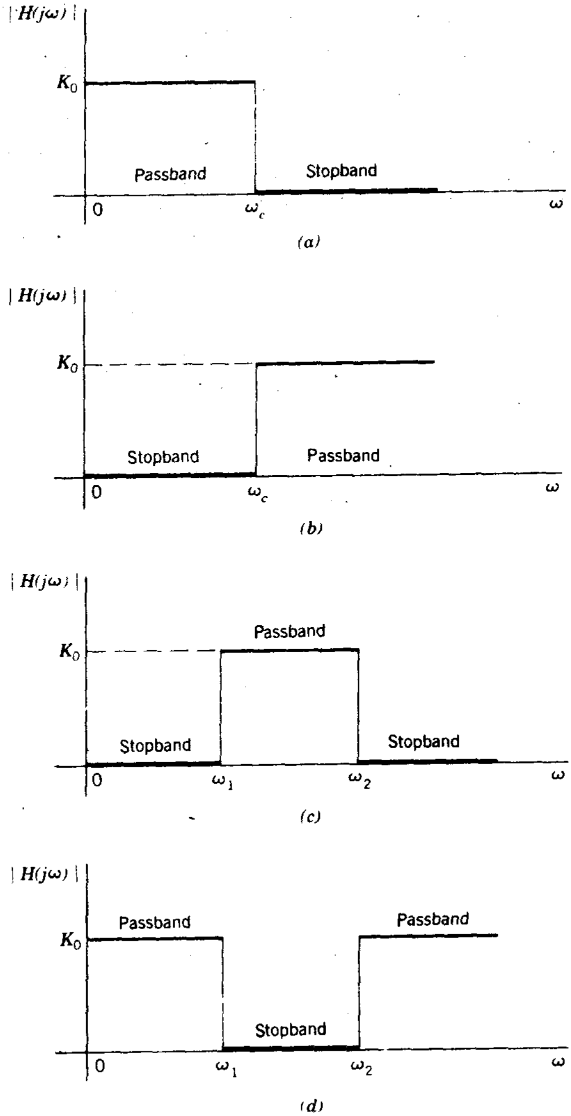

Figure 1.7

band-pass filter, in which radian frequencies extending from ω_1 to ω_2 are passed, while all other frequencies are stopped. Finally, the *ideal band-elimination* characteristic is shown in (*d*), where the radian frequencies from ω_1 to ω_2 are stopped and all others are passed. The band-elimination filters are also known as *notch* filters.

A familiar example that makes use of low-pass, high-pass, and band-pass filters is in the detection of signals generated by a telephone set with push buttons as in TOUCH-TONE dialing. In TOUCH-TONE dialing, the 10 decimal digits from 0 to 9, together with two extra buttons * and # used for special purposes, need to be identified. By using 8 signal frequencies in the frequency band from 697 to 1633 Hz, and arranging them in two groups, 4 low-band frequencies and 4 high-band frequencies, as depicted in Figure 1.8, 16 distinct signals can be identified. Each signal is represented by a pair of tones, one from the low band and one from the high band. Pressing a push button is therefore identified by a unique pair of signal frequencies. As the telephone number is pushed a set of signals is transmitted and then converted to suitable dc signals that are used by the switching system to connect the caller to the party being called. To detect the proper number to be called, it is necessary to identify the individual tones in the respective groups. This is accomplished by the 8 band-pass filters as shown in Figure 1.9. Each of these band-pass filters passes one tone and rejects all others, and is followed by a detector that is energized when its input voltage exceeds a certain threshold. When a detector is energized its output provides the required dc switching signal.

As another example, consider the transmission of a low-frequency signal such as a voice signal over a distance. To do this it is necessary to modulate a high-frequency signal carrier with this low-frequency signal before transmission. At the receiver,

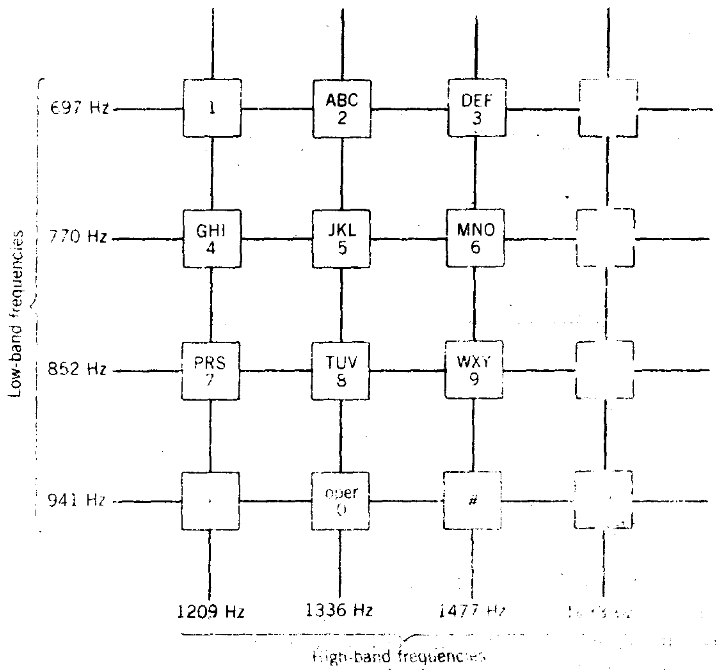

Figure 1.8