OPTICS

OPTICS

PARTS 1 AND 2

BY

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Preface to the English Edition

THE treatment of optics as presented here was developed during the early 1960's as part of a revision of the optics curriculum. Experience in teaching a course based on this material has shown that the material can easily be treated within one academic year.

In order to give new developments sufficient coverage, it has been necessary to dispense with some traditional material. In crystal optics the treatment is limited mainly to uniaxial crystals and only a short treatment is given to wave propagation rather than ray propagation in these crystals. The study of Fresnel diffraction is considerably abridged. A number of interferometers have not been discussed. Only one-electron optical spectra have been treated and the thermodynamic arguments relevant to thermal radiation have been omitted.

It has been found useful to use the electromagnetic theory of light from the outset. This approach is not the only one available, however, and the majority of optics texts do not presume prior knowledge of electromagnetism. This approach does have the advantage of permitting a logical development of the propagation equations, of allowing a precise treatment of phase, and a concise treatment of optical crystals. In addition, one should remember that optics actually extends from the r.f. region to X-rays and numerous demonstration experiments are easily performed in the decimetre wave region; in addition, the general interest in many methods, recently only within the domain of the specialist, is today much wider. These include the use of the Fourier transform, X-ray analysis of crystal structure, and the study of the radiation diagrams of antennae.

It is within the optics sequence that one often chooses to initiate the study of two important modern theories: relativity and quantum mechanics. With the former one limits the study to the special theory and to the derivation of the Lorentz transformation and its simplest kinematic results. The quantum theory plays a much more significant role. It is now time to attempt to elucidate a rigorous quantum atomic and molecular optics in the form which has resulted from 40 years of study and to spare the beginner the early wanderings now of more interest to the historian than to the physicist. However, it seemed that in the first study an axiomatic approach to quantum mechanics, in spite of the advantages it offers, was not the best approach and the wave theory was best suited to show the analogies and differences in the theory of the photon. This approach is certainly not the most general or the most rigorous; but the same can be said of the approach generally used at this level in the study of diffraction.

The author is grateful to M. Zarembovitch, maître-assistant, for help in the preparation of the exercises. The majority of the photographs in Part I were furnished by M. Françon whose help is most gratefully acknowledged.

Principal Physical Constants

Avogadro's number	$\mathcal{N}$	$= 6.025 \times 10^{26}$ molecules/kmole
Volume of a kmole of an ideal gas under standard conditions	V_m	$= 22{,}420$ cubic metres
Ideal gas constant (for 1 kmole)	R	$= 8.3169 \times 10^{3}$ J/K
Boltzmann's constant	k	$= R/\mathcal{N} = 1.380 \times 10^{-23}$ J/K
Permittivity of free-space	ε_0	$= 8.834 \times 10^{-12}$ farad/metre $[\simeq 1/(4\pi \times 9 \times 10^{9})]$
Faraday's constant	$\mathcal{F}$	$= 96522 \times 10^{6}$ coul/kmole
Electron charge	e	$= 1.602 \times 10^{-19}$ coul
Electron rest mass	m_e	$= 9.1083 \times 10^{-31}$ kg
Proton mass	M_{H}	$= 1.6724 \times 10^{-27}$ kg
Specific charge of the electron	e/m_e	$= 1.759 \times 11^{11}$ coul/kg
Planck's constant	h	$= 6.6252 \times 10^{-34}$ J-sec
Free-space velocity of light	c	$= 2.99793 \times 10^{8}$ m/sec
Rydberg constant for H	R_{H}	$= 10967758$ m^{-1}
Ground state radius of H	r_0	$= 0.5292 \times 10^{-10}$ m
Bohr magneton	μ_B	$= eh/(4\pi m_e) = 9.27 \times 10^{-24}$ A-m^2
Compton wavelength (electron)	λ_c	$= h/(m_e c) = 2.4262 \times 10^{-12}$ m

Energy conversion factors

1 calorie = 4.185 joules

1 electronvolt = 1.602×10^{-19} joules = 8068 cm^{-1}($\times hc$)

Principal symbols and variables

A	Absorption factor
α	angle, polarizability
B	magnetic induction
c	velocity of light in free-space
d	distance, electric dipole moment
D	electric displacement
e	thickness, elementary charge
E	electric field
$\mathcal{E}$	illumination
f	frequency, atomic scattering factor
F	force, oscillator strength
Φ	energy flux
g	Lande factor
G	angular momentum
γ	conductivity
Γ	contrast factor
h	Planck's constant
H	magnetic field
i	current density
I	light intensitv, current
$\mathfrak{I}$	source intensity
j	internal quantum number
k	absorption index, Boltzmann's constant
K	absorption coefficient
l	length, azimuthal quantum number
L	length, optical path length
$\mathcal{L}$	luminance
λ	wavelength
m	mass, magnetic quantum number
M	mass, magnetic moment
μ	reduced mass, permeability
n	index of refraction, principal quantum number
N	number of particles per unit volume
$\mathcal{N}$	Avogadro's number
ν	frequency
$\hat{\nu}$	spectroscopic wave vector $(1/\lambda)$
O	aperture
ω	angular frequency
Ω	solid angle
p	momentum
P	electric polarization
ψ, Ψ	wave function
q, Q	electric charge
r	distance, reflection coefficient
R	distance, reflection factor, Rydberg constant
ϱ	charge density, distance, depolarization factor
s	spin quantum number
S	surface area, Poynting vector
σ	wave vector $(2\pi/\lambda)$
t	transmission coefficient, time
T	transmission factor, period
τ	mean lifetime, volume
u	particle velocity, angle
v	wave velocity
V	electric potential, volume
w	electromagnetic energy density
W	energy
Z	impedance

Contents

CHAPTER 1

Definitions and fundamental phenomena

1.1. The object of optics

The study of optics begun with observations made with the eye. The mechanism of vision and the analysis of visual sensation arises in such observations. Now the sensations of light, darkness and colour cannot be described but only named. They have a subjective reality but they bring into the problem anatomical, physiological and even psychological factors which today are the basis of a branch of optics known as *physiological optics*.

This will not be part of this study and we will consider the eye as possessing a certain number of constant properties which will all be taken to have their mean values.

It frequently happens, in fact, that all the observations of a phenomenon give agreement in the description of the visual observation. This agreement permits the objective study of the relations between the light, the agent responsible for the visual sensation, and the properties of their source. This set of relationships is the basis of *physical optics*. A blind person can understand them.

On the one hand in fact, one knows that the eye is only a special kind of detector sensitive to light (§ 1.8.3). On the other, it has been discovered that the objective relationships established for visible light can be extended with certain quantitative modifications to radiation invisible to the human eye known as *electromagnetic radiation* so as to indicate their theoretical connection with their mode of production.

In this chapter we will examine several of the essential properties common to all electromagnetic radiation starting with those of visible light but without initially treating them with all the desirable rigour. The investigation of certain concepts and the limitation of certain far too extensive definitions are in part the consequences of this course as are the development of the basic ideas and the examination of their applications.

1.2. Sources of light

These are made up of luminous bodies (sun, stars, lamps, fireflies, etc.) which emit radiation. One occasionally uses as secondary sources the illuminated objects which reflect the light from a source (moon, mirrors, diffusers, luminescent objects, etc.) (§ 12.6(b)).

The sources are initially characterized by their dimensions, intensity, and colour. These concepts will be given a quantitative basis (§ 1.8).

A *point source* has dimensions which are so small that they can be neglected. This definition depends on the problem at hand; we will come back to this (§ 1.3).

1.3. Transmission of light—light rays—diffraction

1.3.1. The medium situated between the source and the receiver plays an important role in the transmission of light.

This medium may be free-space through which light can pass. If it is a material medium, it can be either *transparent* or *opaque*.

A not very detailed observation shows that for a point P to be illuminated by a point source S in a transparent, homogeneous medium the line SP must not encounter any opaque object. With this degree of precision, the light from a point source S (pin hole) and which passes through the *pupil* or *diaphragm* D (Fig. 1.1) having a width of several centimetres,

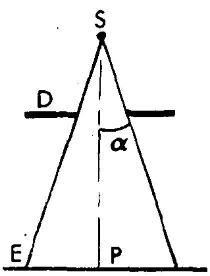

FIG. 1.1. A light beam.

generates a light spot on the screen E corresponding to the cross-section generated at E by the cone with apex at S traced out along the contour of D. The light passing through D forms a *light beam.* If the opening D is circular and normal to the median SP, the solid angle Ω of the conical beam in steradians (sd) is given by

$$\Omega = 2\pi(1 - \cos\alpha) \qquad (1.1)$$

where α is the generator angle of the cone and the angle 2α radians (rd) is the *beam aperture.*

When D is small so is Ω, the light beam is then called a *light pencil,* and expression (1.1) can be written approximately as

$$\Omega \simeq \pi\alpha^2. \qquad (1.2)$$

However, Ω can be small even if D is not; all that is required is that the distance from S to D be very large. When the beam is limited by lines lying in essentially the same direction it is called a *parallel beam* or a *cylindrical beam.*

In addition, it is not necessary that the source be a "point" S having the dimensions of a fraction of a millimetre, it is only necessary that its dimensions are small with respect to the other dimensions which occur in the problem, for example, with respect to the distance SP in Fig. 1.1. What is required then is that the angular diameter β (Fig. 1.2) of the source when viewed from the point of observation P be very small. Thus, the stars are point sources since their angular diameters do not exceed 3×10^{-7} rd (0.05″) in spite of the fact that the diameter of Sirius is, for example, greater than 10^{10} m.

A *collimator,* made up of a diaphragm having a small hole and situated at the principal focus of a converging lens, gives a parallel beam when the opening is illuminated by the light from any source.

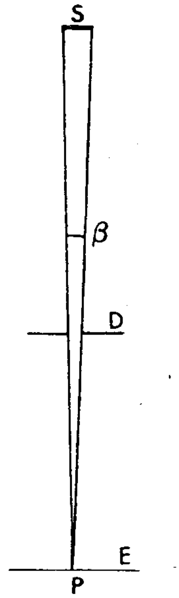

FIG. 1.2. Angular diameter of a source S.

A beam of light as fine as one can conceive of—but of course not get in practice—reduces to a line and this is called a *light ray*. This concept allows one to trace the direction of the propagation of light and it forms the basis of *geometric optics*.

1.3.2. The insufficiency of the concept of light rays is manifested by varying the conditions of the experiment in Fig. 1.1. One seeks to isolate a light ray by progressively shrinking the opening D (Fig. 1.3) which is, in this case, a slit. When one reaches a width which is of the order of a fraction of a millimetre, one observes at E not a luminous spot whose width would be deduced homothetically from D but, rather, a somewhat complicated

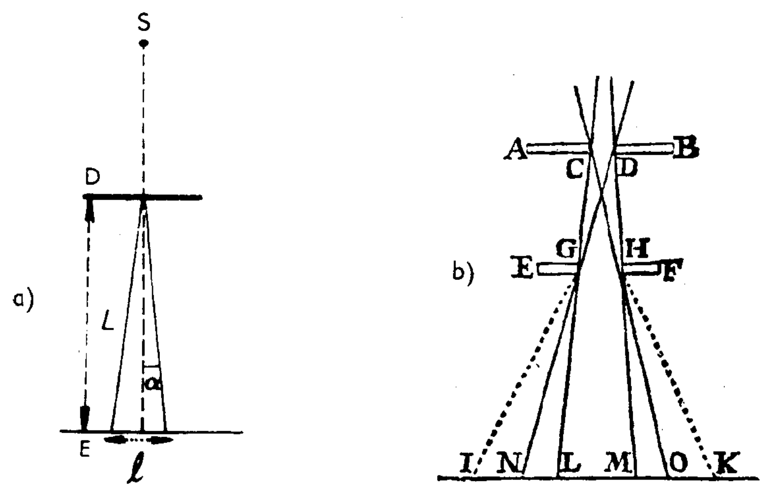

FIG. 1.3. (a) Diffraction of light; (b) Grimaldi's observation of the phenomenon.

light distribution called a *diffraction figure* (Chapter 5). The light is brighter at P and distributed essentially across an interval of width l. For example, for a slit width of 0.05 mm and with L equal to 3 m, l is about 6 cm.† The width l increases when one narrows the slit and at the same time the luminous spot dims. Thus, when one tries to define alight-ray experimentally, it will fade through spreading.

On the other hand, if the slit in Fig. 1.3 is set at 3 mm, one sees on E for SD = 1 m, a luminous band of width 12 mm on the edges of which the diffraction phenomena (observed by Grimaldi in 1665) occupy a space of only about 0.5 mm;† geometric optics give a good approximation in this case.

1.4. Monochromatic and complex radiation

Newton's experiment (Fig. 1.4) shows that the radiation from a source S can generally be broken down with the aid of a collimator SL and the prism P into *simple* or *monochromatic* radiation which forms the *spectrum* RV of the source in the focal plane of the lens L′.

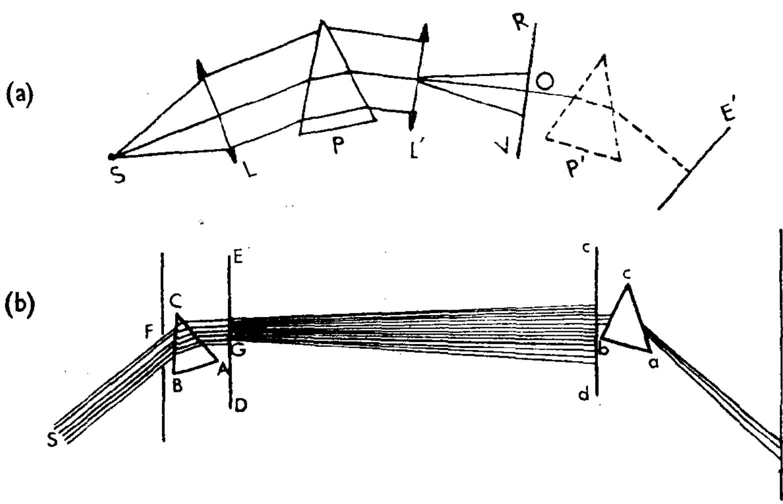

FIG. 1.4. (a) Decomposition of radiation by a prism; (b) Newton's experiment.

A small hole O allows a coloured pencil of light to pass through. This pencil is not further modified when passed through a second prism P′.

Certain sources emit a *continuous spectrum* and their radiation is apparently formed from an infinite number of monochromatic radiations. This is the case with the sun's radiation and the radiation from liquids and solids having a very high temperature (flames, tungsten filaments, carbon arcs, and so on). Other sources have a *discontinuous spectrum* made up of a finite number of monochromatic radiations called *spectral lines*. Radiation of this type is emitted by gases or vapours made luminous by high temperature (metallic salts in flames) or by the passage of an electric current under a voltage lying between several

† The calculations which lead to this value are found in § 5.10.

tens of volts (Na and Hg metallic vapour lamps) and several thousands of volts (hydrogen and the inert gases).

The apparatus which takes the spectrum from a source and selects a very narrow portion of it to serve as a secondary source is called a *monochromator*.

1.5. Speed of light and index of refraction

1.5.1. Light propagates from a source with a finite velocity which one can measure by various methods giving agreeing results (see § 9.4). Here we refer only to the Foucault method (Fig. 1.5). The lens L forms an image of the point source S at S′ on the concave mirror M′ following the reflection on the plane mirror M. The centre of M′ is at I with the

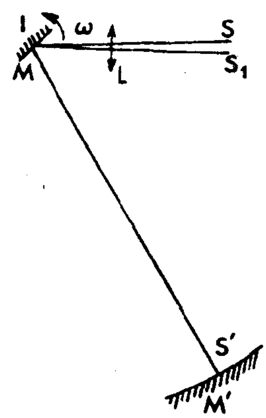

FIG. 1.5. Measurement of the velocity of light by Foucault's method.

result that the light which is reflected forms an image which coincides with S if the mirror M is fixed. M is rotated at a high angular speed ω (of the order of 1000 rd/s) about an axis positioned at I and normal to the figure. It turns through an angle $\theta = \omega t$ during the time $t = 2l/v$ which the light takes to pass twice along the trajectory l = IS′ with speed v. The rays reflected at M make an angle of 2θ with the incident rays as if S′ were displaced by $2\theta l$. The image of S lies at S_1 and one has:

$$SS_1 = 2\theta l \frac{p}{p'}$$

taking p = SL and p' = LIS′. From the measurements of the various lengths and of ω, one finds v.

This method allows one to measure the velocity in air or in a transparent medium placed in a tube between M and M′.

1.5.2. The results are as follows. The most precise methods give the velocity of light in free-space as

$$c = 299{,}793 \pm 0.3 \text{ km/s}$$

or 3×10^8 m/s to within 1 part in 1000. This does not depend on the colour. All electromagnetic radiation propagates with the velocity c in free space (§ 2.2).

In all material media the velocity v of a monochromatic ray is less than c (for yellow light in water $v \simeq \frac{3}{4}c$ and in glass $v \simeq \frac{2}{3}c$). In air, the difference between v and c is very small

$$\frac{c-v}{c} \simeq 3\times10^{-4}.$$

Instead of the value of v, most frequently one gives the value of the ratio

$$n = \frac{c}{v}, \tag{1.3}$$

called the *index of refraction* of the medium with respect to free-space or the *absolute* index of refraction.

In a material medium, the velocity v—and as a result, the index of refraction—of a monochromatic ray depends on its colour. This is the phenomenon of *refractive dispersion*.

1.6. Wave surfaces—optical trajectories

Consider now a monochromatic point source S situated in a homogeneous, isotropic medium, which emits at some time taken as the time origin an extremely brief light pulse. At time t, the light in the direction of any of the rays has passed through a distance $l = vt$. The locus of the points reached at this time for all the rays is a sphere called the *wave surface* for the medium under consideration.

If along certain of the rays, the light which starts at S, encounters any number of isotropic, homogeneous media separated by any kind of surfaces, the locus of the points reached by the light pulses at the end of some time interval is a surface of complex form

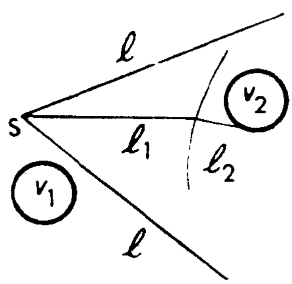

FIG. 1.6. Optical trajectory.

known as a *wave front*. If the light during time t traverses the path l_1 with velocity v_1, then l_2 with the velocity v_2, ... (Fig. 1.6), one has:

$$t = \frac{l}{v_1} = \frac{l_1}{v_1} + \frac{l_2}{v_2} = \sum \frac{l}{v} = \frac{1}{c}\sum nl.^{\dagger}$$

† This relationship assumes that the passage of the light from one medium into the next does not introduce an additional retardation, a fact which is not always the case (Chapter 3).

The product

$$L = ct = \sum nl \tag{1.4}$$

is the optical trajectory, the distance which the light will travel in free-space during time t. The wave front is thus the locus of the points for which the optical trajectories of the various rays are equal.

The wave surface is characteristic of the light propagation in a homogeneous medium in which it is entirely contained. The wave front is less general since its form depends on the trajectory followed by the light leaving the source.

We recall two important theorems from geometric optics related to optical trajectories and wave fronts: the optical trajectory taken by the light between two points is invariant (Fermat's theorem), and, if the media crossed by the light are isotropic, the light rays coming from S are normal to the wave front (Malus' theorem).

1.7. Huygens and Snell constructions

1.7.1. Let Ω (Fig. 1.7) be the wave front at some given instant. About point P on this surface trace a wave surface relative to the medium in which the light propagates from Ω; if this medium is isotropic, this is a sphere whose arbitrary radius $r = vt$ corresponds to an optical trajectory $nr = ct$. At all of the points on Ω trace the corresponding wave sur face

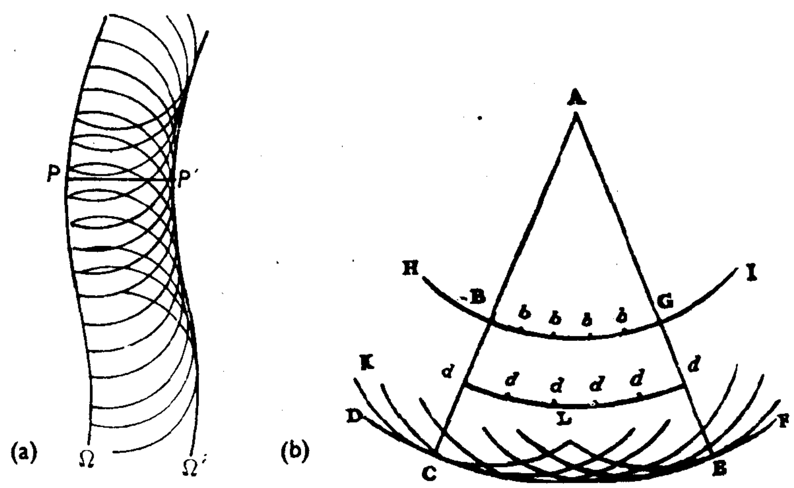

FIG. 1.7. (a) Huygens' principle; (b) Huygens' drawing.

having the same trajectory nr (n can vary from point to point in this case and one should use an infinitesimal radius $dr = v\,dt$). Using the theorem of Malus, the wave front Ω' at time t after its passage from Ω should be perpendicular to all of the rays such as PP′ normal to Ω. This is the *envelope* of all of the spherical wave surfaces.

1.7.2. The construction above allows us to derive the laws of reflection and refraction. Consider a plane wave Ω_1 (Fig. 1.8) normal to the plane of the figure which propagates with the velocity v_1 and falls on the plane surface Σ which separates the first homogeneous

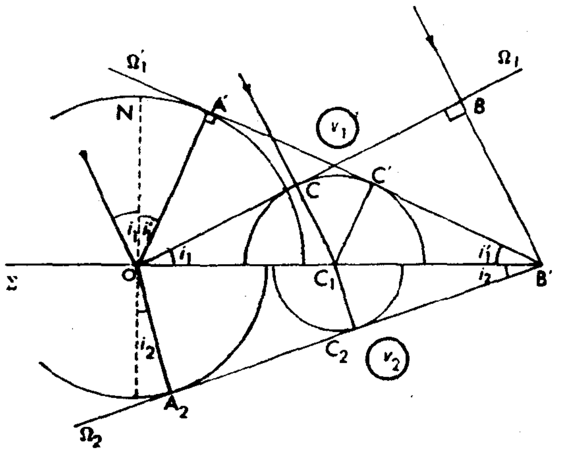

FIG. 1.8. Huygens' construction.

medium from a second where the velocity is v_2. At the given instant the point O of Ω_1 is on Σ. Trace in the first medium the hemispherical wave surface centred about O with radius $r_1 = v_1\,\mathrm{d}t$. At the end of the time interval dt, the point B on Ω_1 has reached B′ on Σ (BB′ $= v_1\,\mathrm{d}t$). The point C, the mid-point of OB, reaches C_1 on Σ at time dt/2. The wave front at time dt in the first medium then passes through B and is tangent to the hemisphere centred at O with radius r_1 and the hemisphere centred at C_1 with radius $r_1/2$. The front of the *reflected wave* Ω'_1, is thus planar and has as its profile A′B′C′ at time dt. The congruence of the triangles OBB′ and B′A′O gives the law of reflection: the reflected rays (normal to Ω'_1, such as OA′) lie in the plane of incidence (plane of the figure) and the angle of incidence i_1 is equal to the angle of reflection i'_1.

In the second medium, the wave surface centred at O corresponding to time dt is a hemisphere with radius $r_2 = v_2\,\mathrm{d}t$. That with C_1 as centre [$CC_1 = v_1(\mathrm{d}t/2)$] is a hemisphere with radius $v_2(\mathrm{d}t/2)$. The *transmitted wave front* in the second medium at time dt is then the plane of the profile Ω_2 passing through B′ and tangent to the hemispheres at A_2 and C_2. Comparison of the triangles OBB′ and B′A_2O gives

$$\frac{\sin i_1}{\sin i_2} = \frac{BB'}{OA_2} = \frac{v_1}{v_2}. \tag{1.5}$$

The Huygens construction then gives the refraction laws of Snell and Descartes: the refracted rays (such as OA_2) lie in the plane of incidence and there exists a constant ratio between the sines of the angles of incidence i_1 and of refraction i_2. Using equation (1.3) this can be written:

$$\frac{\sin i_1}{\sin i_2} = \frac{c}{n_1}\cdot\frac{n_2}{c} = \frac{n_2}{n_1} = n_{2,1} \tag{1.6}$$

$n_{2,1}$ is the *relative refractive index* of the second medium with respect to the first and it is equal to the ratio of the absolute indices n_2 and n_1.

One can note in Fig. 1.8 that to an incident wave Ω'_1 Huygens construction leads to a corresponding reflected wave Ω_1 (reflective inversion). Likewise, to the incident wave Ω_2

in the medium v_2 there corresponds the wave Ω_1 in the medium v_1 (refractive inversion). Thus one has, by the interchange of the angles i_1 and i_2;

$$\frac{\sin i_2}{\sin i_1} = \frac{n_1}{n_2} = n_{1,2} \tag{1.7}$$

hence, through comparison with (1.6),

$$n_{1,2} = \frac{1}{n_{2,1}}. \tag{1.8}$$

1.7.3. A construction due to Snell and equivalent in all respects to that due to Huygens makes use of the rays normal to the waves. Let SO be the incident ray (Fig. 1.9). With O as the centre trace circumferences with radius n_1 and n_2 to act as meridian *indicial surfaces* in the second medium. The extension of the incident ray strikes the circumference n_1 at N_1.

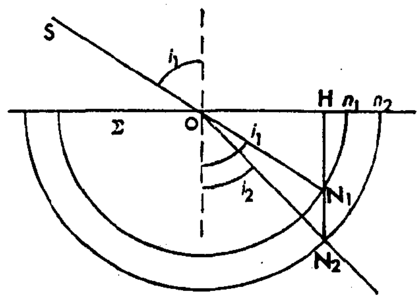

FIG. 1.9. Snell's construction.

Let H be the base of the perpendicular from N_1 on Σ. HN_1 strikes the surface n_2 at N_2. ON_2 is the refracted ray. One has in effect:

$$OH = ON_1 \sin i_1 = ON_2 \sin i_2. \tag{1.9}$$

Figure 1.9 shows that the point N_2 and, therefore, a refracted ray, always exists when n_2/n_1 is greater than one. If the ray comes from the more refractive medium (Fig. 1.10),

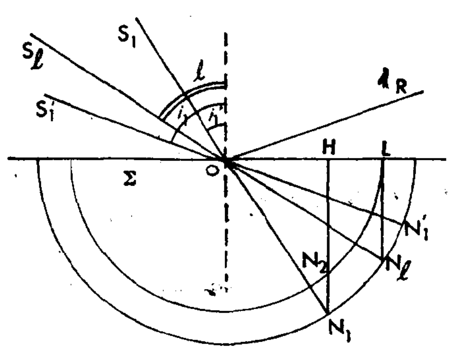

FIG. 1.10. Total reflection (Snell construction).

the construction gives a refracted ray ON_2 for an angle of incidence i_1 lying between O and a *limiting angle* l such that the point H falls at L on the circle of radius n_2. Thus,

$$\sin l = \frac{OL}{ON_l} = \frac{n_2}{n_1}. \tag{1.10}$$

For an angle of incidence i_1' greater than l, the construction no longer leads to an emergent ray. Experiment shows that waves in the direction $S_1'O$ lead to *total reflection* and return in the direction OR according to the laws of reflection.†

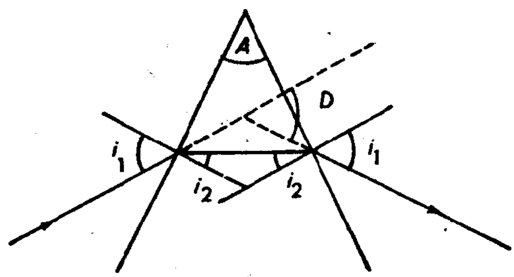

FIG. 1.11. Prism at minimum deviation.

The measurement of the refractive index is made by various methods. Some of these make use of the existence of the limiting angle (§ 3.6.2). We call attention to the method using the angle of minimum deviation of a prism (Fig. 1.11):

$$n_{2,1} = \frac{\sin \dfrac{A+D}{2}}{\sin A/2}. \tag{1.11}$$

We note finally that the laws of reflection and refraction can serve equally well as Fermat's principle as a basis for geometric optics since both systems are equivalent.

1.8. Energy definitions relative to electromagnetic radiation

1.8.1. From the point of view of the energy, a source transforms some other form of energy W into radiant energy (Fig. 1.12). This can be heat (filament lamps) and one calls this

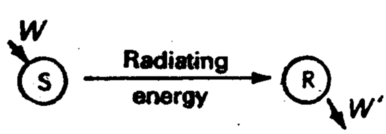

FIG. 1.12. Source S and detector R for radiant energy.

† One notes that the vectors ON_1 and ON_2 are proportional to the *wave vectors* σ (§ B.2) to which Snell's construction then applies.