



VOLUME TWO

SECOND EDITION

UNIVERSITY PHYSICS

ARFKEN GRIFFING KELLY PRIEST

VOLUME

TWO

UNIVERSITY

PHYSICS

Second Edition

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PREFACE

In this second edition of *University Physics* we present students with an authoritative and easy-to-use text. Among the features that we deem important for a beginning physics textbook are:

- a sound pedagogical presentation
- the systematic development of problem-solving skills
- a special sensitivity to students and their goals

A SOUND PEDAGOGICAL PRESENTATION

We present the most important physical principles in the least intimidating way. The unity of physics and the universal character of its principles are emphasized. Basic concepts are illustrated with numerous examples, many drawn from such diverse areas as astrophysics, sports, and the environment.

A SYSTEMATIC DEVELOPMENT OF PROBLEM-SOLVING SKILLS

A step-by-step Problem-Solving Guide is introduced in Chapter 3 and extended in Chapters 6 and 28. Students are shown how to approach and solve problems in a systematic fashion. The Guide is illustrated with numerous examples.

The text of each chapter concludes with a challenging Worked Problem, typical of those found in the end-of-chapter problem sets. Each chapter presents a set of Exercises and Problems that allow students to test their grasp of the principles. Single-concept Exercises reinforce ideas developed in the current chapter. The Exercises are followed by a set of substantive Problems that often illustrate the “vertical” structure of physics and require students to draw on concepts learned in earlier chapters. In this edition, we have provided a wide range of problems, with many problems at the challenging end of the spectrum. Instructors can readily match the abilities of their students to the problems.

A SPECIAL SENSITIVITY TO STUDENTS

Our goal is to create a learning environment that inspires student confidence. We are patient with students. For example, we have considered students who are taking calculus concurrently. The first five chapters avoid calculus and allow

students to develop problem-solving skills and build confidence before being confronted by calculus-based problems.

Also, the liberal use of examples and illustrations, and our Problem Solving Guide, help to produce the sensitive atmosphere for which we strive.

PATHWAYS THROUGH THIS TEXT

There are many ways to structure physics courses, and this text can be used in a variety of ways to meet that diversity. Here at Miami University we use the text in two slightly different sequences. In class sections open only to entering freshmen, the first semester covers Newtonian Mechanics (Chapters 1–13) and Special Relativity (Chapters 39–40). The special relativity is interwoven with Newtonian mechanics. The second semester is devoted to Electromagnetism (Chapters 23–35). Many of the students continue with a third semester that covers Materials and Fluid Mechanics (Chapters 14–15), Waves (Chapters 16–17), Thermal Physics (Chapters 18–22), Optics (Chapters 36–38), and selections from Contemporary Physics (Chapters 41–42).

In the other sequence here, upperclass students form a more heterogeneous audience. The first semester covers Newtonian mechanics, materials, and waves. Special relativity is omitted. The second semester covers electromagnetism and optics.

SUPPLEMENTS

We provide an Instructor's Answer Book, a Student's Solutions Manual containing solutions or hints to approximately twenty percent of the Exercises and Problems, and a set of transparencies. Instructional software is available for IBM compatibles and Apple II series microcomputers. A Study Guide written by T. William Houk, James E. Poth, and John W. Snider offers additional insights and opportunities for students to sharpen their problem-solving skills.

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We are indebted to many reviewers, students, and colleagues for their helpful criticisms during the development of this text. Special thanks go to Bill Adams, Baylor University; Larry Banks, Southwest Missouri State University; James T. Cushing, University of Notre Dame; Patrick Hamill, San Jose State University; Joseph H. Hamilton, Vanderbilt University; James Monroe, Penn State University-Beaver; R. D. Purrington, Tulane University; Eric Sheldon, University of Lowell; K. L. Schick, Union College; and Ken-Hsi Wang, Baylor University. Finally, to Jeff Holtmeier, Debbie Hardin, Chris Nelson, Kim Svetich, Marilyn Britt, Stacy Simpson, and Lynne Bush of Harcourt Brace Jovanovich go our collective thanks for their encouragements, proddings, and zealous attention to detail.

George B. Arfken

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Joseph Priest

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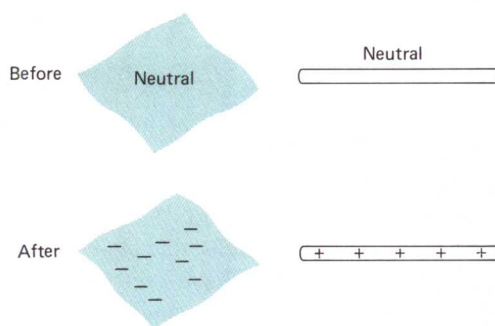
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**FIGURE 23.2**

Separation of electric charge when a glass rod is rubbed with a silk cloth.

Conservation of Electric Charge

The concept that a net electric charge can never be created or destroyed goes back at least to Benjamin Franklin. When a glass rod is rubbed with silk, electrons are transferred from the glass to the silk, giving the silk a negative charge and leaving the glass rod with an equal positive charge. This transfer of electrons is illustrated in Figure 23.2. No change in the total charge of glass-plus-silk occurs. Conservation of charge has been tested repeatedly in the realm of high-energy physics and has been found to hold without exception. The principle of conservation of electric charge can be stated as follows:

The net electric charge remains constant in all processes.

The next two examples illustrate the conservation of electric charge in nuclear processes.

EXAMPLE 1

Uranium 238 Alpha Decay

The radioactive uranium 238 nucleus ${}^{238}_{92}\text{U}$ disintegrates by emitting an alpha particle (helium nucleus). This nuclear reaction may be written as



The superscripts give the combined number of neutrons and protons in each nucleus. The subscripts give the number of protons in each nucleus and therefore measure the positive nuclear charge. There are 92 protons in the uranium nucleus. The decay products contain a total of 92 protons, 2 in helium and 90 in thorium. The balancing of the subscripts, 92 for both sides, describes the exact conservation of electric charge in this nuclear reaction.

In Example 1, electric charge in the form of protons is simply rearranged. Sometimes electric charges are created. When this occurs, positive and negative charges are created in equal amounts, keeping the net charge unchanged.

EXAMPLE 2

Carbon 14 Beta Decay

Carbon 14 (${}^{14}_6\text{C}$) has six protons in its nucleus and is formed in our atmosphere by cosmic ray bombardment of nitrogen. Carbon 14 is unstable and transforms into nitrogen 14 by emitting an electron and an antineutrino (zero mass, zero charge).



In this process one of the 8 neutrons in the carbon 14 nucleus is transformed into three particles: a positively charged proton, a negatively charged electron, and a neutral antineutrino. The proton, the electron, and the antineutrino are *created* in the reaction. Although both positive and negative charges are created, the net charge remains the same (+6 before = -1 + 7 = +6 after).

23.2 COULOMB'S LAW

A quantitative breakthrough in electrostatics occurred in 1785 when the French scientist Charles Augustin de Coulomb measured the force between two small electrically charged spheres. Coulomb found that the force between the charged spheres was inversely proportional to the square of the distance between them and directly proportional to the product of their charges:

$$F \propto \frac{q_1 q_2}{r^2}$$

This proportionality is converted into an equation by introducing a proportionality constant. The result is known as **Coulomb's law of electrostatic force** and may be written as

$$F = k_e \frac{q_1 q_2}{r^2} \quad (23.1)$$

The SI unit of charge is the coulomb (symbol C). The operational definition of the coulomb, based on magnetic effects of electric currents, is presented in Chapter 30. The proportionality constant k_e is

$$k_e = 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \quad (23.2)$$

Like all forces, the electrostatic force obeys Newton's third law. That is, Equation 23.1 describes the magnitude of the equal but oppositely directed forces that the charges q_1 and q_2 exert on each other (Figure 23.3). The Coulomb force is repulsive for like charges and attractive for unlike charges.

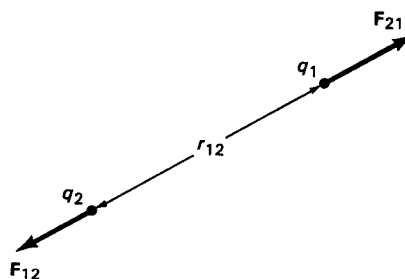


FIGURE 23.3

F_{12} is the force exerted by q_1 on q_2 and F_{21} is the equal but oppositely directed force exerted by q_2 on q_1 . If q_1 and q_2 have the same sign, the forces are repulsive, as shown here. If q_1 and q_2 have opposite signs, the forces are attractive.

EXAMPLE 3**Electrostatic Repulsion**

Let's calculate the force of repulsion between two 1 C charges 1 m apart. From Coulomb's law (Equation 23.1) we have

$$\begin{aligned} F &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \cdot \frac{(1 \text{ C})^2}{(1 \text{ m})^2} \\ &= 8.99 \times 10^9 \text{ N} \end{aligned}$$

This is a force of about 1 million tons. Clearly, one coulomb is an enormous charge. In fact, this example is unrealistic in the sense that we could not get charges of 1 C to stay on small surfaces separated by 1 meter.

Coulomb's law of electrostatic interaction and Newton's law of universal gravitation have the same mathematical form; both are inverse square laws. But what about the relative strength of these two fundamental forces? Let's calculate the ratio of the electrostatic force and the gravitational force between an electron and a proton. These forces are described by Coulomb's law (Equation 23.1) and Newton's law of universal gravitation. The ratio is

$$\frac{F_{\text{elec}}}{F_{\text{grav}}} = \frac{k_e \left(\frac{q_e q_p}{r^2} \right)}{G \left(\frac{m_e m_p}{r^2} \right)} = \frac{k_e q_e q_p}{G m_e m_p}$$

Note that the distance factor cancels out. The value of the ratio is

$$\frac{F_{\text{elec}}}{F_{\text{grav}}} \approx 2 \times 10^{39}$$

This is an enormous number. Imagine grains of sand so fine that you can pack 10^6 grains in 1 cm^3 ; 10^{39} of these grains would occupy the volume of a million earths! Clearly, the electrostatic force is far stronger than gravity.

The fact that Newton's law of gravitation and Coulomb's electrostatic law have the same $1/r^2$ distance dependence has impressed many scientists, including Einstein, as more than mere coincidence. So far, no profound relationship or common origin has been discovered.

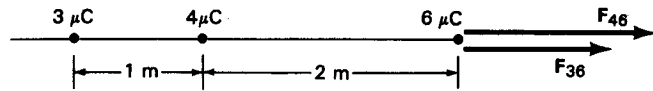
23.3

SUPERPOSITION

Coulomb's law specifies the force between a pair of point charges. When more than two charges interact, experiment shows that the net force on any particular charge is the vector sum of the Coulomb forces exerted on it by the other charges.

EXAMPLE 4**Superposition**

Charges of $3 \mu\text{C}$, $4 \mu\text{C}$, and $6 \mu\text{C}$ are placed along a line (Figure 23.4). Let's use Coulomb's law to calculate the two separate forces exerted on the $6 \mu\text{C}$ charge. First,

**FIGURE 23.4**

The total force on the 6- μC charge is the sum of the forces exerted by the 3- μC and 4- μC charges.

consider the force exerted by the 3- μC charge. From Coulomb's law (Equation 23.1) the force exerted on the 6- μC charge is

$$\begin{aligned} F_{36} &= \frac{8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 (3 \times 10^{-6} \text{ C})(6 \times 10^{-6} \text{ C})}{(3 \text{ m})^2} \\ &= 1.80 \times 10^{-2} \text{ N} \quad (\text{directed to the right}) \end{aligned}$$

Next, we consider the force exerted by the 4 μC charge.

$$\begin{aligned} F_{46} &= \frac{8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 (4 \times 10^{-6} \text{ C})(6 \times 10^{-6} \text{ C})}{(2 \text{ m})^2} \\ &= 5.39 \times 10^{-2} \text{ N} \quad (\text{directed to the right}) \end{aligned}$$

Superposing F_{36} and F_{46} yields the total force on the 6 μC charge:

$$F_6 = F_{36} + F_{46} = 7.19 \times 10^{-2} \text{ N}$$

To within the limits of experimental accuracy the total force on the 6 μC charge has been confirmed to be the sum of F_{36} and F_{46} , or $7.19 \times 10^{-2} \text{ N}$. In other words, experiment shows that the presence of a third charge does not influence the Coulomb force between the other two charges.

We can generalize the experimental result stated in Example 4 by saying that electrical forces obey a **principle of superposition**:

The net force exerted by two or more charges on a single charge Q is the vector sum of the individual forces exerted on Q .

Keep in mind that this principle is the result of experiment.

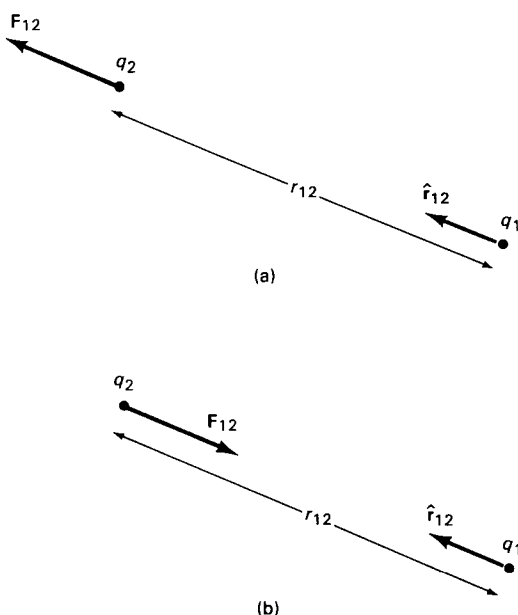
Vector Form of Coulomb's Law

The electrostatic force is a vector quantity—it has direction as well as magnitude. We can write Coulomb's law in vector form by introducing a unit **vector** to indicate direction. In Figure 23.5, \hat{r}_{12} is a unit vector directed from q_1 toward q_2 . The force \mathbf{F}_{12} exerted by q_1 on q_2 is

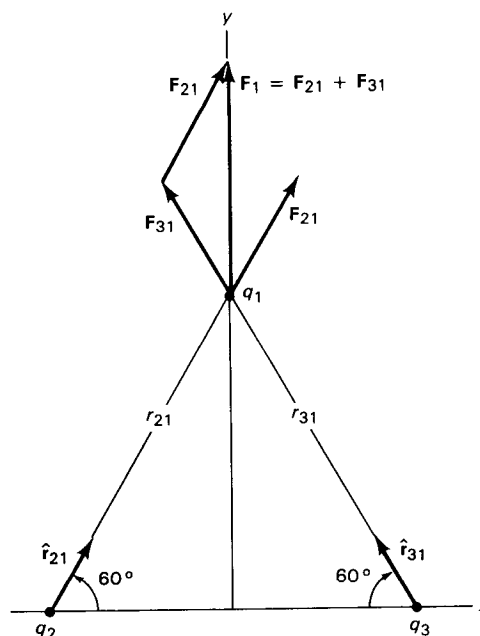
$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} \quad (23.3)$$

If the charges q_1 and q_2 are both positive or both negative, the force is repulsive, and \mathbf{F}_{12} is parallel to \hat{r}_{12} (Figure 23.5a). If q_1 and q_2 have opposite signs, then the force is attractive, indicating that q_1 is urged toward q_2 (Figure 23.5b).

Example 4 involves only parallel forces. Now let's apply Equation 23.3 and the principle of superposition to a system where the forces are not parallel. We build in enough symmetry so that we can check our results.

**FIGURE 23.5**

F_{12} denotes the force exerted by q_1 on q_2 . The unit vector \hat{r}_{12} is directed from q_1 toward q_2 . (a) The direction of F_{12} indicates a repulsive force between like charges. (b) When q_1 and q_2 have opposite signs the force is attractive.

**FIGURE 23.6**

A $1\text{-}\mu\text{C}$ charge is located at each vertex of the equilateral triangle. The net force F_1 on q_1 is the vector sum of the forces F_{31} and F_{21} exerted by q_3 and q_2 .

EXAMPLE 5

Vector Addition for Coulomb Forces

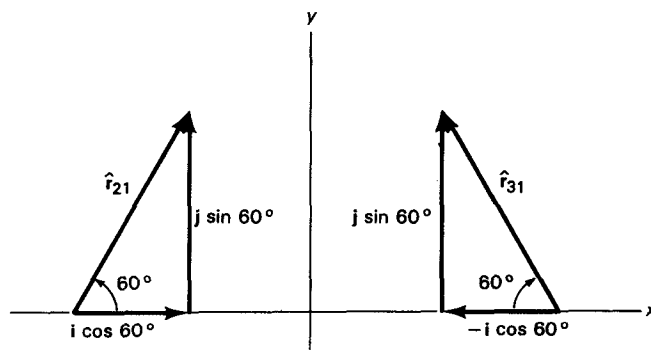
Consider three $1\text{ }\mu\text{C}$ charges at the vertices of an equilateral triangle, 1 m on a side (Figure 23.6). What is the net force that the two bottom charges exert on the top charge (q_1)?

Figure 23.6 shows that the array of three equal charges has left-right symmetry relative to a vertical line through q_1 . We know from this symmetry that the net force on q_1 will be vertical and in the upward direction. (All charges have the same sign; all forces are repulsive.) The net force on q_1 is the vertical component of the force exerted by q_2 plus the vertical component of the force exerted by q_3 . Since the two vertical components are equal by symmetry, the magnitude of the force on q_1 is

$$\begin{aligned} F_1 &= 2k_e \left(\frac{q_1 q_2}{r^2} \right) \cdot \cos 30^\circ \\ &= 2(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \left[\frac{10^{-6} \text{ C} \cdot 10^{-6} \text{ C}}{1 \text{ m}^2} \right] \cdot (0.866) \\ &= 1.56 \times 10^{-2} \text{ N} \end{aligned}$$

Now, let's go through the calculation in detail using the vector form of Coulomb's law. The unit vectors are illustrated in Figure 23.7. For F_{21} —the force that q_2 exerts on q_1 —we have

$$\begin{aligned} F_{21} &= k_e \left(\frac{q_1 q_2}{r_{21}^2} \right) \hat{r}_{21} \\ &= (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \left[\frac{10^{-6} \text{ C} \cdot 10^{-6} \text{ C}}{(1 \text{ m})^2} \right] \hat{r}_{21} \\ &= 8.99 \times 10^{-3} \hat{r}_{21} \text{ N} \end{aligned}$$

**FIGURE 23.7**

The unit vectors \hat{r}_{21} and \hat{r}_{31} can be resolved into x - and y -components.

We can resolve the unit vector \hat{r}_{21} into its Cartesian components (Figure 23.7)

$$\hat{r}_{21} = i \cos 60^\circ + j \sin 60^\circ$$

This gives

$$\begin{aligned} \mathbf{F}_{21} &= (8.99 \times 10^{-3} \text{ N}) \cos 60^\circ i + (8.99 \times 10^{-3} \text{ N}) \sin 60^\circ j \\ &= (4.49 \times 10^{-3} \text{ N})i + (7.78 \times 10^{-3} \text{ N})j \end{aligned}$$

The force \mathbf{F}_{31} is given by

$$\mathbf{F}_{31} = k_e \left(\frac{q_1 q_3}{r_{31}^2} \right) \hat{r}_{31}$$

The unit vector \hat{r}_{31} is given by

$$\hat{r}_{31} = -i \cos 60^\circ + j \sin 60^\circ$$

This gives

$$\mathbf{F}_{31} = (-4.49 \times 10^{-3} \text{ N})i + (7.78 \times 10^{-3} \text{ N})j$$

The net force on q_1 is the vector sum of \mathbf{F}_{21} and \mathbf{F}_{31} . The horizontal components cancel each other because of the symmetry of the system. The result is

$$\begin{aligned} \mathbf{F}_1 &= \mathbf{F}_{21} + \mathbf{F}_{31} \\ &= (1.56 \times 10^{-2} \text{ N})j \end{aligned}$$

This shows that the net force is vertically upward with a magnitude of $1.56 \times 10^{-2} \text{ N}$. This solution agrees with the first calculation.

Using symmetry made it easier to find a solution. If symmetry is not present, however, we can still find the net force by using the vector form of Coulomb's law and the principle of superposition.

Continuously Distributed Charge

All electric charge distributions are collections of discrete charges such as electrons and protons. However, when we consider a large number of closely packed charges, we can treat the distributed discrete charges as continuous. To determine forces exerted by continuous distributions of charge the principle of superposition may be applied, but integrations replace discrete sums.

Figure 23.8 shows a **line charge**, a collection of charges spread continuously along a line. A point charge Q located at the point P experiences forces exerted