LINEAR SYSTEMS

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PREFACE

Linear systems have been under study for a long time, and from several different points of view, in physics, mathematics, engineering, and many other fields. But the subject is such a fundamental and deep one that there is no doubt that linear systems will continue to be an object of study for as long as one can foresee. However, a particular feature of recent engineering studies, and the main focus of this book, is the emphasis on the structure of finite-dimensional linear systems. While such systems have been extensively studied, especially since the early 1930s, the frequency-domain techniques that were commonly used often did not specifically exploit the underlying finite dimensionality of the systems involved. Moreover, almost all this work was for single-input, single-output (or *scalar*) systems and did not seem to extend satisfactorily to the multi-input, multi-output (or *multivariable*) systems that became increasingly important in aerospace, process control, and econometric applications in the late 1950s. This fact, plus the importance of time-variant systems and time-domain characteristics in aerospace problems, led to a resurgence of interest, sparked by the work of Bellman and Kalman, in the state-space description of linear systems. This approach led naturally to more detailed examinations of the structure of finite-dimensional linear systems, or linear dynamical systems as they are often called, and to questions of redundancy, minimality, controllability, observability, etc. The papers [1]† and [2] give a good perspective of the situation around 1960. The state-space

†See the references following the Preface.

formulation led to some new proposals for system design and feedback compensation—pole-shifting controllers, quadratic regulator synthesis, state observers and estimators, noninteracting control, etc. But just as the state-space techniques were being codified into textbooks (References [3] and [4] nicely bracket the books of that period), Popov [5] and Rosenbrock [6] were showing how many of the scalar rational transfer function concepts could be naturally extended to matrix transfer functions and multivariable systems and how several questions could be more readily posed and solved in these terms. Since then these concepts have been effectively pursued by several researchers. By now, it seems to us, the main insight from this work is that transfer function (or high-order differential equation) descriptions and state-space (or first-order differential equation) descriptions are only two extremes of a whole spectrum of possible descriptions of finite-dimensional systems. We can work exclusively with one description or the other, but we can also easily translate results from one framework to the other, and, as expected, there are situations where a hybrid of the two extremes (using the so-called partial-state descriptions) is the most natural.

Our aim in this textbook is to take a beginning student, with some prior exposure to linear system analysis (elementary transform and matrix theory), through a motivated and integrated development of these new and fuller perspectives on linear system theory.

The detailed table of contents will provide a general idea of the scope of the book. Briefly, we start with scalar (single-input, single-output) systems and introduce the notions of state-space realizations, internal and external descriptions, controllability, observability, and their applications to minimal realizations, state-feedback controllers, and observers. While doing this, we also compare and contrast these state-space results with more classical transfer function ideas and gradually build up the awareness that equivalent results could have been obtained by working (carefully) with transfer function descriptions without reference to state variables or controllability or observability. The restriction to constant scalar systems in Chapters 1 to 5 allows one to gain this perspective at a fairly concrete and explicit level, so that the extension to multivariable systems can proceed more rapidly in the rest of the book (Chapters 6 to 9). Particular care was devoted to the selection and arrangement of topics in the scalar case, so that the parallel multivariable development is not only well motivated but, in the author's opinion, also quite insightful and powerful. Thus at many points the development reaches the frontiers of research (see also Chapter 10), equipping the reader for new studies and applications in the many fields where linear system theory can be important—e.g., in signal detection and estimation, system identification, process control, digital filtering, communication systems, and, generally speaking, the broad and exciting field of signal processing.

At Stanford, the material in the first five chapters and in Secs. 6.1, 6.2, and in Chapter 9 is covered in a 40 to 45-hour senior/first-year graduate

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course, with some of the sections indicated by asterisks being left for extra reading. Chapters 6 to 8 provide enough material for another 30-hour course for graduate students, with opportunities for further reading and development as term-paper projects. However, the material can be arranged in various ways, and I have tried to write in a way that will encourage browsing and self-study by students of various ages and backgrounds.

At this point, some explanation of the origins of this book may be helpful. For a variety of reasons, the state-space approach has been largely developed in control theory, and not in communication theory, where most of my own interests lie. In the mid-1960s, Schweppe [7] in the United States and Stratonovich and Sosulin [8] in the USSR began to show the usefulness of state-space methods in signal detection problems. Then Omura showed how the quadratic regulator control algorithm could be applied to certain feedback communication schemes [9]. These papers, and also the patient instruction of some of my early Ph.D. students, especially Jim Omura, Paul Frost, Roger Geesey, Ty Duncan, and B. Gopinath, gave me a greater appreciation of state-space theory and led me to introduce more of it into the Stanford linear systems course. However, it soon became clear that a deeper knowledge was necessary to really exploit the power of state-space methods. Also, the existing fashion in textbooks was largely oriented toward the background mathematics in differential equations and linear algebra, with less attention to the engineering significance and applications of the concepts peculiar to system theory. For example, much attention was devoted to Jordan forms, various ways of evaluating matrix exponentials, and numerous definitions of controllability and observability. The mathematics of all this was clear, but what the books did not really explain was why all this was useful to anyone—engineers or mathematicians.

It was the role of controllability in the pole-shifting problem for time-invariant systems (Chapter 3), and that of observability in the design of asymptotic observers (Chapter 4), that first gave me some meaningful indication of the value of these concepts. Then, as I examined the original research literature, I learned of their role in providing stability results for quadratic regulators [10] and optimum filters [11]. The significance of this stability is that the effect of numerical errors in computation, e.g., round-off errors, does not build up and destroy the calculation—obviously a very important practical consideration. It became clear that controllability and observability first arose as certain technical conditions to resolve existence and uniqueness conditions in certain optimal control and estimation problems. It was only somewhat later that Kalman isolated them and defined them via certain idealized problems [12], which for various reasons came to be overly emphasized in many treatments.

Moreover, as I began to obtain a better appreciation of the state-space point of view by applying it to various detection, estimation, and control problems, the pioneering and extensive studies of Rosenbrock [6] (and then

Popov, Forney, Wolovich, and others) clarified the power of the transfer function approach and the benefits to be gained by a better understanding of the relationships between it and the state-space approach. This book attempts a synthesis of the powerful new perspectives on linear system theory that are now available. The advantages of such a development are already to be seen in various areas, and I believe that a lot more will be done with it.

These background remarks also explain why the contents of this book do not quite follow the "traditional" (since 1963!) order of presentation found in most existing textbooks. One favorite topic in many of them is the time-domain solution of state-space equations. This is an interesting topic and can build well upon earlier knowledge of linear differential equations. However, I feel that the students' sense of accomplishment in mastering this material is somewhat illusory. First, if one really had to solve some equations, there are several readily available computer routines developed just for this purpose. But it is claimed that one should "understand" what one is computing. True, but this understanding comes from numerical analysis and not really from the pretty but particular mathematics learned in the linear systems course (see [13]). In fact, what is lost in dallying with this mathematics is the awareness that many of the things that can be done with state-space equations do not really need explicit time-domain solutions of the state equations. Therefore the solution of state-space equations has been deemphasized in this book. On the other hand, I have tried to show that the notion of explicit realizations of a given set of system equations (or transfer functions) can be a powerful aid in understanding and using linear systems. This theme first appears in Chapter 2 and continues to be developed throughout the book, e.g., in the exploration of multivariable systems (Secs. 6.4 and 6.5) halfway through the book, in the study of general differential systems in Chapter 8, and in the explanation of adjoints of time-variant systems in Chapter 9, and to a certain extent in the brief final Chapter 10. It may take time, and several readings, to adjust to the somewhat different perspectives of this book, and I can only offer my own experience as proof that it might be worthwhile.

While learning this subject and attempting to get some perspective on what was vital and what transient, I have found great help in going back to the original sources. For, as Robert Woodhouse [14] pointed out in 1810 (in the first book in English on the calculus of variations), "the Authors who write near the beginnings of science are, in general, the most instructive: they take the reader more along with them, show him the real difficulties, and, which is the main point, teach him the subject the way by which they themselves learnt it." Therefore, in these notes I have often made a special effort to point out the earliest papers on the different concepts and would encourage the active reader to pursue them independently. More generally, the references have been carefully selected for their significance, readability, and potential for further study and, in several cases, further independent investigation. Similarly, the exercises in this book are of various levels of difficulty and in several instances serve to complement and extend the material in the text. Therefore, all the exercises should at least be read along with each section, even if only a few are actually attempted.

I have also attempted to make the book reasonably self-contained, and every effort has been made to keep the proofs as simple and direct as possible. For example, things have been so arranged so that very little linear algebra is required either as a pre- or corequisite. What is really needed is some exposure to matrix manipulations and, more importantly, a recognition and acceptance by the student that, at this level, no course or textbook on linear algebra (or in fact any mathematical subject) can be a perfect or complete prerequisite for the material in any engineering course—there is no substitute for just buckling down to figure out many things for oneself (with liberal use of 1×1 or 2×2 matrices in the early stages). Of course some guidance is necessary, and therefore, in the Appendix and in Sec. 6.3, I have tried to collect the results from elementary algebra and polynomial matrix theory that are used in this book. However, they are not meant to be mastered before launching into the rest of the book—rather, the explicit references made in later sections to special results, such as determinantal and block matrix identities or the Cayley-Hamilton theorem or the Smith canonical form, are to be used as occasions for a more motivated study of the relevant special topics. Of course this may often be painful and slow, but my experience is that the student thereby achieves a better mastery of the material and, more important, a foretaste of the ability to pick out and learn enough about some special (mathematical) topic to try to resolve particular problems that he may encounter in his later work. The range of mathematics used in present-day engineering problems is so wide that one could spend all one's time taking "prerequisites"—especially since studying well-established material is so much easier than venturing out, even just a little, into some less well-defined territory.

Therefore in this book I have tried to subordinate the mathematical concepts to the system concepts—it is only too easy, and unfortunately only too common, for readers at this level to be led down very entertaining but ultimately deeply frustrating mathematical garden paths. My goal is not the presentation or development of "mathematical" system theory but an effort to introduce and use just as little mathematics as possible to explore some of its basic concepts. I try to follow an admonition of Joshua Chover [15]: "It is time to dispel a popular misconception. The goal of mathematics is *discovery*, not 'proof'." Or to make the point another way, I belong to the school that holds ideas and exposition to be more important than "mere" results [16, 4.B].

Finally, I should also caution that the unavoidable vagueness of real problems, arising from constraints of imperfect knowledge, nonmathematical performance specifications, economic constraints, flexible acceptability criteria, etc., means that solutions of the necessarily clean and specific mathematical problems of any theory can ultimately only serve as "guides" to the actual "resolution" of any engineering problem. Unfortunately this is a distinction that cannot really be conveyed by a textbook and is the reason good teachers (or engineers) can never be replaced by a book (or a computer program).

In this connection I should mention that, especially in the early chapters, the presentation is deliberately loosely organized, with emphasis on discussion and motivation rather than formal development. Several major themes are gradually developed in a spiral fashion, and readers should not expect to find all their questions answered the first time a topic is introduced. *Students will also find it helpful to frequently make up for themselves tables and charts of the major concepts, results and interrelations as they perceive them at various points in the course.* A continuous interplay between skills and knowledge must take place in any successful learning effort. As succinctly put by Edsger Dijkstra [17, p. 211], a scientific discipline is "not any odd collection of scraps of knowledge and an equally odd collection of skills" but "the skills must be able to improve the knowledge and the knowledge must be able to refine the skills." *Therefore, to really understand a subject one has ultimately to make a personal selection and resynthesis, modulated by one's own background and other knowledge, of the material in any given book or course.* My hope is that this book will provide enough material and opportunity for such an educational experience, via self-study and/or classroom instruction.

All this—
was for you, [dear reader].
I wanted to write a [book]
that you would understand.
For what good is it to me
if you can't understand it?
But you got to try hard—

Adapted from "January Morning" by William Carlos Williams†

REFERENCES

1. E. GILBERT, "Controllability and Observability in Multivariable Systems," *SIAM J. Control,* **1**, pp. 128–151, 1963.
2. R. E. KALMAN, "Mathematical Description of Linear Dynamical Systems," *SIAM J. Control,* **1**, pp. 152–192, 1963.

†From *Collected Earlier Poems* by William Carlos Williams, New Directions Publishing Corp., 1938. "January Morning" is published in adapted form (3 words changed), by permission of New Directions.

3. L. A. ZADEH and C. A. DESOER, *Linear System Theory—A State-Space Approach*, McGraw-Hill, New York, 1963.
4. C. T. CHEN, *Introduction to Linear System Theory*, Holt, Rinehart and Winston, New York, 1970.
5. V. M. POPOV, "Some Properties of Control Systems with Matrix Transfer Functions," in *Lecture Notes in Mathematics*, Vol. 144, Springer, Berlin, 1969, pp. 169–180.
6. H. H. ROSENBROCK, *State Space and Multivariable Theory*, Wiley, New York, 1970.
7. F. C. SCHWEPPE, "Evaluation of Likelihood Functions for Gaussian Signals," *IEEE Trans. Inf. Theory*, **IT-11**, pp. 61–70, July 1965.
8. R. L. STRATONOVICH and YU. G. SOSULIN, "Optimal Detection of a Markov Process in Noise," *Eng. Cybern.*, **6**, pp. 7–19, Oct. 1964 (trans. from Russian).
9. J. OMURA, "Optimum Linear Transmission of Analog Data for Channels with Feedback," *IEEE Trans. Inf. Theory*, **IT-14**, pp. 38–43, Jan. 1968. See also Ph.D. dissertation, Stanford University, Stanford, Calif., 1966.
10. R. E. KALMAN and R. KOEPCKE, "Optimal Synthesis of Linear Sampling Control Systems Using Generalized Performance Indexes," *Trans. ASME*, **80**, pp. 1820–1826, 1958.
11. R. E. KALMAN and R. S. BUCY, "New Results in Linear Filtering and Prediction Theory," *Trans. ASME Ser. D J. Basic Eng.*, **83**, pp. 95–107, Dec. 1961.
12. R. E. KALMAN, "On the General Theory of Control Systems," *Proceedings of the First IFAC Congress*, Vol. I, Butterworth's, London, 1960, pp. 481–493.
13. C. B. MOLER and C. VAN LOAN, "Nineteen Dubious Ways to Compute the Exponential of a Matrix," *SIAM Review*, pp. 801–836, Oct. 1978.
14. R. WOODHOUSE, *A History of the Calculus of Variations in the Eighteenth Century*, Cambridge University Press, London, 1810; reprint, Chelsea, New York, 1966.
15. J. CHOVER, *The Green Book of Calculus*, Benjamin, Reading, Mass., 1972.
16. B. PARLETT, "Progress in Numerical Analysis," *SIAM Review*, **20**, pp. 443–456, July 1978.
17. E. W. DIJKSTRA, *A Discipline of Programming*, Prentice-Hall, Englewood Cliffs, N.J., 1976.

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A perceptive author (B. G. Raja Rao in *The Serpent and the Rope*) has written that all books are autobiographical. As the earlier remarks indicate, I am especially conscious of that, and would like to also express my appre-

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ciation to my large family, so happily extended by marriage, and especially to my parents, who have moulded us through faith, discipline and example.

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"The lines have fallen unto me in pleasant places; yea, I have a goodly heritage"

THOMAS KAILATH

Stanford, Calif.,

CONTENTS

*Sections so marked throughout Contents may be skipped without loss of context.

CHAPTER 6 STATE-SPACE AND MATRIX-FRACTION DESCRIPTIONS OF MULTIVARIABLE SYSTEMS 345

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