Feedback Systems

Edited by José B. Cruz, Jr.

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FEEDBACK SYSTEMS

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Frederick Emmons Terman

Preface

Feedback theory has had a significant impact in the design of systems for precision regulation, stabilization, and reduction of noise and distortion. Recent results have revealed deeper insights into known uses of feedback and also have led to new feedback system design concepts. The introduction of feedback modifies system structural properties, and it is this modification that is exploited in analysis and design. For example, feedback offers the possibility of meeting parameter tolerance specifications, simplifying design computation by parameter imbedding, maintaining optimality in control systems, and adapting a system to changes in environmental conditions. This book contains developments of many of the recent results in feedback theory. It is suitable as a main or a supplementary textbook for graduate courses and as a reference or self-study book for engineers in industry.

Chapter 1, "Feedback in Systems," by Jose B. Cruz, Jr., provides a perspective for feedback theory and for the contributions contained in the other chapters. Although the other chapters are somewhat self-contained, it is suggested that Chapter 1 be read first for an overview of the field, and to see the relationships among the contributions. Chapter 2, "Sensitivity Analysis," by William R. Perkins, provides a basis for the study of the dependence of systems on parameters and on disturbance inputs. Chapter 3, "Effects of Feedback on Signal Distortion in Nonlinear Systems," by Jose B. Cruz, Jr., applies the method of sensitivity analysis to the study of nonlinear distortion in feedback systems. Chapter 4, "Feedback Design of Large Linear Systems," by Petar V. Kokotović, describes two methods for reducing computations in the design of large-scale linear feedback systems with a quadratic performance index. Chapter 5, "Comparative Sensitivity of Optimal Control Systems," by Eliezer Kreindler, examines the effects of plant parameter variations on the performance of optimal feedback systems. Chapter 6, "Near-Optimal Feedback Control," by Jose B. Cruz, Jr., discusses the use of expansions to obtain feedback control which maintains near-optimal performance in spite of parameter variations. Chapter 7, "On the Theory of Linear Multiloop Feedback Systems," by Irwin W. Sandberg, generalizes the theory of Black, Nyquist, Blackman, and Bode to multiloop linear feedback systems. Chapter 8, "Applications of Functional Analysis to Nonlinear Control Systems with Unknown Plants," by Philip E. Sarachik, discusses the use of functional analysis in the theory and computation of optimal control. Many of the results on which the above chapters are based have appeared only recently in journal articles.

John G. Truxal reviewed the entire volume, and his helpful suggestions are acknowledged. The undersigned is indebted to Rose Lane for her excellent typing of Chapters 1, 2, 3, 4, and 6.

Jose B. Cruz, Jr.

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1
Feedback in Systems

Jose B. Cruz, Jr.
University of Illinois, Urbana

The successful and widespread introduction of feedback in system design for precision regulation, stabilization, and reduction of noise and distortion has firmly established the importance of feedback. Recent research has revealed not only broader insights in known uses of feedback but also new feedback concepts in system design. In Sec. 1.1 major goals that might be attained with feedback are discussed in order to provide perspective for the book. Section 1.2 summarizes the nature of contributions described in the subsequent chapters.

1.1 WHY USE FEEDBACK?

A system to be controlled, called a *plant*, has a set of outputs represented by the vector y and a set of inputs represented by the vector u. An external input vector r, which usually represents the desired output, may be available also. If the plant input u is obtained as an operation on r, the operator is known as an *open-loop controller* and the system is said to be an *open-loop*

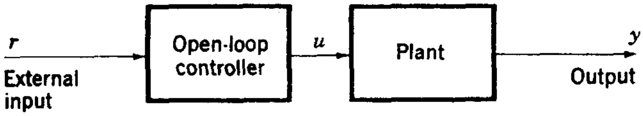

Fig. 1.1 Open-loop system.

system. Figure 1.1 shows the general structure of an open-loop system. If the plant input *u* is an operation on *r* and *y*, the operator is called a *feedback controller* and the system is defined as a *feedback system*. Figure 1.2 shows the general structure of a feedback system. It is assumed that the output is accessible for measurement. In Fig. 1.2, the transducer which senses the output is considered as part of the controller.

The type of dynamic system discussed in this volume is modeled in terms of ordinary differential equations. Such a system is called a lumped time-continuous dynamic system. The basic concepts of feedback apply also to discrete-time systems and distributed systems, but these latter systems are not discussed in the book, for simplicity of presentation of the main theme of feedback.

The use of feedback in systems may not necessarily improve performance. In fact, if the goals are not clearly understood, it may create more problems than it solves. To appreciate and evaluate the trade-offs involved, it is helpful to know what the potential benefits are. Listed below are goals that might be attained by employing feedback.

1.1.1 TO STABILIZE AN UNSTABLE SYSTEM

The stabilization of unstable systems has been one of the major aims of feedback. Without the addition of feedback in rocket booster systems, reference platform systems for navigation and guidance, spacecraft attitude systems, nuclear-reactor systems, electromechanical levitation systems, controlled fusion, controlled growth processes in biological systems, and biasing of certain solid-state electronic circuits, the unavoidable uncertainties in initial conditions and the inaccuracies in the model that would be used for determining an open-loop control would render such systems useless.

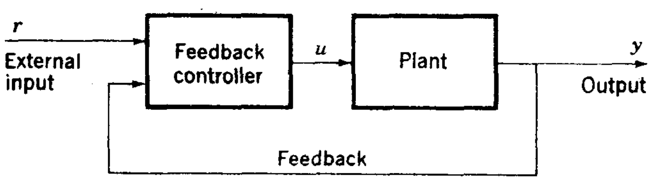

Fig. 1.2 Feedback system.

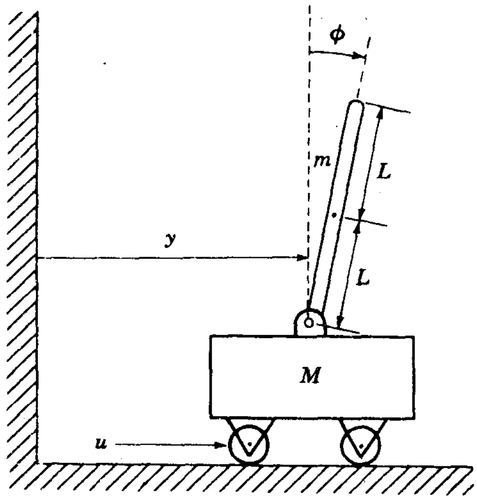

Fig. 1.3 Inverted pendulum, an inherently unstable system that can be stabilized by feedback.

To illustrate the use of feedback to stabilize an inherently unstable system, consider the inverted pendulum in Fig. 1.3, which is a stick of mass m supported by a cart of mass M through a hinge, where the stick motion is constrained to be on a plane, and cart motion is constrained to be along the horizontal y direction only. This simple mechanical system has been extensively studied as a simplified model for many important aerospace applications [1–3]. Applying Newton's laws,

$$\frac{I\,d^2\varphi}{dt^2} = VL \sin \varphi - HL \cos \varphi$$

$$\frac{m\,d^2}{dt^2}(L \cos \varphi) = -mg + V$$

$$\frac{m\,d^2}{dt^2}(y + L \sin \varphi) = H$$

$$\frac{M\,d^2y}{dt^2} = u - H$$

where $I = \frac{1}{3}mL^2$ is the moment of inertia of the stick about its center of gravity, L is one-half the length of the stick, V is the vertical force in the upward direction exerted by the cart on the stick, and H is the horizontal force to the right exerted by the cart on the stick. When the cart is at rest, the stick is in vertical position, and the force u is zero, then the system is in equilibrium. This equilibrium position is unstable, in the sense that with any initial perturbation from this position, no matter how small, the stick will fall down.

Let us now examine the motion for small values of φ if a control force u can be applied. Using the approximation $\sin\varphi \approx \varphi$ and $\cos\varphi \approx 1$,

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = a_{21}x_1 + b_2u$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = a_{41}x_1 + b_4u$$

or

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ a_{41} & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ b_2 \\ 0 \\ b_4 \end{bmatrix} u = Ax + Bu \tag{1.1}$$

where $x_1 = \varphi$, $x_2 = \dot{\varphi}$, $x_3 = y$, $x_4 = \dot{y}$, $a_{21} = 3g(m + M)/L(m + 4M)$, $a_{41} = -3mg/(m + 4M)$, $b_2 = -3/L(m + 4M)$, $b_4 = 4/(m + 4M)$, and x is a four-dimensional vector whose components are x_1, x_2, x_3, and x_4. If there is a small but unknown perturbation of the initial state and if u is an open-loop control, the equilibrium point would be asymptotically stable if all the eigenvalues of A had negative real parts, and it would be unstable if at least one eigenvalue of A had a positive real part. This stability criterion based on the linearized system is Lyapunov's first method. (For example, see Ref. 4 for a treatment of this point.) The eigenvalues of A are the roots of

$$\det(sI - A) = s^2(s^2 - a_{21}) = 0 \tag{1.2}$$

Since a_{21} is positive, there is a real and positive eigenvalue. Thus the equilibrium point of the open-loop system is unstable.

If the control input u can be formed as a linear combination of x_1, x_2, x_3, and x_4,

$$u = k_1x_1 + k_2x_2 + k_3x_3 + k_4x_4 \tag{1.3}$$

where k_1, k_2, k_3, k_4 are constants, u is called a linear state feedback control, and (1.1) becomes

$$\dot{x} = Ax + B[k_1 \quad k_2 \quad k_3 \quad k_4]x = (A + BK)x \tag{1.4}$$

The linearized system with state feedback would be asymptotically stable if all the eigenvalues of $A + BK$, that is, all the roots of

$$\begin{aligned}\det(sI - A - BK) &= s^4 - (b_2k_2 + b_4k_4)s^3 - (b_4k_3 + a_{21} + b_2k_1)s^2 \\ &\quad - (b_2a_{41}k_4 - b_4a_{21}k_4)s - (b_2a_{41}k_3 - b_4a_{21}k_3) \\ &= 0\end{aligned} \tag{1.5}$$

had negative real parts. For any given set of values of a_{21}, a_{41}, b_2, and b_4, it is clear that the feedback coefficients k_3, k_4, k_1, and k_2 could be chosen to make the coefficients of the characteristic equation in (1.5) have any desired values. Hence, not only could the eigenvalues be forced to have negative real parts, but the eigenvalues could have any values whatsoever, provided only that complex eigenvalues occur in conjugate pairs. Thus in this example, linear state feedback could stabilize the system in the sense that for any arbitrary but sufficiently small perturbation of the initial state from the equilibrium position, the motion would tend to the equilibrium point as time goes to infinity.

The above example illustrates the first method of Lyapunov for a nonlinear time-invariant system

$$\dot{x} = f(x,u) \tag{1.6}$$

where $f(0,0) = 0$, x is n-dimensional, u is m-dimensional, and f and $\partial f/\partial x$ are continuous in a neighborhood of the origin. Denote $\partial f/\partial x$ evaluated at $x = 0$ and $u = 0$ by A:

$$A = \left.\frac{\partial f}{\partial x}\right|_{\substack{x=0\\u=0}} \tag{1.7}$$

Note that A is a constant $n \times n$ matrix. If u is implemented as an open-loop control, the equilibrium point $x = 0$ is asymptotically stable if all the eigenvalues of A have negative real parts, and the equilibrium point is unstable if A has at least one eigenvalue with a positive real part [4, 5]. Furthermore, if u is implemented as a feedback control $u = \varphi(x)$, where $\varphi(0) = 0$, $\partial f/\partial u$, φ, and $\partial\varphi/\partial x$ are continuous in a neighborhood of the origin, then the equilibrium point of the nonlinear system is asymptotically stable if all the eigenvalues of $A + BK$ have negative real parts [5]. B and K are constant matrices defined by

$$B = \left.\frac{\partial f}{\partial u}\right|_{\substack{x=0\\u=0}} \tag{1.8}$$

$$K = \left.\frac{\partial \varphi}{\partial x}\right|_{x=0} \tag{1.9}$$

We shall say that the nonlinear system (1.6) is *stabilizable* if there exists a feedback control $u = \varphi(x)$ such that all the eigenvalues of $A + BK$ have negative real parts. For small motion around the equilibrium point, (1.6) can be approximated by the linearized system

$$\dot{x} = Ax + Bu \tag{1.10}$$

If the linearized system in (1.10) is controllable [6], then the matrix K can always be chosen to attain any desired set of eigenvalues of $A + BK$ [6–8].

Hence controllability of the linearized system guarantees that (1.6) is stabilizable.

Not all the state variables of a linearized system may be accessible. But if the linearized system is observable [6] as well as controllable, it can be made stable by feedback through an observer [7] whose input consists of the accessible state variables and the plant input.

Suppose that the p-dimensional output y is given by

$$y = g(x) \tag{1.11}$$

where g and $\partial g/\partial x$ are continuous in a neighborhood of $x = 0$ and $g(0) = 0$. Then the linearized output is

$$y = Cx$$

where the constant matrix C is equal to $\partial g/\partial x$ evaluated at $x = 0$.

Construct a dynamic system called an observer

$$\dot{\hat{x}} = A\hat{x} + Bu - \hat{K}(y - C\hat{x}) \tag{1.12}$$

whose inputs are u and y and whose output is $\hat{x}$, where the matrix $\hat{K}$ is yet to be determined. Subtracting (1.10) from (1.12),

$$\dot{\hat{x}} - \dot{x} = (A + \hat{K}C)(\hat{x} - x) \tag{1.13}$$

Now form

$$u = K\hat{x} \tag{1.14}$$

where K is to be determined. In terms of the error $e = \hat{x} - x$, the state equations of the composite system can now be written as

$$\dot{x} = (A + BK)x + BKe \tag{1.15}$$

$$\dot{e} = (A + \hat{K}C)e \tag{1.16}$$

It can be verified that the eigenvalues of the composite system in (1.15) and (1.16) are the eigenvalues of $A + BK$ and the eigenvalues of $A + \hat{K}C$ [7]. If the system is completely controllable, K can be chosen to yield any set of desired eigenvalues of $A + BK$; and if the system is completely observable, $\hat{K}$ can be chosen to yield any desired set of eigenvalues for $A + \hat{K}C$ [7–11]. Hence K and $\hat{K}$ can be chosen to yield any set of $2n$ eigenvalues, where n is the order of the plant. In particular, the eigenvalues can be chosen to have negative real parts resulting in a stable system. Figure 1.4 shows the block diagram for the realization, when the plant is linear time-invariant.

For the nonlinear system (1.6) and (1.11), if the linearized model is controllable and observable, a linear feedback control given by (1.14) and (1.12) can always be found by suitable choice of K and $\hat{K}$ such that the equilibrium point $x = 0$ is asymptotically stable for sufficiently small initial-state perturbation. Figure 1.5 shows the structure for the feedback system.

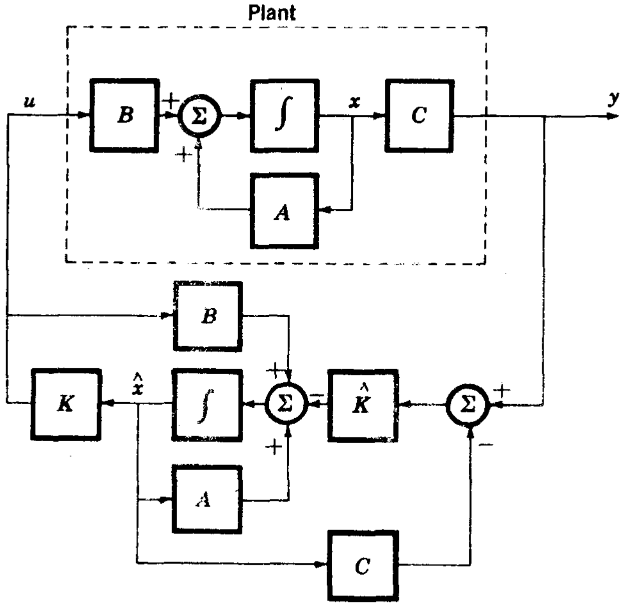

Fig. 1.4 For a completely controllable and completely observable linear time-invariant plant, K and $\hat{K}$ can be chosen to yield any set of $2n$ eigenvalues for the feedback system.

The stabilization of an equilibrium point of an autonomous nonlinear system by stabilizing the associated linearized system is local in the sense that the stabilization is guaranteed only for x_0 sufficiently close to the equilibrium point. Global stability may be investigated by using Lyapunov's second method. For a recent account of this theory, see Ref. 12.

1.1.2 TO REDUCE SENSITIVITY TO PARAMETERS, NOISE, AND NONLINEAR DISTORTION

One of the well-known benefits of feedback is the possibility of achieving a high degree of static accuracy in spite of variations in parameters of the plant [13–16]. Reduction of noise transmitted to the output and reduction of nonlinear distortion are other classical motivations for the use of feedback [13–16].

For a single-input, single-output, single-loop linear time-invariant system, a classic rule of thumb is to make the loop gain as high as possible without incurring instability. This makes the sensitivity to plant parameters and load disturbances low, and at the same time it reduces the effect of a nonlinear perturbation of a nominally linear plant. If noise associated with the feedback measurement is not negligible, a high loop gain tends to make the noise transmitted to the output high. Thus feedback design for high

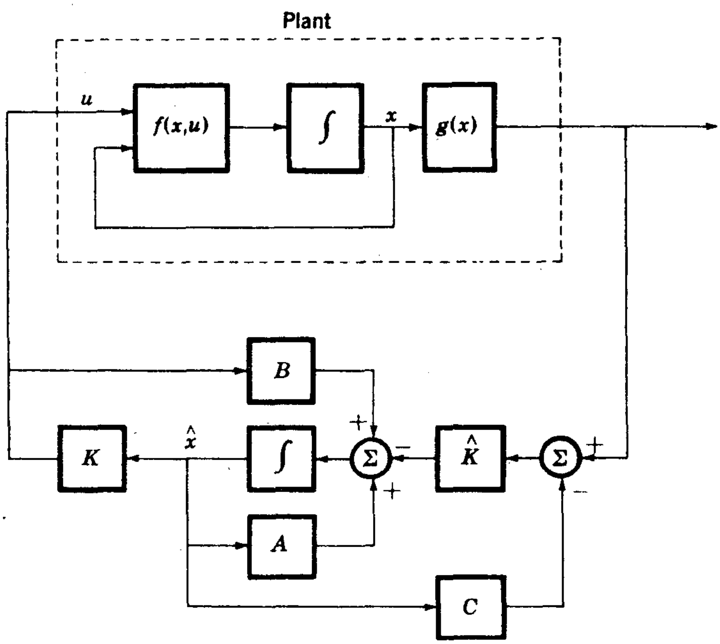

Fig. 1.5 If the linearized plant is controllable and observable, K and $\hat{K}$ can be chosen to stabilize the equilibrium point $x = 0$.

static accuracy, low sensitivity to load disturbance, low nonlinear distortion, low transmission of feedback measurement noise, and stability leads to a compromise choice of gain and frequency characteristic of the loop transmission [13, 14, 16].

For multivariable systems with multiple loops and multiple parameters, it is easy to lose sight of the aim of feedback and end up with an unsatisfactory feedback system. A suitable criterion is needed to assess the degree to which the feedback is able to achieve the stated goal. Such a criterion is discussed in Chaps. 2 and 3, and a general theory for multiloop linear time-invariant systems is described in Chap. 7.

1.1.3 TO MAINTAIN OPTIMALITY

In a system optimization problem, a functional called *performance index* or *cost functional* is minimized with respect to a control-vector function or with respect to control parameters. The performance index might be fuel consumption, time to reach a target, integrated squared error between a desired trajectory and actual trajectory, economic cost, energy expenditure, or a weighted sum of several of these.