RODGER E. ZIEMER

WILLIAM H. TRANTER

D. RONALD FANNIN

SIGNALS AND SYSTEMS

Continuous and Discrete

RODGER E. ZIEMER
Department of Electrical Engineering
University of Missouri-Rolla

WILLIAM H. TRANTER
Department of Electrical Engineering
University of Missouri-Rolla

D. RONALD FANNIN
Department of Electrical Engineering
University of Missouri-Rolla

SIGNALS AND SYSTEMS
Continuous and Discrete

Macmillan Publishing Co., Inc.
New York

Collier Macmillan Publishers
London

The following figures and table appear in *Principles of Communication* by R. E. Ziemer and W. H. Tranter, copyright 1976 by Houghton Mifflin Co., Boston; they are reprinted by permission: fig. 1-7, p. 11; fig. 1-8, p. 12; fig. 1-12, p. 19; fig. 3-5, p. 102; fig. 3-6, p. 110; table 3-3, p. 125; fig. 3-14, p. 134; fig. 3-18, p. 140; and fig. 3-19, p. 143

DS16/29

Macmillan Publishing Co., Inc.
866 Third Avenue, New York, New York 10022

Collier Macmillan Canada, Inc.

Library of Congress Cataloging in Publication Data

Ziemer, Rodger E.
Signals and systems.
Includes bibliographies and index.
1. System analysis. 2. Discrete-time systems.
3. Digital filters (Mathematics) I. Tranter, William H. II. Fannin, D. Ronald, III. Title.
QA402.Z53 003 82-6616
ISBN 0-02-431650-4 AACR2

Printing: 1 2 3 4 5 6 7 8 Year: 3 4 5 6 7 8 9 0

ISBN 0-02-431650-4

PREFACE

This textbook provides an introduction to the tools and mathematical techniques necessary for understanding and analyzing both continuous-time and discrete-time linear systems. We have attempted to give an insight into the application of these tools and techniques for solving practical engineering problems. Our philosophy has been to adopt a systems approach throughout the book for the introduction of continuous-time signal and system analysis, rather than use the framework of traditional circuit theory. We believe that the systems viewpoint provides a more natural approach to introducing this material in addition to broadening the horizons of the student. Furthermore, the topics of discrete-time signal and system analysis are most naturally introduced from a systems viewpoint, which lends overall consistency to the development. We have, of course, relied heavily on the students' circuit theory background to provide illustrative examples.

The organization of the book is straightforward. The first six chapters deal with continuous-time linear systems in both the time domain and the frequency domain. The principal tool developed for time-domain analysis is the convolution integral. Frequency-domain techniques include the Fourier and the Laplace transforms. An introduction to state variable techniques is also included. The remainder of the book deals with discrete-time systems including z-transform analysis techniques, digital filter analysis and synthesis, and the discrete Fourier transform and fast Fourier transform (FFT) algorithms.

This organization allows the book to be covered in two three-semester-hour courses, with the first course being devoted to continuous-time signals and systems and the second course being devoted to discrete-time signals and systems. Alternatively, the material can be used as a basis for three quarter-length courses. With this format, the first course would cover time- and frequency-domain analysis of continuous-time systems. The second course would cover state variables, sampling, and an introduction to the z-transform and discrete-time systems. The third course would deal with the analysis and synthesis of digital filters and provide an introduction to the discrete Fourier transform and its applications.

The assumed background of the student is mathematics through differential equations and the usual introductory circuit theory course or courses. Knowledge of the basic concepts of matrix algebra would be helpful but is not essential. Appendix A is included to bring together the pertinent matrix relations that are used in Chapters 5 and 6. We feel that in most electrical engineering curricula the material presented in this book is best taught at the junior level.

We begin the book by introducing the basic concepts of signal and system models and system classifications. The idea of spectral representations of periodic signals is first introduced in Chapter 1 because we feel that it is important for the student to think in terms of both the time and the frequency domains from the outset.

The convolution integral and its use in fixed, linear system analysis by means of the principle of superposition are treated in Chapter 2. The evaluation of the convolution integral is treated in detailed examples to provide reinforcement of the concepts. Calculation of the impulse response and its relation to the step and ramp responses of a system are discussed. Chapter 2 also contains optional sections* and examples regarding writing the governing equations for lumped, fixed, linear systems and the solution of linear, constant coefficient differential equations. These are intended as review and may be omitted without loss of continuity.

The Fourier series and Fourier transform are introduced in Chapter 3. We have emphasized the elementary approach of approximating a periodic function by means of a trigonometric series and obtaining the expansion coefficients by using the orthogonality of sines and cosines. We do this because this is the first time most of our students have been introduced to Fourier series. The alternative generalized orthogonal function approach is included as a nonrequired reading section at the end of this chapter for those who prefer it. The concept of the transfer function in terms of sinusoidal steady-state response of a system is discussed in relation to signal distortion. The Fourier transform is introduced next, with its applications to spectral analysis and systems analysis in the frequency domain. The concept of an ideal filter, as motivated by the idea of distortionless transmission, is also introduced at this point. The Gibbs phenomenon, window functions, and convergence properties of the Fourier coefficients are treated in optional closing sections.

The Laplace transform and its properties are introduced in Chapter 4. Again, we have tried to keep the treatment as simple as possible because this is assumed to be a first exposure to the material for a majority of students, although a

*Optional sections are denoted by an asterisk.

summary of complex variable theory is provided in Appendix B so that additional rigor may be used at the instructor's option. The derivation of Laplace transforms from elementary pairs is illustrated by example, as is the technique of inverse Laplace transformation using partial fraction expansion. Optional sections on the evaluation of inverse Laplace transforms by means of the complex inversion integral and an introduction to the two-sided Laplace transform are also provided.

The application of the Laplace transform to network analysis is treated in detail in Chapter 5. The technique of writing Laplace transformed network equations by inspection is covered and used to review the ideas of impedance and admittance matrices, which the student will have learned in earlier circuits courses for resistive networks. The transfer function is treated in detail, and the Routh test for determining stability is presented. The chapter closes with a treatment of Bode plots and block diagram algebra for fixed, linear systems.

In Chapter 6, the concepts of a state variable and the formulation of the state variable approach to system analysis are developed. The state equations are solved using both time-domain and Laplace transform techniques, and the important properties of the solution are examined. Finally, as an example, we show how the state-variable method can be applied to the analysis of circuits.

The final three chapters provide coverage of the topics of discrete-time signal and system analysis. Chapter 7 begins with a study of sampling and the representation of discrete-time systems. The sampling operation is covered in considerable detail. This is accomplished in the context of formulating a model for an analog-to-digital (A/D) converter so that the operation of quantizing can be given some physical basis. A brief analysis of the effect of quantizing sample values in the A/D conversion process is included as an introduction to quantizing errors. As a bonus, the student is given a basis upon which to select an appropriate wordlength of an A/D converter. The z-transform, difference equations, and discrete-time transfer functions are developed with sufficient rigor to allow for competent problem solving but without the complications of contour integration.

Chapter 8 allows the student to use his knowledge of discrete-time analysis techniques to solve an important class of interesting problems. The idea of system *synthesis*, as opposed to system *analysis*, is introduced. Discrete-time integration is covered in considerable detail for several reasons. First, the idea of integration will be a familiar one. Thus the student can appreciate the different information gained by a frequency-domain analysis as opposed to a time-domain analysis. In addition, the integrator is a basic building block for many analog systems. Finally, the relationship between trapezoidal integration and the bilinear z-transform is of sufficient importance to warrant a discussion of trapezoidal integration. The synthesis techniques for digital filters covered in this chapter are the standard ones. These are synthesis by time-domain invariance, the bilinear z-transform synthesis, and synthesis through Fourier series expansion. Through the application of these techniques, the student is able to gain confidence in the previously developed theory. Since several synthesis techniques depend on knowledge of analog filter prototypes, Appendix B, which discusses several different prototypes, is included.

The discrete Fourier transform (DFT) and its realization through the use of fast Fourier transform (FFT) algorithms is the subject of Chapter 9. Both decimation-in-time and decimation-in-frequency algorithms are discussed. Several examples are provided to give the student practice in performing the FFT operations. We believe that this approach best leads to a good understanding of the FFT algorithms and their function. Basic properties of the DFT are summarized and a comparison of the number of operations required for the FFT as compared to the DFT is made. Several applications of the DFT are summarized and the use of windows in suppressing leakage is discussed. This chapter closes with a discussion and illustration of FFT algorithms with arbitrary radixes and the chirp-z transform.

A complete solutions manual, which contains solutions to all problems, is available from the publisher as an aid to the instructor. Answers to selected problems are provided in Appendix E as an aid to the student.

The authors wish to express their thanks to the many people who have contributed, both knowingly and unknowingly, to the development of this textbook. First, thanks go to our long-suffering students, who have been forced to study from our notes, often while they were still in various stages of development. Their many comments and criticisms have been invaluable and are gratefully appreciated. Many of our colleagues in the Electrical Engineering Department at the University of Missouri–Rolla taught courses that used the book in note form and provided many suggestions for improvement. In this regard, we thank Professors Gordon E. Carlson, Kenneth H. Carpenter, Ralph S. Carson, David R. Cunningham, Thomas J. Herrick, Frank J. Kern, Earl F. Richards, John A. Stuller, and Thomas P. Van Doren. Professors Carlson, Carson, and Stuller critically reviewed much of the manuscript and provided valuable suggestions for improvement. Additionally, we would like to thank the reviewers at other institutions who provided valuable criticism, especially K. Ross Johnson, Michigan Technological University, Bruce Johansen, Ohio Northern University, Saleem Kassan, University of Pennsylvania, and Neal Gallagher, Purdue University. However, any shortcomings of the final result are solely the responsibility of the authors. We also are indebted to John Liebetreu, Graduate Teaching Fellow in the EE Department at UMR, who produced an extremely neat master for the solutions manual. A most sincere thanks goes to our secretaries whose great care and expert typing skills allowed us to generate the final manuscript with a minimum of headaches. The National Engineering Consortium is also due thanks since it was through their series of seminars that much of the material in Chapters 7 and 8 was originally taught.

Last, but not least, we thank our wives and families for putting up with a project whose end at times seemed nonexistent.

R.E.Z.
W.H.T.
D.R.F.

CONTENTS

2 SYSTEM ANALYSIS IN THE TIME DOMAIN 44

3 FOURIER SERIES AND THE FOURIER TRANSFORM 86

APPENDICES

CHAPTER

Signal and System Modeling Concepts

1-1

INTRODUCTION

This book deals with *systems* and the interaction of *signals* in such systems. A system, in its most general form, is defined as a combination and interconnection of several components to perform a desired task.† Such a task might be the control of liquid level in a tank or the transmission of a message from New York to Los Angeles. A liquid-level controller might make use of a human operator who closes a valve once the liquid reaches the desired level. An equally unsophisticated solution to the message delivery problem might make use of a horse and rider. Obviously, more complex solutions are possible (and probably better). Note, however, that our definition is sufficiently general to include them all.

We will be concerned primarily with *linear* systems. Such a restriction is reasonable because many systems of engineering interest are closely approximated by linear systems and very powerful techniques exist for analyzing them. We consider several methods for analyzing linear systems in this book. Although each of the methods to be considered is general, not all of them are equally

†The Institute of Electrical and Electronics Engineers Dictionary defines a system as "an integrated whole even though composed of diverse, interacting structures or subjunctions."

convenient for any particular case. Therefore, we will attempt to point out the usefulness of each.

A *signal* may be considered to be a function of time which represents a physical variable of interest associated with a system. In electrical systems, signals usually represent currents and voltages, whereas in mechanical systems, they might represent forces and velocities.† In the liquid-level control problem mentioned above, one of the signals of interest represents the level of liquid in the tank.

Just as there are several methods of systems analysis, there are several different ways of representing and analyzing signals. They are not all equally convenient in any particular situation. As we study methods of signal representation and analysis we will attempt to point out useful applications of the techniques.

So far, the discussion has been rather general. In order to be more specific and to fix more clearly the ideas we have introduced, we will expand on the liquid-level control and the message delivery problems already mentioned.

1-2 TWO EXAMPLES

Liquid-Level Controller

Shown in Figure 1-1 is a simple electromechanical control system for controlling the liquid level in a tank. Such a system is often used for sump pumps. Two floats are suspended on a wire that is attached to a spring-loaded switch. The weight of the floats is chosen such that the toggle action switch closes if the liquid level is above the higher float, and stays closed until the liquid level goes below the lower float, whereupon the switch opens.

To analyze this system, the first thing an engineer would do is to replace the actual system with a *model*. Such a model is an attempt at representing only the essential details of the actual system: mathematically, or pictorially, or both. To illustrate the concept of a model, we assume that a model is desired which describes the tank's liquid level for all time.

For example, letting the liquid level in the tank be x centimeters from the bottom float and the volumetric flow rate out of the tank be f liters per second when the pump is on, a set of equations that would be a possible model for the system of Figure 1-1 when the pump is on is

$$f(x) = \begin{cases} 10 \text{ liters/s}, & 0 \leq x \leq 50 \text{ cm} \\ 0, & x \leq 0 \end{cases}$$

where the top float is assumed to be at $x = 50$ cm and the lower one is assumed to be at $x = 0$ cm. Futhermore, if the tank is 100 cm in diameter, the volume

†More generally, a signal can be a function of more than one independent variable, such as the pressure on the surface of an airfoil, which is a function of three spatial variables *and* time.

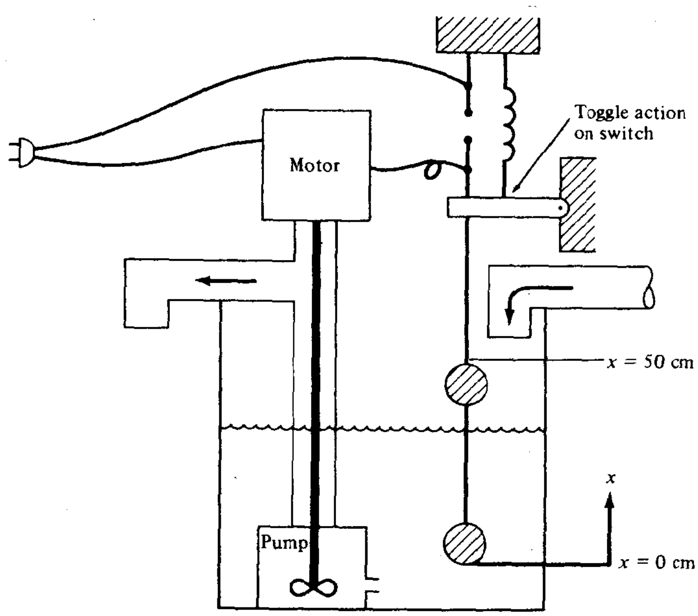

FIGURE 1-1. Liquid-level controller.

at height x cm is

$$V = \pi(50)^2 x$$
$$= 2500\pi x \text{ cm}^3$$
$$= 2.5\pi x \text{ liters}$$

Thus the time required for the liquid level to decrease 1 cm is

$$\Gamma = (2.5\pi \text{ liters/cm})/(10 \text{ liters/s})$$
$$= 0.25\pi \text{ s/cm}$$

or the rate at which the tank will empty is

$$r = \frac{1}{\Gamma} = \frac{4}{\pi} \text{ cm/s}$$

The total time required for the liquid to reach the bottom float if it began at the top float is

$$T_f = (0.25\pi \text{ s/cm})\ (50 \text{ cm})$$
$$= 12.5\pi \text{ s}$$

Clearly, when the pump is on, the change in x with time must be linear, since the volumetric flow rate f and the tank diameter are constants. A plot showing liquid level versus time, assuming that the tank started to empty at $t = 0$, is shown in Figure 1-2.

In order to proceed further with this problem, we might hypothesize a mechanism by which the tank fills so that we could mathematically describe the liquid

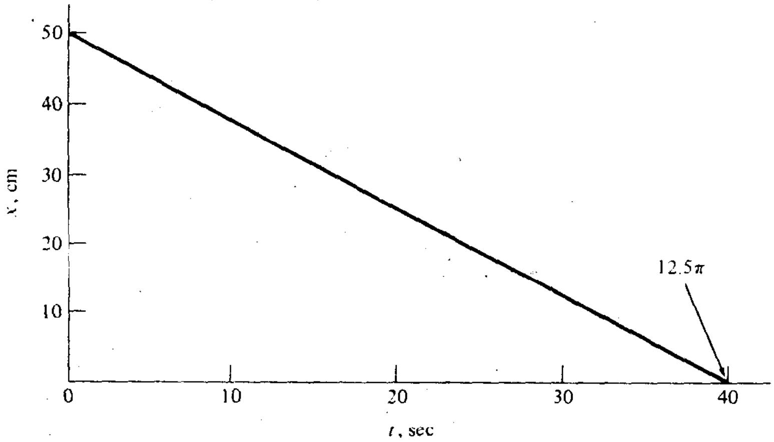

FIGURE 1-2. Liquid level versus time.

level for all time. We could also include information about the pump motor starting and stopping dynamics, or the dynamics of the opening and closing of the switch. We will consider some of these refinements in this and later chapters. All such refinements to the original model would be attempts at representing more closely the real-world situation. Regardless of how refined the model becomes, however, it is still an idealization of the actual set of circumstances. Since it is the model that is analyzed, it therefore follows that the graph showing liquid level versus time is also an idealization.

EXAMPLE 1-1

Suppose that water flows into the tank continuously at a constant rate of $f_{in} = 5$ liters/s. Modify the analysis above to obtain expressions for x that are valid for all time.

Solution: If $t = 0$ is taken as an instant in time at which the tank is full, then $\Gamma = (2.5\pi \text{ liters/cm})/[(10 - 5)\cdot\text{liters/s})] = 0.5\pi$ s/cm is the time required for the tank to empty 1 cm. Thus $T_f = (0.5\pi \text{ s/cm})\ (50 \text{ cm}) = 25\pi$ s for the tank to empty. The tanks fills at the same rate, so that x is a triangular function of time that repeats every $2(25\pi) = 50\pi$ s. You should sketch the waveform for x and dimension it fully as in Figure 1-2. ■

To reemphasize, the starting point of any systems analysis problem is a *model* which, no matter how refined, is *always an idealization of a real-world (physical) system.* Hence the result of any systems analysis is an idealization of the true state of affairs. Nevertheless, if the model is sufficiently accurate, the results obtained will portray the operation of the actual system sufficiently accurately to be of use.

Communications Link

Figure 1-3 is a pictoral representation of a two-way communications link as might exist, for example, between New York and Los Angeles. It might consist