FOUNDATIONS OF OPTIMUM DESIGN IN CIVIL ENGINEERING

Andrzej M. BRANDT

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Preface

The present book is intended for readers who are interested in designing building structures and especially for those who wish to approach new problems in this field in an active and creative way.

The development of the optimization methods of structures has gathered impetus in recent years. This is due to different causes, the most important of them being the increasing importance of economy in the use of materials, the impossibility of erecting many kinds of unconventional structures without a careful choice of theirs hapes and finally, the development of the mathematical optimization theory and its applications in other branches of technology. Optimal solutions are already indispensable in designing outer space vehicles and their trajectories. Undoubtedly, in the nearest future, the designing of many kinds of building structures without determining by objective methods their optimal geometrical and strength parameters may prove to be too expensive or hazardous.

Thus, in preparing this book, the authors' aim was to facilitate the use of optimization in building by offering the constructors an ordered presentation of the existing methods and solutions.

Optimization was initiated and developed in Poland by the late Professor Zbigniew Wasiutyński, who died prematurely at the beginning of 1974. The main achievements of Professor Wasiutyński consisted in formulating, as early as in 1939, the optimum design criterion for minimum potential, obtaining numerous valuable solutions, working out fundamental theorems and stimulating interest in optimization problems among his disciples to whom some of the authors of the present monograph had the honour to belong. Thus the influence of his thoughts and creative ideas can clearly be seen in the book. Professor Wasiutyński not only stressed the importance of optimization as a means of reducing costs but also pointed to other technological and aesthetic advantages resulting from an orderly arrangement of internal forces. It would not be out of place to recall Professor Wasiutyński's words on the links between structural optimization and man's natural tendencies: "In the optimum design of structures, of paramount importance are two tendencies characteristic of our behaviour. The first may be called endeavouring to satisfy our needs the largest possible extent, and the second—aspiration for aesthetic values. Each of them imparts significant features to structural forms... In producing a structural form we aim at satisfying human needs to the highest degree with the least expenditure of labour, materials, tooling and machinery. In the attempt to achieve aesthetic values we are inclined to emphasize the exposure of certain features of the form, to put them in an ordered arrangement, to supplement and enrich them".[1]

The above remarks do not contradict in any way the advancing penetration of mathematical methods into structural optimization; on the contrary, they lead to more extensive applications and more effective solutions. Modern mathematical methods and the use of computer technology are indispensable for the further development of optimization and have been fully treated in the present monograph.

The present book is the outcome of the common work of a group of authors. Its starting point was the preparation, under the guidance of Professor Wasiutyński, of materials for a scientific and training conference which took place at Jabłonna near Warsaw in 1968. The first of the conference proceedings, enlarged and properly arranged, was published as a book on optimization criteria and methods[2]. The present monograph, written by a slightly larger team of authors, develops and extends the earlier one so as to comprise applications of modern criteria and methods of optimization to various types of civil engineering and building structures. It includes a great deal of new material, which has not appeared in print before and so far has not been published in any form. The book considers also a number of solutions and examples collected from other Polish and foreign publications. In spite of the authors' attempts at a uniform treatment of all types of structures and a similar way of presenting their views certain unavoidable differences have appeared. Differences in treatment result from the unequal degree of development of optimization methods for the different types of structures; for instance, the problems concerning beams or columns could be treated extensively and in full detail, whereas the chapter on the optimization of plates had to be restricted. A complete unification of style has also proved impossible and so each of the chapters bears the individual stamp of its author.

The book contains ten chapters.

Chapters 1 and 2 comprise an introduction to the subject and an explana-

[1] WASIUTYŃSKI, Z., On the use of exact sciences for determining shapes in the construction industry, (in Polish), *Nauka Polska*, VIII, 1, 29, 1960.

[2] *Criteria and Methods of Structural Optimization*, edited by A. M. Brandt, PWN–Polish Scientific Publishers, Warsaw, and Martinus Nijhoff Publishers BV, The Hague, The Netherlands, 1984.

tion of the goals and scope of the monograph, and also give a short survey of the problems which arise and are solved in the domain in question.

Chapter 3 presents the criteria and methods applied to modern problems of structural optimization.

Chapter 4 contains formulations and solutions of the optimization of beams and, to a lesser extent, of frames.

Chapters 5 and 6 deal with problems concerning the optimization of columns and arches. The search for the optimal shapes of this type of structures has a long history; but here we confine ourselves to providing solutions which can be applied in the present-day engineering practice.

Chapter 7, concerning trusses, presents both the basic theorems and the latest techniques based on mathematical programming.

Chapter 8 contains solutions obtained for cable-suspended structures and parts of support structures.

Optimization of a number of types of plates and slabs is considered in Chapter 9, including several original solutions illustrated by examples.

Chapter 10, devoted to plane-loaded plate structures, contains the fundamental solutions which can be used for further development of this field of optimization.

As can be seen, the chapters, starting from Ch. 4, are arranged according to subjects and concern the successive types of structures. This arrangement should make it easier for the reader to resort to the book while solving definite problems which may arise in designing a structure. The authors have thus left out of account the most general optimization problems, in which the designer formulates only the data concerning the service conditions and loads and optimization consists also in choosing the type of the structure. Problems of this kind, with attempting logical system, are neither formulated nor solved as yet, and research in this direction would go beyond the scope of the present book.

The limited size of the book made it impossible to include the problems of the optimization of shells and of structures made of steel-reinforced concrete. On account of the freedom of optimization and the design and economic importance of a proper choice of the shapes of shells, this topic should be developed in the nearest future. Let us also note that there exist solutions which are valuable in this field, but a general approach and fundamental criteria seem to be lacking. Also the optimization of concrete steel reinforced structures lacks a general formulation. It may be hoped therefore that the next monograph on optimization will deal with the above two topics.

The division of labour between the authors of the present monograph has been as follows: Prof. Dr Hab. Andrzej M. Brandt wrote Chapters 1 and 2 and contributed to Chapter 4; Doc. Dr Hab. Wojciech Dzieniszewski wrote Sections 9.1 and 9.4, Dr Eng. Jacek Gierliński—Sections 9.2 and 9.3, Doc. Dr Hab. Eng. Jan Holnicki-Szulc—Chapter 10; Doc. Dr. Hab. Stefan Jedno and Doc. Dr Hab. Eng. Antoni Stachowicz were the co-authors of Chapter 8; Doc. Dr Hab. Eng. Wojciech Marks wrote Chapters 3 and 6 and contributed to Chapter 4; Dr Eng. Andrzej Niemierko wrote Chapter 7 and Dr Eng. Stefan Owczarek—Chapter 5.

Before translation into English the text was subjected to some modifications and corrections in all chapters, in particular in chapters concerning postcompression of concrete beams and optimization of cable-suspended structures, new concepts, methods and references have been added and certain examples have been modified.

Since the authors intend to continue their research in structural optimization, they will be grateful to the readers for any critical remarks and suggestions concerning the book.

ANDRZEJ M. BRANDT

Contents

1

Subject, Goal and Scope of the Book

The present book deals with the ways of applying optimization methods in designing various kinds of building and engineering structures.

The search for the optimum shapes of structures is becoming more and more important for two reasons: firstly, modern industrial and civil engineering structures are very large and expensive; and secondly, they are often erected in accordance with the same typical documentations. The optimization of structural shapes and sizes, thanks to the saving of material and introducing order in the internal forces, may bring considerable advantages. What is more, to rely solely on the abilities and intuition of the designers proves inappropriate or even impossible.

All over the world new research teams emerge specializing in structural design optimization. The accessibility of highly effective computing techniques and up-to-date analytical methods favours obtaining solutions that are interesting and useful for design work. Although investigations of structural design problems have a long history, many of them still remain unsolved. New areas of studies continually emerge in which previously disregarded effects are taken into account. Optimization problems are formulated in new engineering domains, e.g., in seeking the proper ingredients and internal structures of composite materials.

It is possible to observe a characteristic tendency to undertake more and more extensive general problems. They concern not only particular structural members but also their complete assemblies, e.g., bridge spans together with their supports and foundations. This leads to a less abstract understanding of the concept of the objective function which is in most cases the cost formulated in a general fashion.

Optimization is also oriented towards the problems involving multiple criteria which should be satisfied at the same time by some structures. Studies are also developed of the so far neglected effects of instability, delayed deformations and dynamic behaviour.

Constraint conditions and objective functions concern more and more frequently not only the general properties characterizing the whole optimized element but also local quantities, e.g., the maximum displacement, stress and strain.

1 Subject, goal and scope of the book

Problems involving the probabilistic treatment of all the design parameters appear very frequently in optimization. This follows from the fact that not only the loads and sizes of the structures and their members but also the properties of the structural materials can be treated as random variables. Such an approach is directly related to the commonly adopted design procedure based on the method of critical states with separated safety factors, derived—to a greater or lesser extent—from probabilistic reasoning.

In spite of such multidirectional development, optimization methods are still rather rarely used in structural design.

In other fields of engineering, e.g. in telecommunication and transportation systems, in the design of parts of aircrafts and spacecrafts, and also in military and economic strategies, optimization is more extensively used than in the construction industry. This can be explained by the fact that in these fields the use of optimal solutions is simply indispensable in view of, among other things, the large number of parameters and variables involved, which makes intuitive methods unworkable.

The present state of art in optimization is characterized not by a development free from conflicts but by a serious discrepancy between the high level of the methods employed in formulating and solving individual problems on the one hand and the relatively rare cases of application in design practice on the other hand. Thus, there are obstacles which, occurring in different degrees in the individual countries, impede the use of objective methods in choosing structural shapes. These obstacles include, among other things, too complicated and labourious optimization methods and also insufficient interest of designers in finding the optimal solution, partly as a result of the inadequate way in which their work is organized and paid for.

One of the methods of formulating and solving optimization problems is to reduce them to forms which would be directly applicable in designing practice. This is just what the present book is expected to achieve.

The book presents the current state of the optimization of building structures in such a way as to make the available methods and solutions more easily accessible to designers. Accordingly, the book discusses efficient optimization methods as they are applied to particular types of structures and illustrates them by means of examples, whereas the general methods and criteria dealt with in Chapter 3, are treated briefly.

Chapter 4 deals with the optimization problems of beams and frames, and Chapter 5 with the optimization of columns.

Chapter 6 contains some basic solutions in the field of arch and vault optimization, which is now regarded as classical. The next three chapters comprise problems of the optimization of truss, cable-suspended and plate systems including solid, three-layer and network plates. Chapter 10 presents the basic equations and conditions involved in the optimization problems of plane-loaded plates.

It has not been possible to treat all the types of structures in an equally detailed fashion. This is due to the present state of researches in the world's leading centres, where new methods are developed and their possible applications are envisaged. It should also be borne in mind that some kinds of structures have been optimized for centuries and others only for a few years. Distinction should be made between such elements as arches and columns, with regard to which classical design predominates, and more modern elements, such as beams, frames, plates, trusses and cable-structures, the optimization of which absorbs today the attention of numerous researchers. A different situation can be observed with regard to plane-loaded plates and shells. Optimization of these structures is vital not only for economic reasons; it is often decisive for their rational construction, e.g. in problems resulting from the action of concentrated forces applied to thin shells. In spite of this the number of optimized designs of shells and, in particular, of plane-loaded plates is rather small and for many versions of structures no verified analytical or numerical techniques have been worked out so far. Consequently, Chapter 10 is restricted to the general considerations and principles and to the discussion of very few selected examples of the optimization of plane-loaded plates, while the optimization problems of shells have been left beyond of the scope of the book.

Chapter 3 describes optimization methods and criteria but excludes the derivation of relations: it only aims at presenting the relations and concepts which are indispensable to understand the subsequent chapters. This approach is justified by the fact that the basic criteria and methods of optimization have already been presented in numerous publications from different view points, e.g. their use in numerical solutions. The most important works in this field are listed at the end of Chapter 3; its direct continuation is the monograph *Criteria and Methods of Structural Optimization* published jointly by Martinus Nijhoff Publishers and PWN—Polish Scientific Publishers, Warsaw in 1984, in which the readers will find the explanations and derivations of some of the relations introduced here.

The monograph in question and the present book, written partly by the same authors, are complementary to each other in their treatment of the foundations and applications of structural optimization.

The present book contains little information on the historical development of optimization methods and solutions. Information of this kind is very interesting, but it might overload the book with matter which is useless from the point of view of solving present day problems of structural optimization; on the other hand, an effort has been made to draw the reader's attention to the importance of the electronic computer technique, without which effective solutions of problems posed by structural designers is hardly imaginable. For this reason, examples and solutions are presented in the forms suitable for use in digital computers; in some cases, detailed data are given about the existing programs. The fact is that the further development of structural optimization is undoubtedly bound with mathematical programming and numerical methods applied to more and more sophisticated systems, representing complete constructions or sets of constructions.

In spite of the clearly interdisciplinary nature of optimization methods and criteria, which can be used with minor modifications in a large variety of problems, the specific features of each individual area of application should not be underestimated. By making use, in structural optimization, of all the data so far available, e.g., on the unchangeability of some quantities or the obvious reduction of others, we can often considerably restrict the scope of analytic or numerical work necessary to obtain the solution. In this sense, the present book concerns mainly civil engineering structures.

2

Survey of Structural Optimization Problems

2.1 Tasks and goals of optimization

Structural optimization concerns the problems of the selection of geometric parameters and mechanical strength properties of structural elements and whole constructions. This selection consists in a search for the extremal solutions to satisfy the prescribed criteria, the search being conducted in an objective and rational way, that is, without depending on the intuition or special abilities of the designer.

Thus, optimization takes over that part of the design process which consists in selecting shapes and sizes and subsequently checking that the mechanical strength criteria and other requirements have been met.

The question arises whether optimization can and should fully replace the traditional designing procedures, that is, whether the task of optimization is to embrace all structural parameters so that the solution of an optimization problem should be equivalent to obtaining a complete design of a structure. It does not seem possible to give an unambiguous answer to this question.

In the case of simple elements or even whole structures it is probably possible and even necessary to take into account all the parameters in the formulation of an optimization problem. This case is exemplified in the classical problems of the optimum design of an arch or a column. If the structure is more complex, however, the problem of optimization comprising all parameters may prove impossible to solve by the methods known so far, or a search for such a solution may be inexpedient, for example, on account of the cost and labour involved.

In such cases it is advisable to include only some of the parameters in the formulation of the optimization problem, leaving the remaining ones to the care of the designer. Such an approach is justified because almost always there exist parameters whose choice is obvious, for example, the straight axis of an axially compressed column or the location of all the joints of a truss in the plane of the forces acting on it. Thus it can be said that on the whole the optimization does not replace the entire design process but is only part of it. Indeed, some parameters of shape are initially assumed by the designer as given and others are obtained from solving of the optimization problem in detail and from designing elements which are neutral with respect to optimization or those which—with a certain approximation—can be regarded as neutral.

Structural optimization is connected with structural mechanics and mathematical optimization theory. The first traces of structural optimization can be found in Galileo's reflections concerning a beam subject to bending loads. Also the further development of structural optimization was based on structural mechanics. Consequently, optimization dealt successively with various kinds of structures which are at present the subject of structural mechanics, and its basic concepts and hypotheses were also taken over from the theory of structures. However, the recent rapid development of structural optimization follows—to a large extent—from its connections with mathematical optimization theory and from the use of modern methods of solving mathematical problems.

Thus, optimization is one of those branches of present-day knowledge which make use of methods and results arrived at in various traditional disciplines.

The goals of structural optimization are diverse and find expression in various criteria or the so-called objective functions. In most cases the problem is to find the minimum volume or weight of a structure, its minimum cost, or finally the extremal values of such quantities as energy or stiffness. As regards the criteria of minimum volume and weight the basic quantity is the amount of constructional materials. In seeking the minimum cost the amount of material is essential, because other items in the total cost of a project, such as assembly and transport, are usually proportional to the amount of constructional materials. It can be shown that the minimum strain energy leads to the minimum amount of materials. Thus, it turns out that optimization is largerly based on the saving of materials, which results not only from the minimum amount of materials used in optimized structures but also from the ordered arrangement of internal forces and deformations appearing in them. Indeed, it cannot be claimed that the only and fundamental aim of structural optimization is to save material and obtain other economic advantages. Optimization of structures by determining the relations between shape parameters, internal forces and distribution of material, eliminates arbitrariness in choosing the shape of the structure. This brings many-sided engineering and economic advantages.

It can thus be concluded that although structural optimization may be regarded as belonging to structural mechanics or to general optimization theory and not to economics, its effects, just as any other engineering activity, have also economic aspects.

2.2 Basic formulations of optimization problems

The classical approach in seeking optimal shapes consists in making use of the solutions of variational calculus. In the simplest case, it is an isoperimetric problem, which can be presented in the following form:

Find a function $F(x_i)$

for which the functional $\int F(x_i)\,dx$ attains its extremum and the functional $\int G(x_i)\,dx_i$ is constant.

Additional conditions in the form of equalities can be taken into account by introducing Lagrange multipliers. These conditions can be stated in such a way as to include the equalization of stresses, in certain prescribed regions, which is advantageous on account of mechanical strength.

Examples of the application of the isoperimetric problem of the variational calculus are described in greater detail in Chapter 4.

Analytical methods of formulating and solving the problems of strength optimum design and optimization have evolved in line with the general development of mathematical methods and the state of knowledge on structural mechanics and physical properties of materials. Methods worked out in other areas and making up the general theory of optimization are now the basis for various applications, including structural optimization.

This approach permits us to define the optimal structure in the form adopted in mathematical programming: a structure described by the parameters $x_i(x_1, x_2, \ldots, x_n)$ is said to be *optimal* if the following condition is satisfied:

$$\max F(x_i) \quad \text{or} \quad \min F(x_i)$$

where $F(x_i)$ denotes the objective function; this function should thus take the extremum values. The shape parameters x_i are called the *design* or *decision variables* the values of which are constrained by equalities or inequalities, resulting from the kinds of materials and technology employed and the intended use of the construction. Examples of constraining conditions are the values of permissible stresses, strains or displacements, minimum or maximum sizes of the structure and finally the flatness of some surfaces or the rectilinearity of the axes. The constraining conditions set up a multidimensional domain of feasible solutions which contains the required solutions of the optimization problem under consideration.

A detailed description of various objective functions, called also the *optimization criteria*, and of diverse cases of formulating the constraining conditions is given in Chapter 3.