

SECOND EDITION

# **SYSTEM SIMULATION**

**GEOFFREY GORDON**

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*IBM Corporation*

PRENTICE-HALL, INC., ENGLEWOOD CLIFFS, NEW JERSEY 07632

*Library of Congress Cataloging in Publication Data*

GORDON, GEOFFREY.

System simulation.

Includes bibliographies and index.

1. Digital computer simulation. 2. System analysis. I. Title.

QA76.5.G63 1978 001.4'24 77-24579

ISBN 0-13-881797-9

© 1978, 1969 by Prentice-Hall, Inc., Englewood Cliffs, New Jersey 07632

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10 9 8 7 6 5 4 3 2 1

Printed in the United States of America

PRENTICE-HALL INTERNATIONAL, INC., *London*

PRENTICE-HALL OF AUSTRALIA PTY. LIMITED, *Sydney*

PRENTICE-HALL OF CANADA, LTD., *Toronto*

PRENTICE-HALL OF INDIA PRIVATE LIMITED, *New Delhi*

PRENTICE-HALL OF JAPAN, INC., *Tokyo*

PRENTICE-HALL OF SOUTHEAST ASIA PTE. LTD., *Singapore*

WHITEHALL BOOKS LIMITED, *Wellington, New Zealand*

# PREFACE

The application of simulation continues to expand, both in terms of the extent to which simulation is used and the range of applications. This second edition of *System Simulation* reflects the changes.

The ties between analog and digital simulation, represented by digital-analog simulators, have become a matter of history. The section originally devoted to a specific digital-analog language has, therefore, been omitted. On the other hand, interest in socio-economic problems has greatly expanded the type of study originally described as Industrial Dynamics. The chapter on System Dynamics is devoted to this broader topic. The DYNAMO programming language was a pioneer in this type of study. It is still an active language, but other continuous-system simulation languages are now extensively used for the same purpose. The chapter devoted specifically to DYNAMO has, therefore, been omitted. Similarly, while FORTRAN is still being used for simulation programming, the general availability of programming languages designed for simulation makes the chapter that was devoted to that language unnecessary.

The development of discrete-system simulation languages continues. In fact, there are now languages that bridge the gap between continuous and discrete system simulation. GPSS and SIMSCRIPT, however, remain prominent. They have been retained as representatives of the dominant techniques for implementing discrete-system simulation.

The prime purpose of the book remains to provide material for a one-semester course introducing students to the topic and techniques of system simulation.

Some familiarity with programming concepts is assumed, but no knowledge of any particular programming language is needed as a prerequisite. In addition, familiarity with the concepts of statistics and probability theory will be helpful, although two chapters review the required theory. Many areas of application are discussed, but the technical details are kept simple so that the book will be useful to students of different backgrounds. A course based on the text could be introduced at the senior undergraduate or graduate level in many disciplines.

The first chapter discusses the types of models that are basic to any simulation. The second chapter considers the topic of organizing a system study. The nature of the simulation technique is then introduced in Chapter 3. Continuous-system simulation is examined in Chapter 4, and its application to System Dynamics studies is illustrated in Chapter 5. Chapters 6 and 7 are the ones that introduce the necessary statistics and probability theory.

The application of discrete-system simulation is demonstrated in Chapter 8, using a hand-worked example of a simple telephone-system. Two chapters are then devoted to GPSS, followed by two chapters describing SIMSCRIPT. In both cases, the first of the two chapters is self-contained, and would be sufficient for students needing only a brief introduction. The second chapter in each case includes, as a worked-out example, the same telephone-system problem presented in Chapter 8, thus giving an opportunity to compare the three approaches.

Chapter 13 discusses the programming techniques involved in the design and construction of simulation programming systems. Its prime interest will be to students concerned with the evaluation and design of simulation languages. The last chapter, also, is of a more technical nature, since it discusses the topic of analyzing statistical outputs of simulation runs.

I would like to thank CACI, Inc., for providing the SIMSCRIPT listings used in Chapters 11 and 12, and Dr. E. C. Russell for his generous help in both correcting the SIMSCRIPT examples and reviewing those chapters.

GEOFFREY GORDON

# CONTENTS

## PREFACE *xi*

## SYSTEM MODELS **1**

- 1-1 The Concepts of a System **1**
- 1-2 System Environment **3**
- 1-3 Stochastic Activities **4**
- 1-4 Continuous and Discrete Systems **5**
- 1-5 System Modeling **6**
- 1-6 Types of Models **8**
- 1-7 Static Physical Models **10**
- 1-8 Dynamic Physical Models **11**
- 1-9 Static Mathematical Models **13**
- 1-10 Dynamic Mathematical Models **16**
- 1-11 Principles Used in Modeling **17**

**2****SYSTEM STUDIES 21**

- 2-1 Subsystems 21
- 2-2 A Corporate Model 22
- 2-3 Environment Segment 23
- 2-4 Production Segment 24
- 2-5 Management Segment 25
- 2-6 The Full Corporate Model 26
- 2-7 Types of System Study 27
- 2-8 System Analysis 28
- 2-9 System Design 31
- 2-10 System Postulation 34

**3****SYSTEM SIMULATION 38**

- 3-1 The Technique of Simulation 38
- 3-2 The Monte Carlo Method 40
- 3-3 Comparison of Simulation and Analytical Methods 41
- 3-4 Experimental Nature of Simulation 43
- 3-5 Types of System Simulation 43
- 3-6 Numerical Computation Technique for Continuous Models 44
- 3-7 Numerical Computation Technique for Discrete Models 46
- 3-8 Distributed Lag Models 48
- 3-9 Cobweb Models 50
- 3-10 Progress of a Simulation Study 52

**4****CONTINUOUS SYSTEM SIMULATION 58**

- 4-1 Continuous System Models 58
- 4-2 Differential Equations 59
- 4-3 Analog Computers 61
- 4-4 Analog Methods 63
- 4-5 Hybrid Computers 65
- 4-6 Digital-Analog Simulators 66
- 4-7 Continuous System Simulation Languages (CSSLs) 66

- 4-8 CSMP III 67
- 4-9 Hybrid Simulation 71
- 4-10 Feedback Systems 71
- 4-11 Simulation of an Autopilot 74
- 4-12 Interactive Systems 78
- 4-13 Real-Time Simulation 78

## **SYSTEM DYNAMICS 82**

- 5-1 Historical Background 82
- 5-2 Exponential Growth Models 83
- 5-3 Exponential Decay Models 87
- 5-4 Modified Exponential Growth Models 88
- 5-5 Logistic Curves 89
- 5-6 Generalization of Growth Models 91
- 5-7 System Dynamics Diagrams 92
- 5-8 Simple System Dynamics Diagrams 94
- 5-9 Multi-Segment Models 96
- 5-10 Representation of Time Delays 98
- 5-11 Feedback in Socio-Economic Systems 99
- 5-12 A Biological Example 103
- 5-13 World Models 105
- 5-14 The DYNAMO Language 106

## **PROBABILITY CONCEPTS IN SIMULATION 112**

- 6-1 Stochastic Variables 112
- 6-2 Discrete Probability Functions 113
- 6-3 Continuous Probability Functions 115
- 6-4 Measures of Probability Functions 117
- 6-5 Numerical Evaluation of Continuous Probability Functions 122
- 6-6 Continuous Uniformly Distributed Random Numbers 125
- 6-7 Computer Generation of Random Numbers 128
- 6-8 A Uniform Random Number Generator 130
- 6-9 Generating Discrete Distributions 131
- 6-10 Non-Uniform Continuously Distributed Random Numbers 133
- 6-11 The Rejection Method 138



**ARRIVAL PATTERNS AND SERVICE TIMES 144**

- 7-1 Congestion in Systems 144
- 7-2 Arrival Patterns 145
- 7-3 Poisson Arrival Patterns 146
- 7-4 The Exponential Distribution 150
- 7-5 The Coefficient of Variation 153
- 7-6 The Erlang Distribution 154
- 7-7 The Hyper-Exponential Distribution 155
- 7-8 Service Times 157
- 7-9 The Normal Distribution 158
- 7-10 Queuing Disciplines 162
- 7-11 Measures of Queues 163
- 7-12 Mathematical Solutions of Queuing Problems 164
- 7-13 Utilization as a Design Factor 166
- 7-14 Grade of Service 167

**DISCRETE SYSTEM SIMULATION 173**

- 8-1 Discrete Events 173
- 8-2 Representation of Time 174
- 8-3 Generation of Arrival Patterns 175
- 8-4 Simulation of a Telephone System 177
- 8-5 Delayed Calls 181
- 8-6 Simulation Programming Tasks 185
- 8-7 Gathering Statistics 188
- 8-8 Counters and Summary Statistics 188
- 8-9 Measuring Utilization and Occupancy 189
- 8-10 Recording Distributions and Transit Times 191
- 8-11 Discrete Simulation Languages 193

**INTRODUCTION TO GPSS 197**

- 9-1 GPSS Programs 197
- 9-2 General Description 198

- 9-3 Action Times 201
- 9-4 Succession of Events 203
- 9-5 Choice of Paths 203
- 9-6 Simulation of a Manufacturing Shop 204
- 9-7 Facilities and Storages 208
- 9-8 Gathering Statistics 212
- 9-9 Conditional Transfers 215
- 9-10 Program Control Statements 217

## **GPSS EXAMPLES 222**

- 10-1 Priorities and Parameters 222
- 10-2 Standard Numerical Attributes 223
- 10-3 Functions 226
- 10-4 Simulation of a Supermarket 227
- 10-5 Transfer Modes 232
- 10-6 Logic Switches 234
- 10-7 Testing Conditions 234
- 10-8 GPSS Model of a Simple Telephone System 235
- 10-9 Set Operations 238

## **INTRODUCTION TO SIMSCRIPT 246**

- 11-1 SIMSCRIPT Programs 246
- 11-2 SIMSCRIPT System Concepts 247
- 11-3 Organization of a SIMSCRIPT Program 248
- 11-4 Names and Labels 249
- 11-5 SIMSCRIPT Statements 250
- 11-6 Defining the System 251
- 11-7 Defining the Telephone System Model 253
- 11-8 Referencing Variables 255
- 11-9 The MAIN Routine 256
- 11-10 The Arrival Event 258
- 11-11 The Timing Routine 260
- 11-12 The Disconnect Event 261
- 11-13 The Closing Event 261

**12****MANAGEMENT OF SETS IN SIMSCRIPT 264**

- 12-1 Definition of Sets in SIMSCRIPT 264
- 12-2 Set Organization 265
- 12-3 Set Controls 266
- 12-4 Telephone System Model 2 267
- 12-5 Gathering Statistics in SIMSCRIPT 271
- 12-6 Searching Arrays 272
- 12-7 Searching Sets 274

**SIMULATION PROGRAMMING TECHNIQUES 276**

- 13-1 Entity Types 276
- 13-2 List Processing 277
- 13-3 Data Structures in SIMSCRIPT 282
- 13-4 Data Structures in GPSS 283
- 13-5 Implementation of Activities 284
- 13-6 Simultaneous Events 285
- 13-7 Conditional Events 287
- 13-8 Event Scanning 288
- 13-9 Execution of Simulation Algorithm in SIMSCRIPT 290
- 13-10 Execution of Simulation Algorithm in GPSS 293

**ANALYSIS OF SIMULATION OUTPUT 299**

- 14-1 Nature of the Problem 299
- 14-2 Estimation Methods 300
- 14-3 Simulation Run Statistics 303
- 14-4 Replication of Runs 305
- 14-5 Elimination of Initial Bias 307
- 14-6 Batch Means 308
- 14-7 Regenerative Techniques 310
- 14-8 Time Series Analysis 314
- 14-9 Spectral Analysis 315
- 14-10 Autoregressive Processes 316

**INDEX 320**

# SYSTEM MODELS

## 1-1

### The Concepts of a System

The term *system* is used in such a wide variety of ways that it is difficult to produce a definition broad enough to cover the many uses and, at the same time, concise enough to serve a useful purpose, (6), (12), and (20).<sup>1</sup> We begin, therefore, with a simple definition of a system and expand upon it by introducing some of the terms that are commonly used when discussing systems. A *system* is defined as an aggregation or assemblage of objects joined in some regular interaction or interdependence. While this definition is broad enough to include static systems, the principal interest will be in dynamic systems where the interactions cause changes over time.

As an example of a conceptually simple system, consider an aircraft flying under the control of an autopilot (see Fig. 1-1). A gyroscope in the autopilot detects the difference between the actual heading and the desired heading. It sends a signal to move the control surfaces. In response to the control surface movement, the airframe steers toward the desired heading.

As a second example, consider a factory that makes and assembles parts into a product (see Fig. 1-2). Two major components of the system are the fabrication

<sup>1</sup> Parenthetical numbers in text refer to items in bibliography at end of chapter.

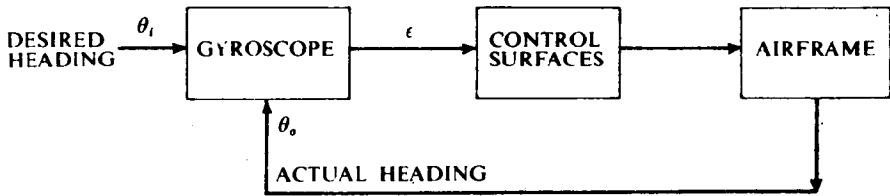


Figure 1-1. An aircraft under autopilot control.

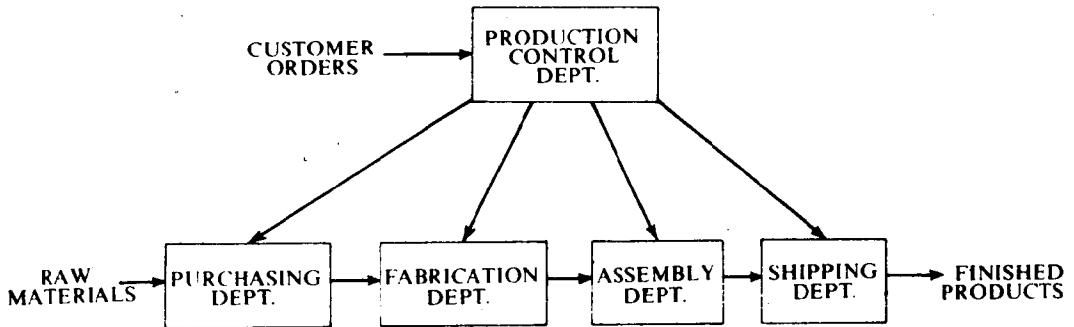


Figure 1-2. A factory system.

department making the parts and the assembly department producing the products. A purchasing department maintains a supply of raw materials and a shipping department dispatches the finished products. A production control department receives orders and assigns work to the other departments.

In looking at these systems, we see that there are certain distinct objects, each of which possesses properties of interest. There are also certain interactions occurring in the system that cause changes in the system. The term *entity* will be used to denote an object of interest in a system; the term *attribute* will denote a property of an entity. There can, of course, be many attributes to a given entity. Any process that causes changes in the system will be called an *activity*. The term *state of the system* will be used to mean a description of all the entities, attributes, and activities as they exist at one point in time. The progress of the system is studied by following the changes in the state of the system.

In the description of the aircraft system, the entities of the system are the airframe, the control surfaces, and the gyroscope. Their attributes are such factors as speed, control surface angle, and gyroscope setting. The activities are the driving of the control surfaces and the response of the airframe to the control surface movements. In the factory system, the entities are the departments, orders, parts, and products. The activities are the manufacturing processes of the departments.

Attributes are such factors as the quantities for each order, type of part, or number of machines in a department.

Figure 1-3 lists examples of what might be considered entities, attributes, and activities for a number of other systems. If we consider the movement of cars as a traffic system, the individual cars are regarded as entities, each having as attributes its speed and distance traveled. Among the activities is the driving of a car. In the case of a bank system, the customers of the bank are entities with the balances of their accounts and their credit statuses as attributes. A typical activity would be the action of making a deposit. Other examples are shown in Fig. 1-3.

| SYSTEM         | ENTITIES  | ATTRIBUTES               | ACTIVITIES   |
|----------------|-----------|--------------------------|--------------|
| TRAFFIC        | CARS      | SPEED<br>DISTANCE        | DRIVING      |
| BANK           | CUSTOMERS | BALANCE<br>CREDIT STATUS | DEPOSITING   |
| COMMUNICATIONS | MESSAGES  | LENGTH<br>PRIORITY       | TRANSMITTING |
| SUPERMARKET    | CUSTOMERS | SHOPPING LIST            | CHECKING-OUT |

Figure 1-3. Examples of systems.

The figure does not show a complete list of all entities, attributes, and activities for the systems. In fact, a complete list cannot be made without knowing the purpose of the system description. Depending upon that purpose, various aspects of the system will be of interest and will determine what needs to be identified.

## 1-2

### System Environment

A system is often affected by changes occurring outside the system. Some system activities may also produce changes that do not react on the system. Such changes occurring outside the system are said to occur in the *system environment*. An important step in modeling systems is to decide upon the boundary between the system and its environment. The decision may depend upon the purpose of the study.

In the case of the factory system, for example, the factors controlling the arrival of orders may be considered to be outside the influence of the factory and therefore part of the environment. However, if the effect of supply on demand is to be considered, there will be a relationship between factory output and arrival of orders, and this relationship must be considered an activity of the system. Similarly, in the case of a bank system, there may be a limit on the maximum interest rate that can be paid. For the study of a single bank, this would be regarded as a constraint imposed by the environment. In a study of the effects of monetary laws on the banking industry, however, the setting of the limit would be an activity of the system.

The term *endogenous* is used to describe activities occurring within the system and the term *exogenous* is used to describe activities in the environment that affect the system. A system for which there is no exogenous activity is said to be a *closed* system in contrast to an *open* system which does have exogenous activities.

### 1-3

#### Stochastic Activities

One other distinction that needs to be drawn between activities depends upon the manner in which they can be described. Where the outcome of an activity can be described completely in terms of its input, the activity is said to be *deterministic*. Where the effects of the activity vary randomly over various possible outcomes, the activity is said to be *stochastic*.

The randomness of a stochastic activity would seem to imply that the activity is part of the system environment since the exact outcome at any time is not known. However, the random output can often be measured and described in the form of a probability distribution. If, however, the *occurrence* of the activity is random, it will constitute part of the environment. For example, in the case of the factory, the time taken for a machining operation may need to be described by a probability distribution but machining would be considered to be an endogenous activity. On the other hand, there may be power failures at random intervals of time. They would be the result of an exogenous activity.

If an activity is truly stochastic, there is no known explanation for its randomness. Sometimes, however, when it requires too much detail or is just too much trouble to describe an activity fully, the activity is represented as stochastic. For example, in modeling elevator service in a building, the re-entry of people into the elevator, after they have been taken to a floor, could be connected with their having left the elevator, by assigning the time they stay on the floor. In most models, however, leaving and re-entry would be treated as separate stochastic

activities, connected only by the fact that the mean rates at which they transfer people are equal.

Assembling the data for a model will often involve an element of uncertainty that arises from sampling or experimental error. A value for some attribute of a model, which is known to be fixed, must be selected from a number of recorded values that contain random errors. Deciding on the best estimate is a statistical exercise. Usually, an arithmetic average will be considered sufficiently accurate.

## 1-4

### Continuous and Discrete Systems

The aircraft and factory systems used as examples in Sec. 1-1 respond to environmental changes in different ways. The movement of the aircraft occurs smoothly, whereas the changes in the factory occur discontinuously. The ordering of raw materials or the completion of a product, for example, occurs at specific points in time.

Systems such as the aircraft, in which the changes are predominantly smooth, are called *continuous systems*. Systems like the factory, in which changes are predominantly discontinuous, will be called *discrete systems*. Few systems are wholly continuous or discrete. The aircraft, for example, may make discrete adjustments to its trim as altitude changes, while, in the factory example, machining proceeds continuously, even though the start and finish of a job are discrete changes. However, in most systems one type of change predominates, so that systems can usually be classified as being continuous or discrete.

The complete aircraft system might even be regarded as a discrete system. If the purpose of studying the aircraft were to follow its progress along its scheduled route, with a view, perhaps, to studying air traffic problems, there would be no point in following precisely *how* the aircraft turns. It would be sufficiently accurate to treat changes of heading at scheduled turning points as being made instantaneously, and so regard the system as being discrete.

In addition, in the factory system, if the number of parts is sufficiently large, there may be no point in treating the number as a discrete variable. Instead, the number of parts might be represented by a continuous variable with the machining activity controlling the rate at which parts flow from one state to another. This is, in fact, the approach of a modeling technique called System Dynamics, which will be discussed in Chap. 5.

There are also systems that are intrinsically continuous but information about them is only available at discrete points in time. These are called *sampled-data systems*, (15). The study of such systems includes the problem of determining the



effects of the discrete sampling, especially when the intention is to control the system on the basis of information gathered by the sampling.

This ambiguity in how a system might be represented illustrates an important point. The description of a system, rather than the nature of the system itself, determines what type of model will be used. A distinction needs to be made because, as will be discussed later, the general programming methods used to simulate continuous and discrete models differ. However, no specific rules can be given as to how a particular system is to be represented. The purpose of the model, coupled with the general principle that a model should not be more complicated than is needed, will determine the level of detail and the accuracy with which a model needs to be developed. Weighing these factors and drawing on the experience of knowledgeable people will decide the type of model that is needed.

## 1-5

### System Modeling

To study a system, it is sometimes possible to experiment with the system itself. The objective of many system studies, however, is to predict how a system will perform before it is built. Clearly, it is not feasible to experiment with a system while it is in this hypothetical form. An alternative that is sometimes used is to construct a number of prototypes and test them, but this can be very expensive and time-consuming. Even with an existing system, it is likely to be impossible or impractical to experiment with the actual system. For example, it is not feasible to study economic systems by arbitrarily changing the supply and demand of goods. Consequently, system studies are generally conducted with a model of the system. For the purpose of most studies, it is not necessary to consider all the details of a system; so a model is not only a substitute for a system, it is also a simplification of the system, (14).

We define a *model* as the body of information about a system gathered for the purpose of studying the system.<sup>2</sup> Since the purpose of the study will determine the nature of the information that is gathered, there is no unique model of a system. Different models of the same system will be produced by different analysts interested in different aspects of the system or by the same analyst as his understanding of the system changes.

The task of deriving a model of a system may be divided broadly into two subtasks: establishing the model structure and supplying the data. Establishing the structure determines the system boundary and identifies the entities, attributes, and

<sup>2</sup>In the case of a physical model, the information is embodied in the properties of the model, in contrast to the symbolic representation in a mathematical model.