Heinz Georg Schuster

Deterministic Chaos

An Introduction

Second revised edition

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Prof. Dr. Heinz Georg Schuster
Institut für Theoretische Physik
und Sternwarte der Universität
Olshausenstraße – Physikzentrum
D-2300 Kiel
Federal Republic of Germany

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Preface

Daily experience shows that for many physical systems small changes in the initial conditions lead to small changes in the outcome. If we drive a car and turn the steering wheel only a little, our course will differ only slightly from that which the car would have taken without this change.

But there are cases for which the opposite of this rule is true: For a coin which is placed on its rim, a slight touch is sufficient to determine the side on which it will fall. Thus the sequence of heads and tails which we obtain when tossing a coin exhibits an irregular or chaotic behavior in time, because extremely small changes in the initial conditions can lead to completely different outcomes.

It has become clear in recent years, partly triggered by the studies of nonlinear systems using high-speed computers, that a *sensitive dependence on the initial conditions*, which results in a chaotic time-behavior, is by no means exceptional but a *typical property of many systems*. Such behavior has, for example, been found in periodically stimulated cardiac cells, in electronic circuits, at the onset of turbulence in fluids and gases, in chemical reactions, in lasers, etc. Mathematically, all nonlinear dynamical systems with more than two degrees of freedom, i.e. especially many biological, meteorological or economic models, can display chaos and, therefore, *become unpredictable over longer time scales.*

"Deterministic chaos" is now a very active field of research with many exciting results. Methods have been developed to classify different types of chaos, and it has been discovered that many systems show, as a function of an external control parameter, similar transitions from order to chaos. This universal behavior is reminiscent of ordinary second-order phase transitions, and the introduction of renormalization and scaling methods from statistical mechanics has brought new perspectives into the study of deterministic chaos.

It is the aim of this book to provide a self-contained introduction to this field from a physicist's point of view. The book grew out of a series of lectures, which I gave during the summer terms of 1982 and 1983 at the University of Frankfurt, and it requires no knowledge which a graduate student in physics would not have. A glance on the table of contents shows that new concepts such as the Kolmogorov entropy, strange attractors, etc., or new techniques such as the functional renormalization group are introduced on an elementary level. On the other hand, I hope that there is enough material for research workers who want to know, for example, how deterministic chaos can be distinguished experimentally from white noise, or who want to learn how to apply their knowledge about equilibrium phase transitions to the study of (nonequilibrium) transitions from order to chaos.

During the preparation of this book the manuscripts, preprints and discussion remarks of G. Eilenberger, K. Kehr, H. Leschke, W. Selke, and M. Schmutz were of great help. P. Bergé, M. Dubois, W. Lauterborn, W. Martienssen, G. Pfister and their coworkers supplied several, partly unpublished pictures of their experiments. H. O. Peitgen, P. H. Richter and their group gave the permission to include some of their most fascinating computer pictures into this book (see cover and Sect. 5.4). All contributions are gratefully appreciated. Furthermore, I want to thank W. Greulich, D. Hackenbracht, M. Heise, L. L. Hirst, R. Liebmann, I. Neil, and especially I. Procaccia for carefully reading parts of the manuscript and for useful criticism and comments. I also acknowledge illuminating discussions with V. Emery, P. Grassberger, D. Grempel, S. Grossmann, S. Fishman, and H. Horner.

It is a pleasure to thank R. Hornreich for the kind hospitality extended to me during a stay at the Weizmann Institute, where several chapters of this book were written, with the support of the Minerva foundation.

Last but not least, I thank Mrs. Boffo and Mrs. Knolle for their excellent assistance in preparing the illustrations and the text.

Frankfurt, October 1984 H. G. Schuster

Preface to the Second Edition

This is a revised and updated version of the first edition, to which new sections on sensitive parameter dependence, fat fractals, characterization of attractors by scaling indices, the Farey tree, and the notion of global universality have been added. I thank P. C. T. de Boer, J. L. Grant, P. Grassberger, W. Greulich, F. Kaspar, K. Pawelzik, K. Schmidt, and S. Smid for helpful hints and remarks, and Mrs. Adlfinger and Mrs. Boffo for their patient help with the manuscript.

Kiel, August 1987 H. G. Schuster

Legends to Plates I–XVII

Many of these plates are part of chapter 5. Accordingly, references mentioned in the legends are to be found on pages 257–258.

I *Biperiodic flow in a Bénard experiment:* Figs. 1–8 show interferometric pictures of a Bénard cell in the biperiodic régime; that is, there are two incommensurate frequencies in the power spectrum (see also pages 9–11). The time between successive pictures is 10 s. The first period lasts 40 s after which the "mouth" in the middle of the pictures repeats itself (see Figs. 1 and 5). But the details, e.g. in the upper right corners of Figs. 1 and 5 are not the same; that is, the motion is *not* simply periodic. (From a film taken by P. Bergé and M. Dubois, CEN Saclay, Gif-sur-Yvette, France.)

II *Nonlinear electronic oscillator* (see also Fig. 46 on page 75): The current-versus-voltage phase portraits (at the nonlinear diode) are shown on the oscilloscope screen. For increasing driving voltage one observes the period-doubling route. The nonlinearity of the diode that has been used in this experiment differs from eq. (3.121). (Picture taken by W. Meyer-Ilse, after Klinker et al., 1984.)

III *Taylor instability:* a) Formation of rolls, b) the rolls start oscillating, c) a more complicated oscillatory motion, d) chaos. (After Pfister, 1984; see also pages 152–154.)

IV *Disturbed heartbeats:* The voltage difference (black) across the cell membrane of one cell of an aggregate of heart cells from embryonic chicken shows a) phase looking with the stimulating pulse and b) irregular dynamics, displaying escape or interpolation beats if the time between successive periodic stimuli (red) is changed from 240 ms in a) to 560 ms in b). (After Glass et al., 1983; see also page 177.)

V *Chaotic electrical conduction in BSN crystals:* The birefringence pattern of a ferroelectric BSN crystal shows domain walls which mirror the charge transport near the onset of chaos (see also Fig. 115 on page 176). For clarity, the dark lines in the original pattern have been redrawn in red. (After Martin et al., 1984.)

VI *Power spectra of cavitation noise:* The noise amplitude is depicted in colors, and the input pressure is increased linearly in time. One observes (with increasing pressure) a subharmonic route $f_0 \rightarrow f_0/2 \rightarrow f_0/4 \ldots \rightarrow$ chaos. This picture is the colored version of Fig. 47c on page 76. (Picture taken by E. Suchla, after Lauterborn and Cramer, 1981.)

VII *The Cassini division:* The ring of Saturn (a) shows a major gap (b), the so-called Cassini division, because the motion on this orbit is unstable (see also Fig. 141 on page 199). (NASA pictures no. P-23068 and P-23207 with permission from Bildarchiv, Baader Planetarium.)

Plates VIII–XV show fractal boundaries in the complex plane:

VIII Newton's algorithm for $f(z) = z^3 - 1 = 0$. The basins of attraction for the three roots of $z^3 = 1$ are shown in red, green and blue (after Peitgen and Richter, 1984; see also pages 140–142).

IX Mandelbrot's set (black) in the complex plane (after Peitgen and Richter, 1984; see also page 143).

X–XII Enlargements of regions A, D, E in Plate IX (after Peitgen and Richter, 1984).

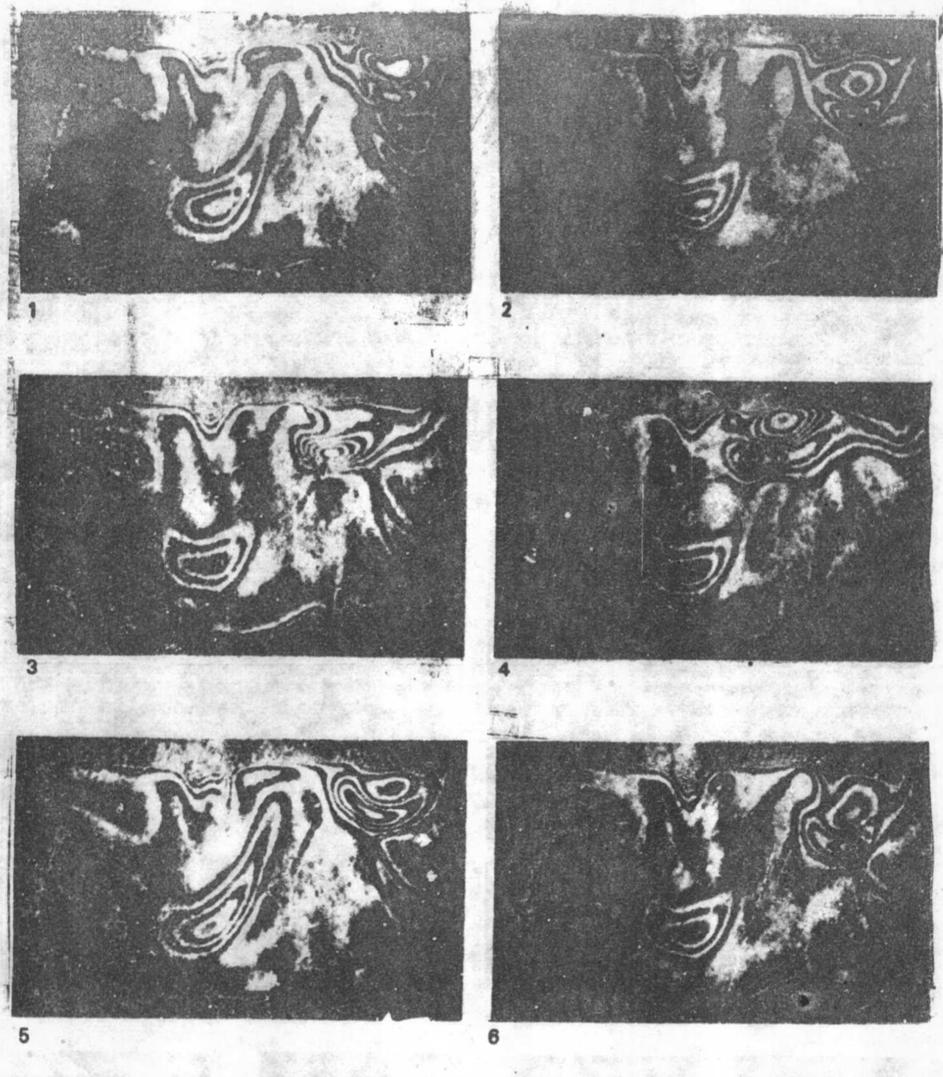
XIII Enlargement of the "tail of the seahorse" in Plate IX (after Peitgen and Richter, 1984).

XIV "Eye of the seahorse" in Plate IX (after Peitgen and Richter, 1984).

XV Detail of the "tail" in Plate IX (after Peitgen and Richter, 1984).

XVI Liapunov exponent λ (depicted in colors) of the circle map ($\theta_{n+1} = \theta_n + \Omega - (K/2\pi)\sin(2\pi\theta_n)$) as a function of the parameters $K = 0\ldots10$ (y-axis) and $\Omega = 0\ldots1$ (x-axis). The arrow indicates the critical line at $K = 1$. (After K. Schmidt, priv. comm.; see also Fig. 106 on page 158.)

XVII Liapunov exponent λ (depicted in colors) of the driven pendulum with additional external torque ($\ddot{\theta} + 1.58\dot{\theta} + K\sin\theta = \Omega\cos(1.76t) + \Omega$) as a function of the parameters $K = 0\ldots20$ (y-axis) and $\Omega = 0\ldots20$ (x-axis). (After K. Schmidt, priv. comm.; see also pages 173–175.)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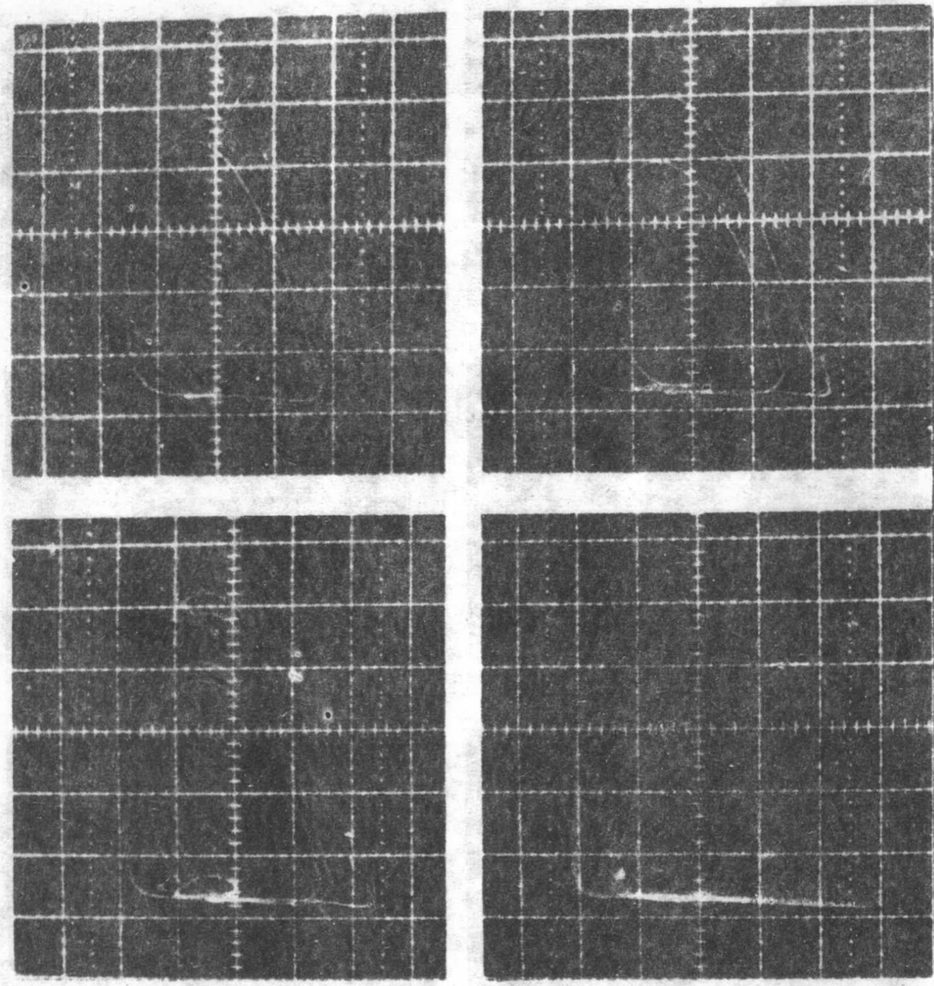

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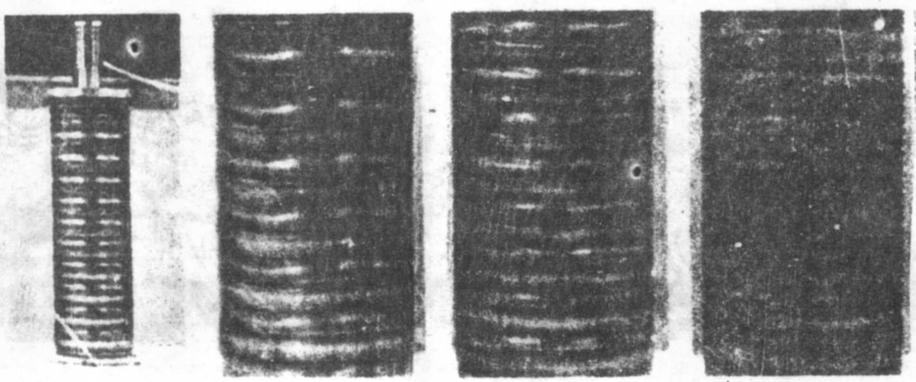

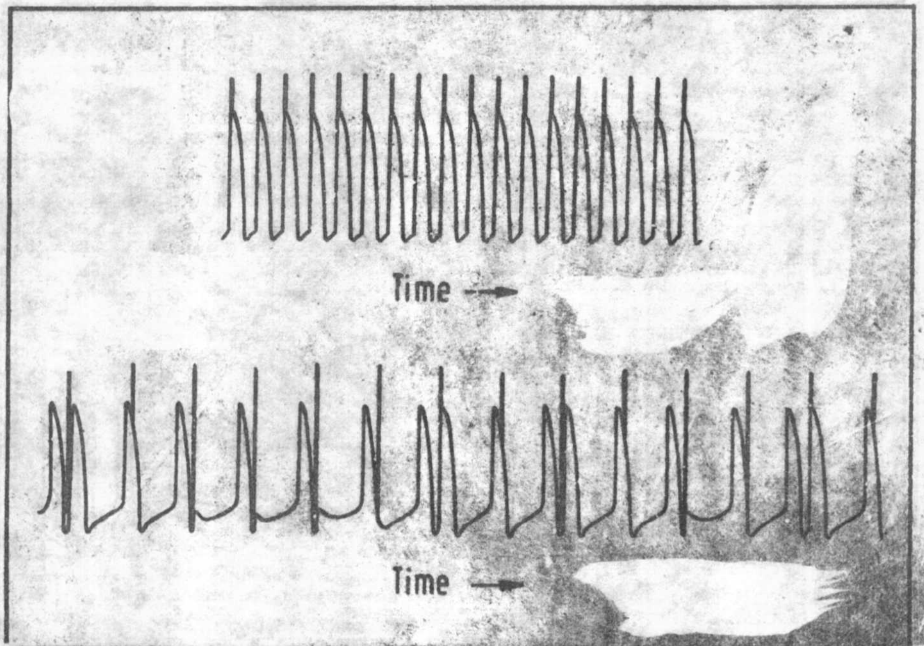
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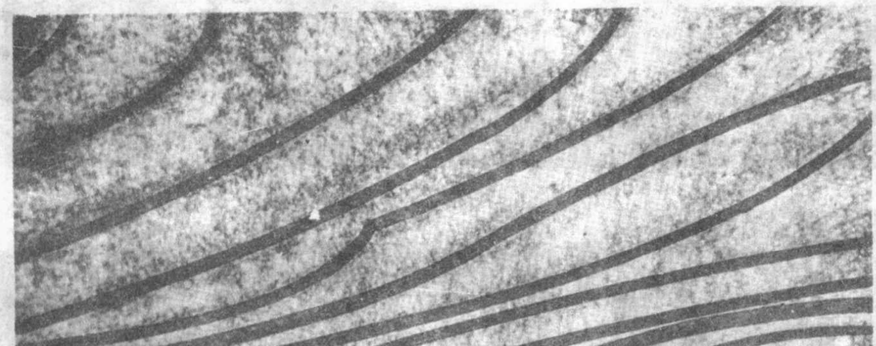

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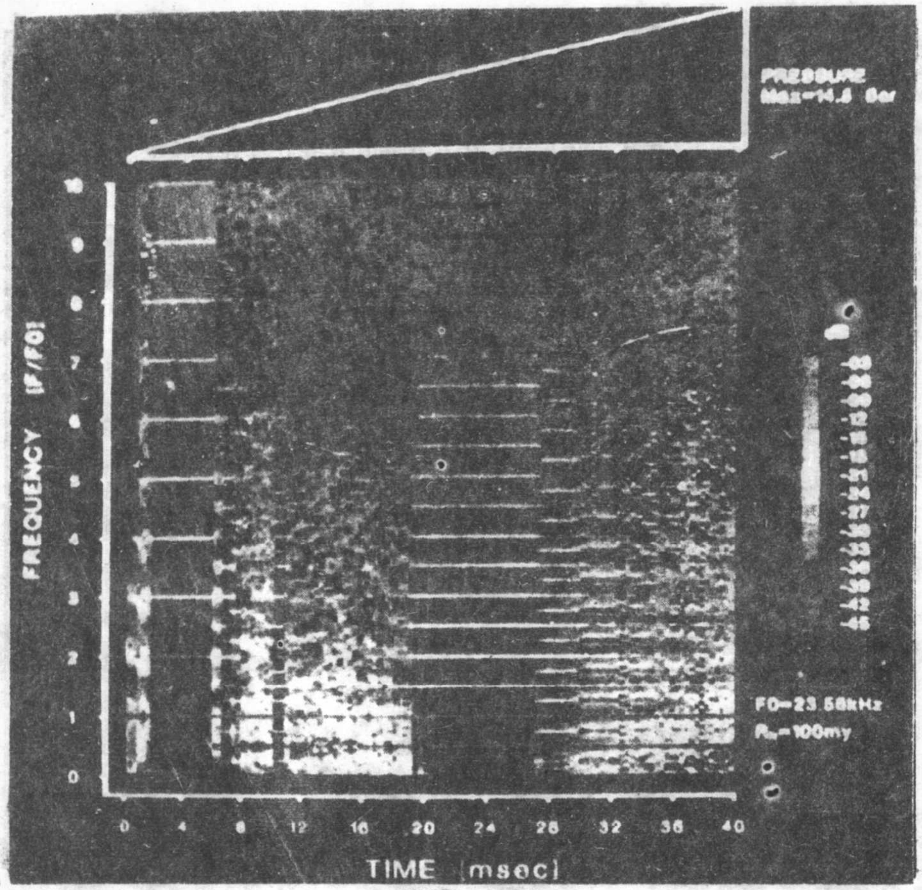
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2
3
4
5
6
7
8
9
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4
8
12
16
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28
32
36
40
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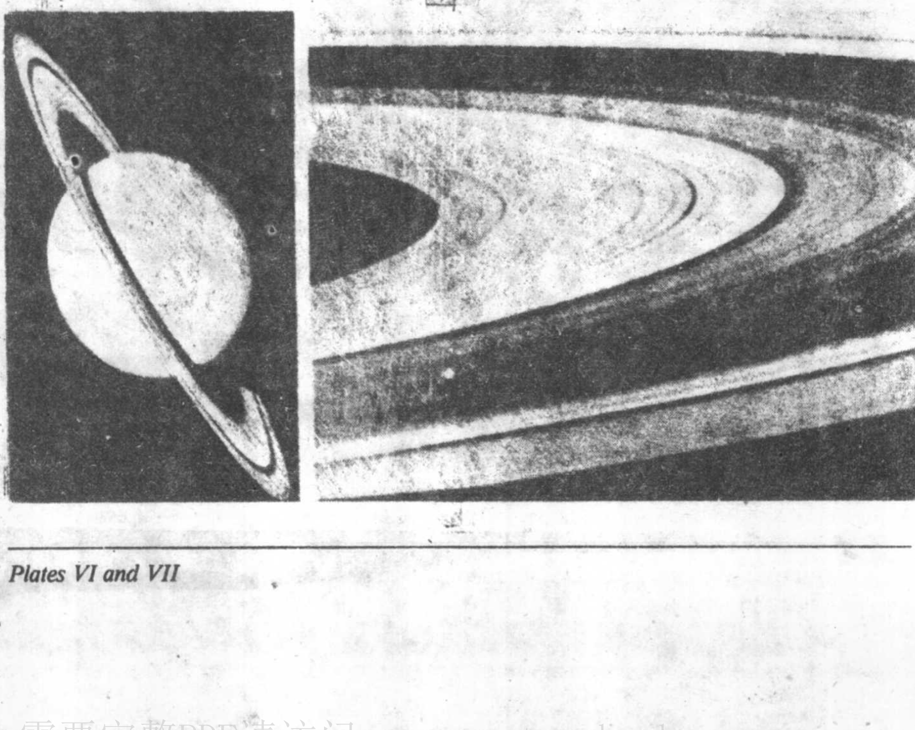
VII



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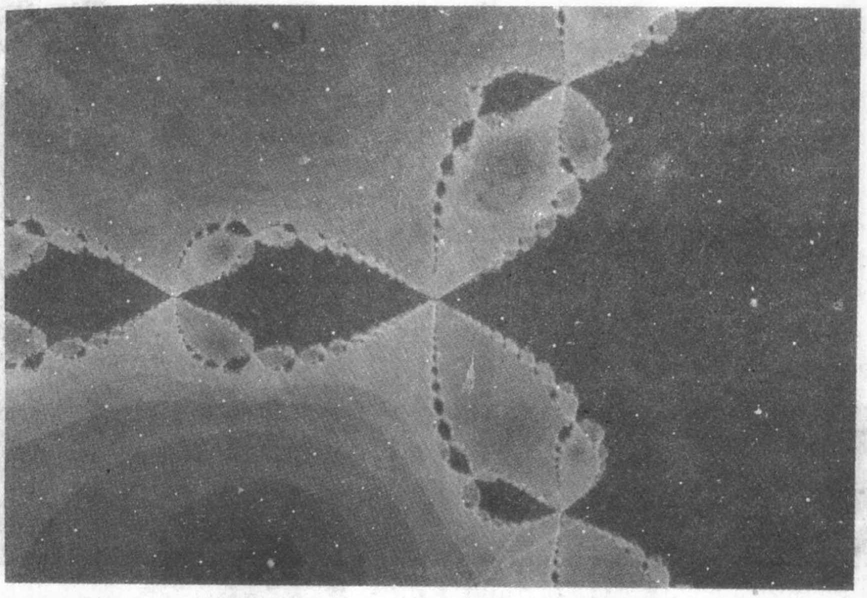

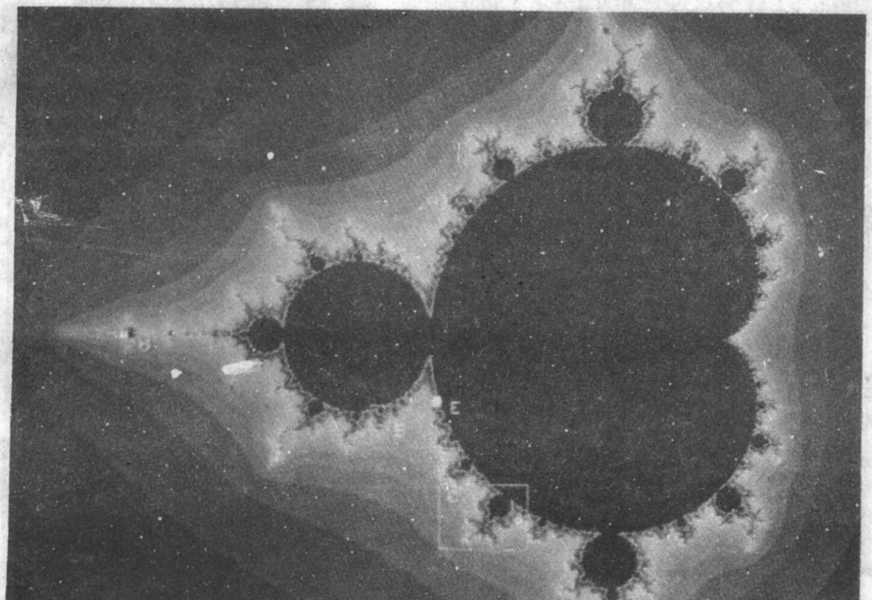

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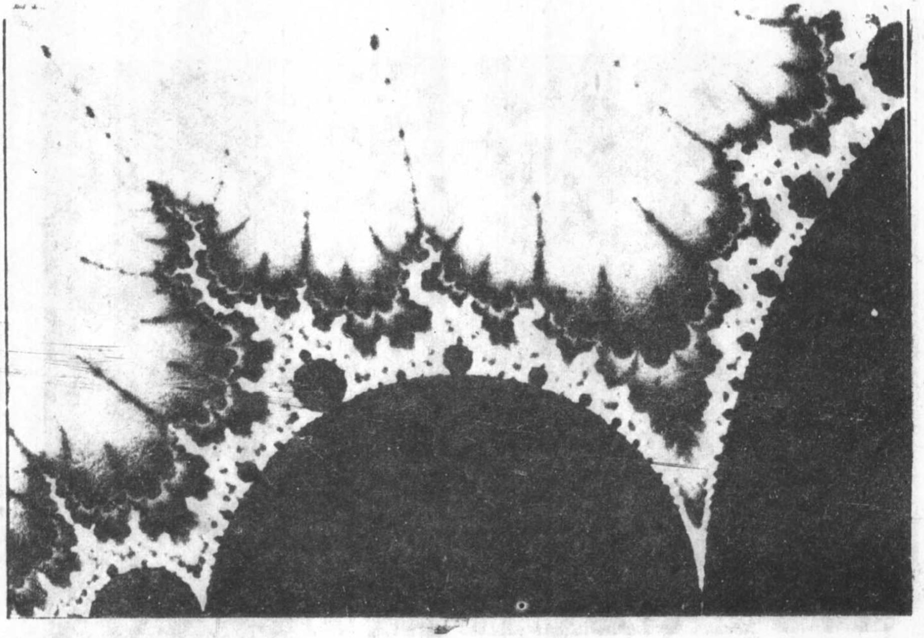

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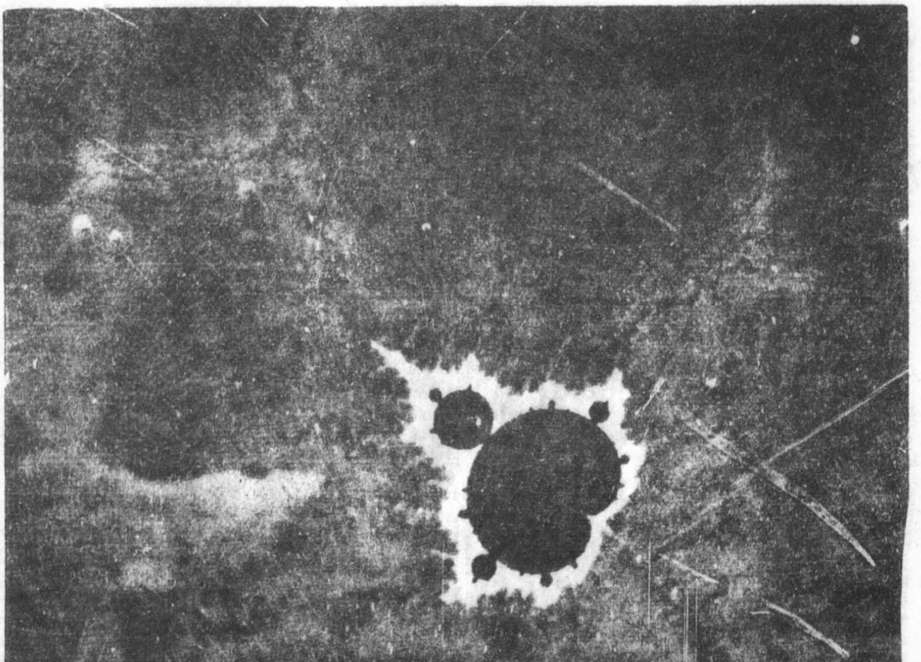
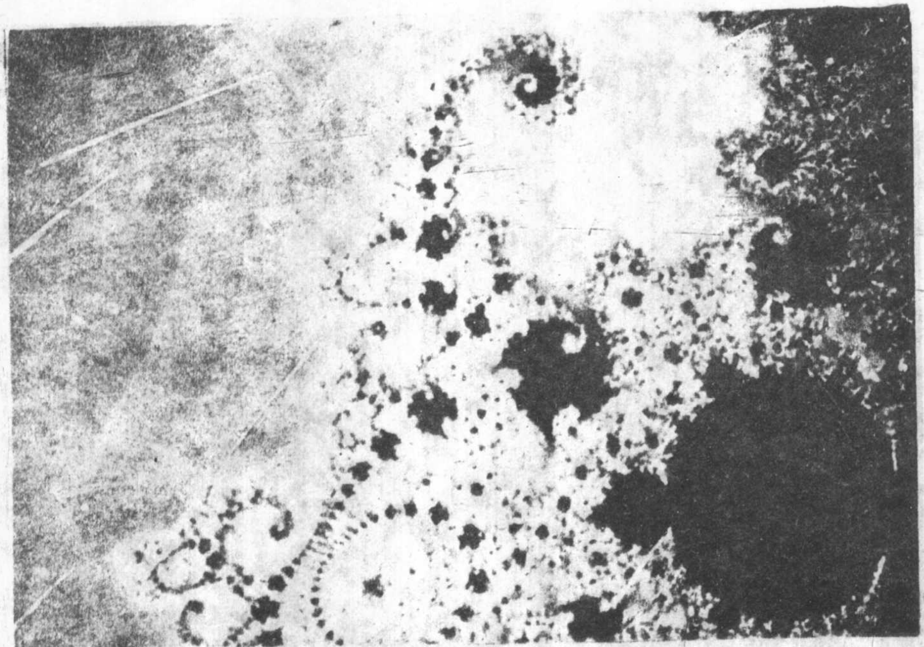


 Plates X and XI

XII



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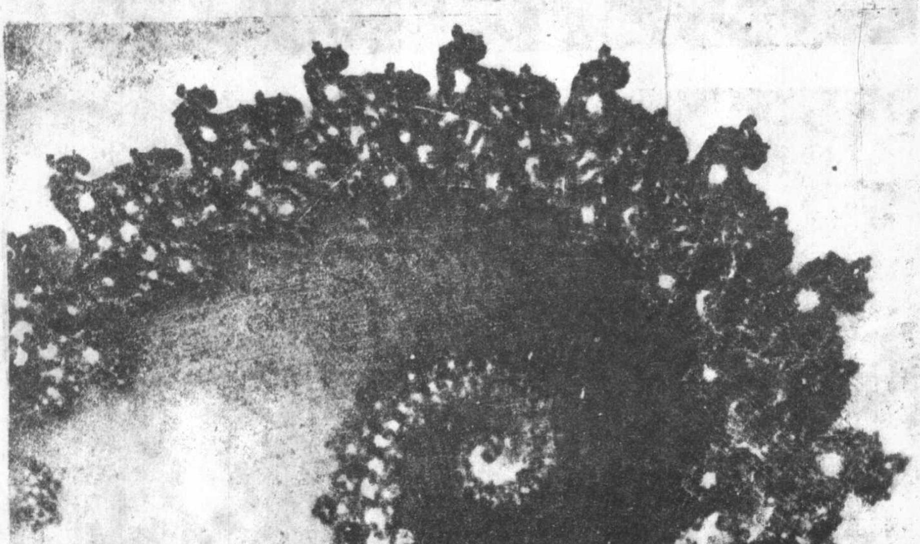

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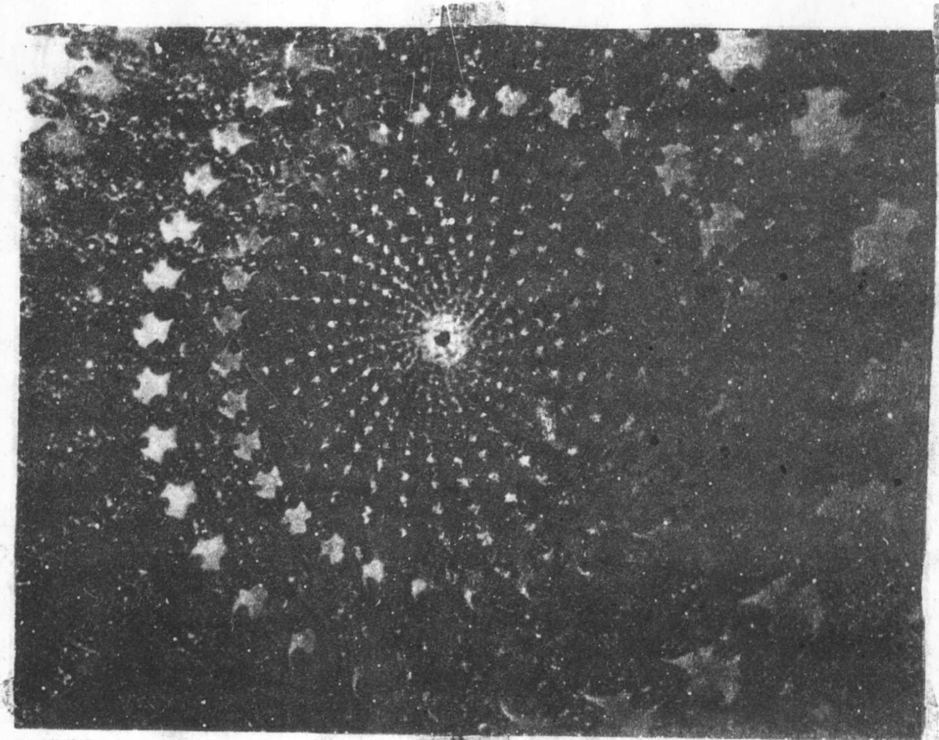

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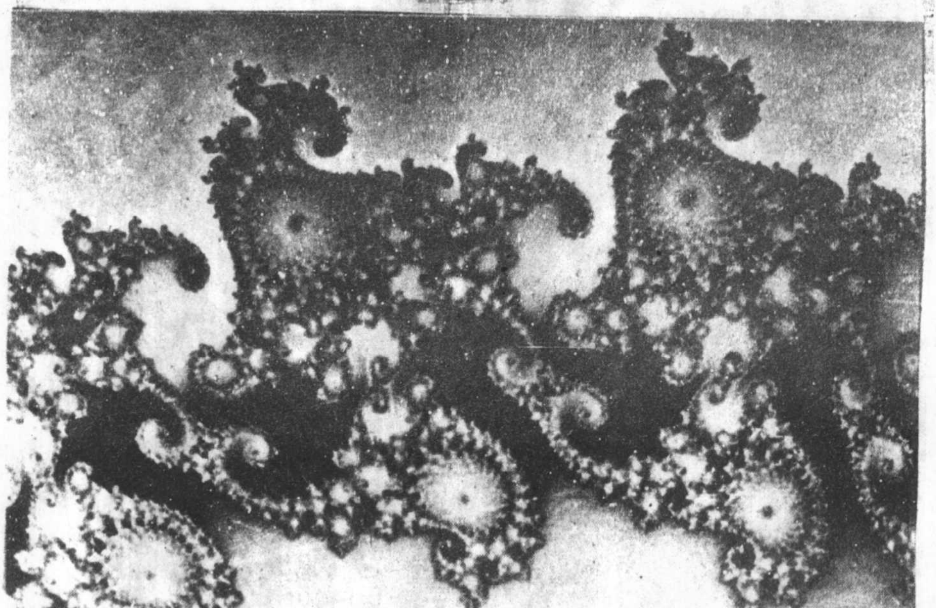
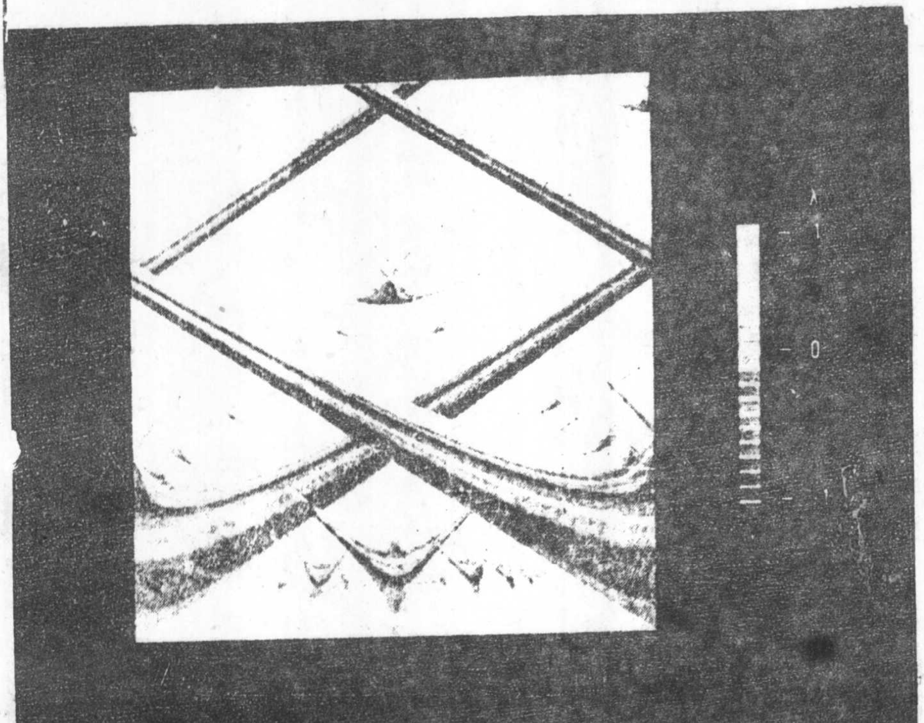


Plates XIV and XV

XVI



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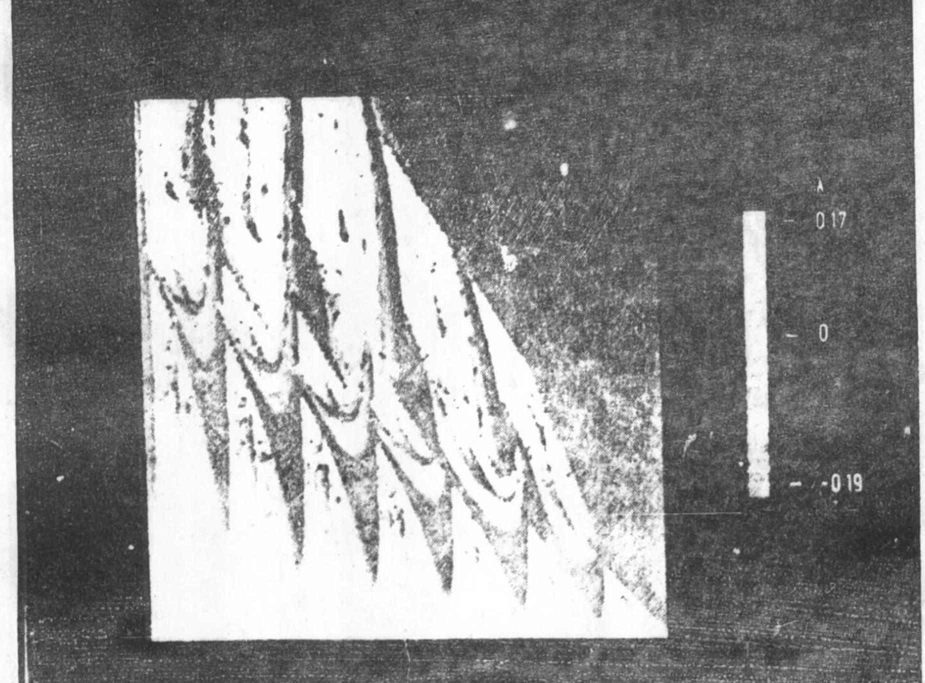

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