PHYSICAL ACOUSTICS

Principles and Methods

Edited by WARREN P. MASON

BELL TELEPHONE LABORATORIES, INCORPORATED,
MURRAY HILL, NEW JERSEY

VOLUME I—PART A

1964

ACADEMIC PRESS
NEW YORK AND LONDON

ACADEMIC PRESS INC.
111 Fifth Avenue, New York 3, New York

United Kingdom Edition published by
ACADEMIC PRESS INC. (LONDON) LTD.
Berkeley Square House, London W.1

LIBRARY OF CONGRESS CATALOG CARD NUMBER: 63-22327

PRINTED IN THE UNITED STATES OF AMERICA

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VOLUME I—PART A

PHYSICAL ACOUSTICS

PRINCIPLES AND METHODS

Volume I Methods and Devices, Part A
Methods and Devices, Part B

Volume II Properties of Gases, Liquids, and Solutions, Part A
Properties of Polymers and Nonlinear Acoustics, Part B

Volume III Applications to the Study of Imperfections and Lattice Dynamics

Volume IV Applications to Quantum and Solid State Physics

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PREFACE

High frequency sound waves in gases, liquids, and solids have proved to be powerful tools for analyzing the molecular, defect, domain wall, and other types of motions that can occur in these media. Furthermore, low and high amplitude waves in these media have considerable device applications. These include such uses as delay lines for storing information, mechanical and electromechanical filters for separating communication channels, ultrasonic cleaning, testing, inspection, measuring, machining, welding, soldering, polymerization, homogenizing, medical diagnosis, surgery, and therapy. Both the analytical and device uses are increasing at an explosive rate. Some of the phenomena analyzed in the last five years are acoustic attenuation due to phonon-phonon interaction, phonon-electron-magnetic field interactions, nuclear-spin and electron-spin interactions with acoustic waves, attenuation caused by the motion of point and line defects (dislocations), as well as such large-scale motions as polymer segments and chains, and domain walls. Hence, it is evident that this general field, which has been given the name of Physical Acoustics, is a powerful investigational tool as well as a source of device application.

Since the field is growing at such a rapid rate it has been thought desirable to produce a series of books which provides an integrated treatment of the techniques, applications, and analytical results obtainable by the use of physical acoustic methods. Since all the applications and analytical uses depend on the tools and techniques used to generate and measure stresses and stress waves in gases, liquids, and solids, this first volume deals with the production, measurement, and application of acoustic waves in these media. As far as the applications go, emphasis has been placed on the physical aspects rather than on the engineering details. However, a complete set of references is provided for such applications. Since a considerable amount of material is required to cover these objectives, this first volume is divided into two parts, A and B. Volume IA covers the propagation of infinitesimal and finite waves in fluids and normal solids, the modifications caused by boundaries, transducers required to generate low and high amplitude waves, methods for measuring the properties of such waves, and their uses in dispersive and nondispersive delay lines as well as in mechanical and electromechanical filters and in the control of the frequencies of oscillators and time standards.

Volume IB specializes on the use of high amplitude waves in liquids and solids and on a new series of semiconductor devices which are receiving wide use in the measurement of pressures, forces, and strains. Very sensitive pressure-measuring devices using transistors provide means for coupling air waves to electrical circuits and hence act as microphones. They provide an amplification of the acoustic energy picked up which is larger than that obtainable with carbon microphones and with a greater efficiency of conversion of dc power input to ac power output. Semiconductor transducers of the depletion layer, diffusion layer, or epitaxial layer type are producing very high frequency devices capable of generating shear or longitudinal waves in the kilomegacycle range. These are of use in device applications and in the fundamental investigation of very rapid liquid and solid state motions. The final chapter considers new methods for producing large motions and strains in solid bodies.

The next three volumes, which are in the process of being written and edited, apply the principles of Volume I to the analysis of molecular interactions in gases, liquids, polymers, and other types of solids and crystals. Volume II discusses the effects and analysis of wave propagation in gases, liquids, solutions, and polymers.

Volume IIIA deals with the effects of point, line (dislocations), and surface (grain boundary) imperfections on the acoustic loss and acoustic velocities in polycrystal and single crystal metals and in insulating crystals. Volume IIIB deals with lattice dynamics, and the final chapter in this part deals with loss mechanisms in that largest solid body, the earth.

Volume IV is devoted primarily to those subjects which contribute to an understanding of solid state physics.

The theories in these volumes are treated in a systematic way and it is hoped that they will be of permanent value even after the topics are further advanced. While the primary purpose is to produce a reference book covering all the principal topics in Physical Acoustics, it is hoped that the books will be useful as advanced texts in graduate schools, or for readers with advanced training who are entering the Physical Acoustics field.

The Editor owes a debt of gratitude to the many contributors who have made the volumes of this treatise possible and to the publishers for their unfailing help and advice.

December, 1963

WARREN P. MASON

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Wave Propagation in Fluids and Normal Solids

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I. Introduction

This chapter develops the basic equations needed to describe waves in fluids and solids and applies them to simple situations in order to illustrate elementary wave phenomena, unobscured by too many complicating effects.

With rare exceptions, the medium in which waves propagate is conveniently regarded as a *continuum*. Even when using waves to probe the molecular or atomic structure of the medium, one often relates the structural parameters to properties of an equivalent continuous medium.

In this chapter we first outline some of the fundamentals of continuum mechanics and then discuss small-amplitude waves in relatively simple situations.

II. Fundamentals of Continuum Mechanics

In the continuum approach, we postulate *fields* of density, stress, velocity, etc. These fields must satisfy the basic conservation laws or *equations of balance* of mass, momentum, angular momentum, and energy. The basic equations of balance apply in any medium. In addition, there are *constitutive relations* which characterize a particular medium. The constitutive relations relate the stress to other variables, specify the flux of nonmechanical energy, and relate thermodynamic variables to each other. Examples are Hooke's law, Newton's law of viscosity, Fourier's law of heat conduction, and the ideal gas equation of state. Special equations for viscous or nonviscous fluids, elastic or viscoelastic solids, etc., may be obtained by substituting appropriate constitutive equations into the basic equations of balance.

A. Material and Spatial Descriptions

Picture a fixed rectangular Cartesian coordinate system with axes x_i, $i = 1, 2, 3$. Any particular position vector $\mathbf{r}$ of components (x_1, x_2, x_3) denotes a point in space. A point which always moves with the material is called a *particle* or *material point*. Lines or surfaces composed of particles are called *material* lines or surfaces. The material inside a closed material surface is called a *body*.

Let every particle be identified by its coordinates at some reference time t_0. These reference coordinates, referred to the same Cartesian system, will be denoted by (a_1, a_2, a_3), and the corresponding position vector by **a**. A particular vector **a** can serve as a name for the particle there at the reference time t_0. The reference values t_0 and **a** will sometimes be called *initial*.

The vectors **r** and **a** both specify position in a fixed Cartesian frame of reference. At any time t, we associate each **r** with an **a** by the rule that **r** is the present position vector of the particle initially at **a**. This connection between **r** and **a** is written symbolically as

$$\mathbf{r} = \mathbf{r}(t, \mathbf{a}) \qquad \text{or} \qquad x_i = x_i(t, a_1, a_2, a_3) \tag{1}$$

where

$$\mathbf{a} = \mathbf{r}(t_0, \mathbf{a}) \qquad \text{or} \qquad a_i = x_i(t_0, a_1, a_2, a_3). \tag{2}$$

The coordinates a_i which identify the particles are called *material* coordinates. A description which like Eq. (1) uses (t, a_1, a_2, a_3) as independent variables is called a *material* description.

The inverses of Eqs. (1) and (2) may be written

$$\mathbf{a} = \mathbf{a}(t, \mathbf{r}) \qquad \text{or} \qquad a_i = a_i(t, x_1, x_2, x_3) \tag{3}$$

where

$$\mathbf{r} = \mathbf{a}(t_0, \mathbf{r}) \qquad \text{or} \qquad x_i = a_i(t_0, x_1, x_2, x_3). \tag{4}$$

A *spatial* description uses the independent variables (t, x_1, x_2, x_3), the x_i being called *spatial* coordinates. When used as independent variables, the x_i's merely specify a point in space. One is frequently interested in a spatial description of pressure or velocity fields but not at all in the initial positions of particles. In such cases, a spatial description would not be pushed to the point of determining the functions a_i in Eq. (3), but would be stopped when the fields of interest are determined.

The terms *Lagrangian* and *Eulerian* have been commonly used for *material* and *spatial*, respectively, even though Euler preceded Lagrange in using both kinds of coordinates (*1*). We follow Truesdell and Toupin (*2*) in using the more descriptive terms material and spatial.

1. *Velocity*

If we fix the material coordinates (a_1, a_2, a_3) in Eq. (1), then (x_1, x_2, x_3) denote the time-dependent coordinates of the particular particle initially at **a**. Now the coordinates of any particular particle depend only on the time, since the vector **a** merely tells which particle is under consideration. Hence, if we limit attention to a single particle, its velocity components are the ordinary time derivatives $\dot{x}_i = dx_i/dt$. But now recalling the presence of other particles, we see that these derivatives are really partial derivatives with the a_i's held constant:

$$v_i = \dot{x}_i \equiv \frac{dx_i}{dt} \equiv \left(\frac{\partial x_i}{\partial t}\right)_{a_1, a_2, a_3} \tag{5}$$

The velocity vector is

$$\dot{\mathbf{r}} \equiv \mathbf{v} \equiv \mathbf{i}_i v_i \qquad (i \text{ summed}) \tag{6}$$

where $\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3$ denote unit vectors in the coordinate directions. In Eq. (6) and hereafter, any term is understood to be summed over all values of any subscript which appears twice. That is, $\mathbf{i}_i v_i$ means

$$\sum_{i=1}^{3} \mathbf{i}_i v_i$$

and $v_j(\partial F/\partial x_j)$ means $\mathbf{v}\cdot\text{grad}\, F$.

2. *Interpretations of* $\oiint \mathbf{v}\cdot\mathbf{n}\,dA$ *and* div v

Let us compute the rate at which material crosses a fixed surface element of unit normal vector **n** and area dA. In a short time interval

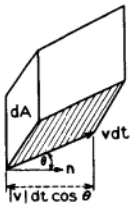

FIG. 1. The meaning of $\mathbf{v}\cdot\mathbf{n}$. $(\mathbf{v}\cdot\mathbf{n})$ is (1) the volume of material crossing a fixed surface, per unit area per unit time, or (2) the volume of space being swept out by a material surface, per unit area per unit time. $\mathbf{v}$ = material velocity; $\mathbf{n}$ = unit vector normal to dA; volume $= |\mathbf{v}|\cos\theta\, dt\, dA = \mathbf{v}\cdot\mathbf{n}\, dA\, dt$.

dt, each particle undergoes a displacement given approximately by $\mathbf{v}\,dt$. Hence, as seen from Fig. 1, the material which crosses the area element from the $-\mathbf{n}$ side to the $+\mathbf{n}$ side in the time interval dt is contained in an approximately prismatic volume element of base area dA and slant height $\mathbf{v}\,dt$. The volume of this prism is $\mathbf{v}\cdot\mathbf{n}\,dA\,dt$, and its mass is $\rho\mathbf{v}\cdot\mathbf{n}\,dA\,dt$, where ρ is the density. The volume flow rate through the element is thus $\mathbf{v}\cdot\mathbf{n}\,dA$ and the mass flow rate is $\rho\mathbf{v}\cdot\mathbf{n}\,dA$. It should be clear that $\mathbf{v}\cdot\mathbf{n}$ and $\rho\mathbf{v}\cdot\mathbf{n}$ are exact expressions for the local rates of volume and mass flow per unit area, even when $\mathbf{v}$ and ρ vary.

Now if **n** denotes the unit normal vector *outward* from a *closed* surface as in Fig. 2, the net volume flow rate out of the surface is $\oiint \mathbf{v}\cdot\mathbf{n}\,dA$ and the net outward mass flow rate is $\oiint \rho\mathbf{v}\cdot\mathbf{n}\,dA$.

By reinterpreting Fig. 1, we see that $\mathbf{v}\cdot\mathbf{n}\,dA\,dt$ is also the volume swept out by a material surface element in the short time interval dt.

The volume swept out is positive if it lies on the $-\mathbf{n}$ side of the moving area element at the end of the time interval.

Figure 3 shows the positions of a closed material surface at times t and $t+dt$. The two positions of the material surface define three regions: a volume (1) which is left behind, a common volume (2), and a

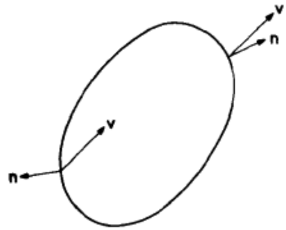

FIG. 2. The net rate at which material flows out of a fixed closed surface is $\oiint \mathbf{v}\cdot\mathbf{n}\,dA$. $\mathbf{v}$ = material velocity; $\mathbf{n}$ = unit normal vector.

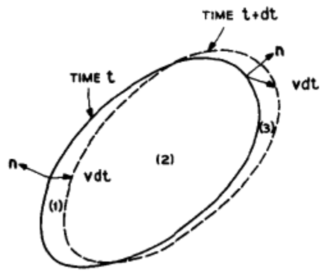

FIG. 3. The net rate of increase of volume inside a closed material surface is $\oiint \mathbf{v}\cdot\mathbf{n}\,dA$. $\mathbf{v}$ = velocity of surface; $\mathbf{n}$ = unit normal vector.

newly acquired volume (3). The volume acquired is obtained by integrating $\mathbf{v}\cdot\mathbf{n}\,dA\,dt$ over that part of the surface for which $\mathbf{v}\cdot\mathbf{n}$ is positive, and the volume left behind is subtracted if the same integral is continued over the remainder of the surface. Hence $\oiint \mathbf{v}\cdot\mathbf{n}\,dA$ is the net rate of increase of volume inside a closed material surface of unit outward normal $\mathbf{n}$, velocity $\mathbf{v}$. In equation form,

$$\frac{d}{dt}\iiint_V dV = \oiint \mathbf{v}\cdot\mathbf{n}\,dA \tag{7}$$

where V denotes the region occupied by the moving body defined by the closed material surface.

We now have two interpretations of $\oiint \mathbf{v}\cdot\mathbf{n}\,dA$: (1) the net volume flow rate out of a fixed closed surface, and (2) the net rate of increase of volume of a body. We obtain two corresponding interpretations of div **v** by applying the divergence theorem

$$\oiint \mathbf{v}\cdot\mathbf{n}\,dA = \iiint \operatorname{div}\mathbf{v}\,dV \tag{8}$$

and noting that the theorem must hold for any arbitrarily small region. These interpretations are (1) the limit, as the volume tends to zero, of the net volume flow rate per unit volume, outward from a fixed volume at the point where div **v** is evaluated, and (2) the rate of expansion per unit volume of a particle at the point where div **v** is evaluated. If dV denotes the time-dependent volume of a small body, the second interpretation says

$$\operatorname{div}\mathbf{v} = \operatorname*{limit}_{dV\to 0}\frac{1}{dV}\frac{d}{dt}(dV). \tag{9}$$

3. *Material Derivative*

It is important to distinguish between the time variation of a field quantity at a fixed point in space and the time variation *following a particle*. The rate of change following a particle is of fundamental importance, and is called the *material derivative*.

We follow a particle by fixing **a**. Hence, in a material description the material derivative is the partial derivative with respect to time. In a spatial description, the material derivative will be denoted by d/dt or by a dot over the variable.

Let the same field be denoted by F in the spatial description, and f in the material description. Then

$$f(t,a_1,a_2,a_3) = F[t, x_1(t,a_1,a_2,a_3), x_2(t,a_1,a_2,a_3), x_3(t,a_1,a_2,a_3)]. \tag{10}$$

By definition, the material derivative of F is

$$\dot{F} \equiv \frac{dF}{dt} \equiv \left(\frac{\partial f}{\partial t}\right)_{a_1,a_2,a_3} \tag{11}$$

To obtain a formula for the material derivative in the spatial description, we differentiate Eq. (10) and use Eq. (11):

$$\frac{dF}{dt} \equiv \frac{\partial f}{\partial t} = \frac{\partial F}{\partial t} + \frac{\partial x_i}{\partial t}\frac{\partial F}{\partial x_i} = \frac{\partial F}{\partial t} + \mathbf{v}\cdot\operatorname{grad} F. \tag{12}$$

The partial derivatives of F are in the spatial description, while those of f and x_i are in the material description. $\partial F/\partial t$ is that part of the