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ACTIVE NETWORK ANALYSIS

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PREFACE

Since Bode published his classical text "Network Analysis and Feedback Amplifier Design" in 1945, very few books have been written that treat the subject in any reasonable depth. The purpose of this book is to bridge this gap by providing an in-depth, up-to-date, unified, and comprehensive treatment of the fundamentals of the theory of active networks and its applications to feedback amplifier design. The guiding light throughout has been to extract the essence of the theory and to discuss the topics that are of fundamental importance and that will transcend the advent of new devices and design tools. Intended primarily as a text in network theory in electrical engineering for first-year graduate students, the book is also suitable as a reference for researchers and practicing engineers in industry. In selecting the level of presentation, considerable attention has been given to the fact that many readers may be encountering some of these topics for the first time. Thus, basic introductory material has been included. The background required is the usual undergraduate basic courses in circuits and electronics as well as the ability to handle matrices.

The book can be conveniently divided into three parts. The first part, comprising the first three chapters, deals with general network analysis. The second part, composed of the next four chapters, is concerned with feedback amplifier theory. The third part, consisting of the last two chapters, discusses the state-space and topological analyses of active networks and their relations to feedback theory.

Chapter 1 introduces many fundamental concepts used in the study of linear active networks. We start by dealing with general n-port networks and define passivity in terms of the universally encountered physical quantities *time* and *energy*. We then translate the time-domain passivity criteria into the equivalent frequency-domain passivity conditions. Chapter 2 presents a useful description of the external behavior of a multiterminal network in terms of the indefinite-admittance matrix and demonstrates how it can be employed effectively for the computation of network functions. The significance of this approach is that the indefinite-admittance matrix can usually be written down directly from the network by inspection and that the transfer functions can be expressed compactly as the ratios of the first-and/or second-order cofactors of the elements of the indefinite-admittance matrix. In Chapter 3 we consider the specialization of the general passivity condition for n-port networks in terms of the more immediately useful two-port parameters. We introduce various types of power gains, sensitivity, and the notion absolute stability as opposed to potential instability.

Chapters 4 and 5 are devoted to a study of single-loop feedback amplifiers. We begin the discussion by considering the conventional treatment of feedback amplifiers based on the ideal feedback model and analyzing several simple feedback networks. We then present in detail Bode's feedback theory, which is based on the concepts of return difference and null return difference. Bode's theory is formulated elegantly and compactly in terms of the first- and second-order cofactors of the elements of the indefinite-admittance matrix, and it is applicable to both simple and complicated networks, where the analysis by conventional method for the latter breaks down. We show that feedback may be employed to make the gain of an amplifier less sensitive to variations in the parameters of the active components, to control its transmission and driving-point properties, to reduce the effects of noise and nonlinear distortion, and to affect the stability or instability of the network. The fact that return difference can be measured experimentally for many practical amplifiers indicates that we can include all the parasitic effects in the stability study and that stability problems can be reduced to Nyquist plots.

The application of negative feedback in an amplifier improves its overall performance. However, we are faced with the stability problem in that, for sufficient amount of feedback, at some frequency the amplifier tends to oscillate and becomes unstable. Chapter 6 discusses various stability criteria and investigates several approaches to the stabilization of feedback amplifiers. The Nyquist stability criteria, the Bode plot, the root-locus technique, and root sensitivity are presented. The relationship between gain and phase shift and Bode's design theory is elaborated. Chapter 7 studies the multiple-loop feedback amplifiers that contain a multiplicity of inputs, outputs, and feedback loops. The concepts of return difference and null return difference for a single controlled source are now generalized to the notions of return difference matrix and null return difference matrix for a multiplicity of controlled sources. Likewise, the scalar sensitivity function is generalized to the sensitivity matrix, and formulas for computing multiparameter sensitivity functions are derived.

In Chapter 8, we formulate the network equations in the time domain as a system of first-order differential equations that govern the dynamic behavior of a network. The advantages of representing the network equations in this form are numerous. First of all, such a system has been widely studied in mathematics and its solution, both analytical and numerical, is known and readily available. Secondly, the representation can easily and naturally be extended to time-varying and nonlinear networks. In fact, nearly all time-varying and nonlinear networks are characterized by this approach. Finally, the first-order differential equations are easily programmed for a digital computer or simulated on an analog computer. We then formulate the general feedback theory in terms of the coefficient matrices of the state equations of a multiple-input, multiple-output and multiple-loop feedback amplifier, and derive expressions relating the zeros and poles of the determinants of the return difference matrix and the null return difference matrix to the eigenvalues of the coefficient matrices of the state equations under certain conditions. Finally, in Chapter 9 we study topological analysis of active networks and conditions under which there is a unique solution. These conditions are especially useful in computer-aided network analysis when a numerical solution does not converge. They help distinguish those cases where a network does not possess a unique solution from those where the fault lies with the integration technique. Thus, when a numerical solution does not converge, it is important to distinguish network instability, divergence due to improper numerical integration, and divergence due to lack of the existence of a unique solution.

The book is an outgrowth of notes developed over the past twenty-five years while teaching courses on active network theory at the graduate level at Ohio University and University of Illinois at Chicago. There is little difficulty in fitting the book into a one-semester or two-quarter course in active network theory. For example, the first four chapters plus some sections of Chapters 5, 6 and 8 would be ideal for a one-semester course, whereas the entire book can be covered adequately in a two-quarter course.

A special feature of the book is that it bridges the gap between theory and practice, with abundant examples showing how theory solves problems. These examples are actual practical problems, not idealized illustrations of the theory. A rich variety of problems has been presented at the end of each chapter, some of which are routine applications of results derived in the text. Others, however, require considerable extension of the text material. In all there are 286 problems.

Much of the material in the book was developed from my research. It is a pleasure to acknowledge publicly the research support of the National Science Foundation and the University of Illinois at Chicago through the Senior University Scholar Program. I am indebted to many graduate students who have made valuable contributions to this book. Special thanks are due to my doctoral student Hui Tang, who helped proofread Chapters 8 and 9, and to my secretary, Ms. Barbara Wehner, who assisted me in preparing the index. Finally, I express my appreciation to my wife, Shiao-Ling, for her patience and understanding during the preparation of the book.

Wai-Kai Chen

Naperville, Illinois
January 1, 1991

CONTENTS

CHAPTER

ONE

CHARACTERIZATIONS OF NETWORKS

Over the past two decades, we have witnessed a rapid development of solid-state technology with its apparently unending proliferation of new devices. Presently available solid-state devices such as the transistor, the tunnel diode, the Zener diode, and the varactor diode have already replaced the old vacuum tube in most practical network applications. Moreover, the emerging field of integrated circuit technology threatens to push these relatively recent inventions into obsolescence. In order to understand fully the network properties and limitations of solid-state devices and to be able to cope with the applications of the new devices yet to come, it has become increasingly necessary to emphasize the fundamentals of active network theory that will transcend the advent of new devices and design tools.

The purpose of this chapter is to introduce many fundamental concepts used in the study of linear active networks. We first introduce the concepts of portwise linearity and time invariance. Then we define passivity in terms of the universally encountered physical quantities *time* and *energy*, and show that causality is a consequence of linearity and passivity. This is followed by a brief review of the general characterizations of n-port networks in the frequency domain. The translation of the time-domain passivity criteria into the equivalent frequency-domain passivity conditions is taken up next. Finally, we introduce the discrete-frequency concepts of passivity. The significance of passivity in the study of active networks is that passivity is the formal negation of activity.

1.1 LINEARITY AND NONLINEARITY

A network is a structure comprised of a finite number of interconnected elements with a set of accessible terminal pairs called *ports* at which voltages and currents may be measured and the transfer of electromagnetic energy into or out of the structure can be made. Fundamental to the concept of a port is the assumption that the instantaneous current entering one terminal of the port is always equal to the instantaneous current leaving the other terminal of the port. A network with n such accessible ports is called an *n-port network* or simply an *n-port*, as depicted symbolically in Fig. 1.1. In this section we review briefly the concepts of linearity and nonlinearity and introduce the notion of portwise linearity and nonlinearity.

Refer to the general representation of an n-port network N of Fig. 1.1. The port voltages $v_k(t)$ and currents $i_k(t)$ (where $k = 1, 2, \ldots, n$) can be conveniently represented by the *port-voltage* and *port-current vectors* as

$$\mathbf{v}(t) = [v_1(t), v_2(t), \ldots, v_n(t)]' \tag{1.1a}$$

$$\mathbf{i}(t) = [i_1(t), i_2(t), \ldots, i_n(t)]' \tag{1.1b}$$

respectively, where the prime denotes the matrix transpose. There are $2n$ port signals, n port-voltage signals $v_k(t)$, and n port-current signals $i_k(t)$, and each port is associated with two signals $v_k(t)$ and $i_k(t)$. The port vectors $\mathbf{v}(t)$ and $\mathbf{i}(t)$ that can be supported by the n-port network N are said to constitute an *admissible signal pair* for the n-port network. Any n independent functions of these $2n$ port signals, taking one from each of the n ports, may be regarded as the *input* or *excitation* and the remaining n signals as the *output* or *response* of the n-port network. In Fig. 1.1 we may take, for example, $i_1(t), i_2(t), \ldots, i_k(t), v_{k+1}(t), \ldots, v_n(t)$ to be the input or excitation signals. Then $v_1(t), v_2(t), \ldots, v_k(t), i_{k+1}(t), \ldots, i_n(t)$ are the

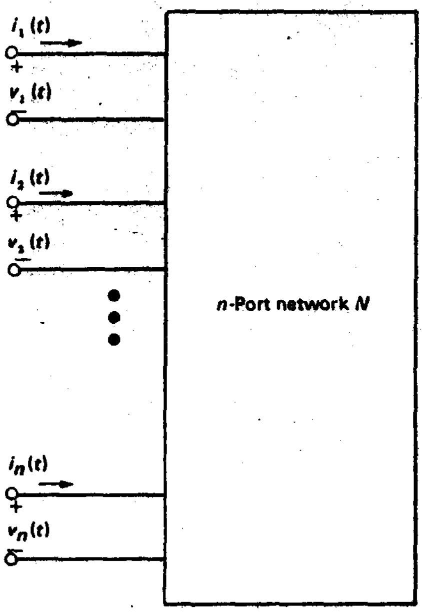

Figure 1.1 The general symbolic representation of an n-port network.

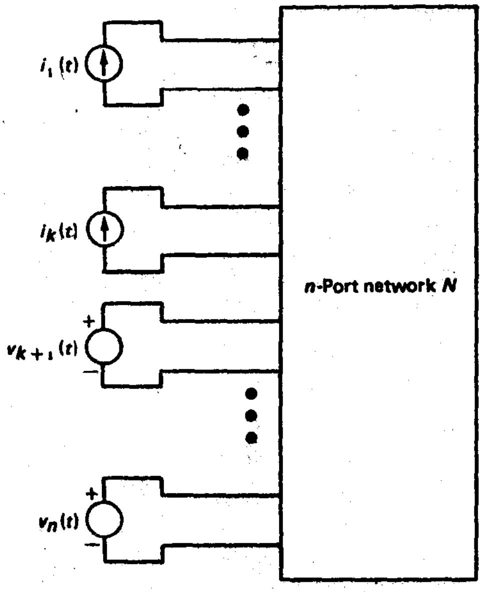

Figure 1.2 A specific input excitation of an n-port network.

output or response signals. This input-output or excitation-response situation is shown in Fig. 1.2. To facilitate our discussion, let $\mathbf{u}(t)$ be the *excitation vector* associated with the excitation signals, and $\mathbf{y}(t)$ the *response vector* associated with the response signals. For the excitation-response situation of Fig. 1.2, the excitation and response vectors are given by

$$\mathbf{u}(t) = [i_1(t), i_2(t), \ldots, i_k(t), v_{k+1}(t), \ldots, v_n(t)]' \tag{1.2a}$$

$$\mathbf{y}(t) = [v_1(t), v_2(t), \ldots, v_k(t), i_{k+1}(t), \ldots, i_n(t)]' \tag{1.2b}$$

respectively. When we speak of zero excitation of an n-port, we mean that every excitation signal is zero; that is, $\mathbf{u}(t) = \mathbf{0}$. On the other hand, a nonzero excitation is meant a set of n excitation signals, not all of them being zero; that is, $\mathbf{u}(t) \neq \mathbf{0}$.

Generally speaking, a network is said to be *linear* if the superposition principle holds. This implies that the response resulting from all independent sources acting simultaneously is equal to the sum of the responses resulting from each independent source acting one at a time. In this sense, any network comprised of linear network elements (linear resistors, linear inductors, linear capacitors, linear transformers, or linear controlled sources) and independent sources is a linear network. Thus, to verify the linearity of a network by this definition, we must have the complete knowledge of the internal structure of the network. For an n-port, the accessible part of the network may be only at its n ports. For this reason, the above definition of linearity may not be adequate for an n-port. For our purposes, we introduce, in addition to the above definition, the notation of portwise linearity.

Definition 1.1: Linearity and nonlinearity An n-port network is said to be *portwise linear* or simply *linear* if the superposition principle holds at its n ports. An n-port is *portwise nonlinear* or simply *nonlinear* if it is not portwise linear.

In other words, if $\mathbf{y}_a(t)$ and $\mathbf{y}_b(t)$ are the responses of the excitations $\mathbf{u}_a(t)$ and $\mathbf{u}_b(t)$ of an n-port, respectively, then the n-port is portwise linear if and only if for any choice of real scalars α and β, the vector $\mathbf{y}(t) = \alpha\mathbf{y}_a(t) + \beta\mathbf{y}_b(t)$ represents the response of the excitation $\mathbf{u}(t) = \alpha\mathbf{u}_a(t) + \beta\mathbf{u}_b(t)$.

The network of Fig. 1.3 is linear in the usual sense. Let us form a one-port from this network as shown in Fig. 1.4. The port voltage and current are described by the equation

$$v(t) = \left(R_1 + \frac{R_2R_3}{R_2 + R_3}\right) i(t) + \frac{ER_2}{R_2 + R_3} \tag{1.3}$$

Suppose that we take $i(t)$ to be the excitation and let $i_a(t) = i_b(t) = 1$ A be two excitations. Assume, for simplicity, that $\alpha = \beta = 1$. Then the corresponding responses $v_a(t)$, $v_b(t)$, and $v_{a+b}(t)$ of the excitations $i_a(t)$, $i_b(t)$, and $i_a(t) + i_b(t)$ are given by

$$v_a(t) = v_b(t) = R_1 + \frac{R_2R_3}{R_2 + R_3} + \frac{ER_2}{R_2 + R_3} \tag{1.4a}$$

$$v_{a+b}(t) = 2R_1 + \frac{2R_2R_3}{R_2 + R_3} + \frac{ER_2}{R_2 + R_3} \tag{1.4b}$$

Since $v_{a+b}(t) \neq v_a(t) + v_b(t)$, the one-port is nonlinear in the portwise sense. Instead of forming a one-port, suppose that we form a two-port from the network of Fig. 1.3. The resulting two-port network is shown in Fig. 1.5; its port voltages and currents are characterized by

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} \tag{1.5}$$

It is straightforward to demonstrate that the two-port is now portwise linear. Thus, a portwise nonlinear network need not contain any nonlinear network elements and can often be rendered portwise linear by extracting internal sources at newly formed ports.

As another example, consider the one-port of Fig. 1.6, in which the capacitor is

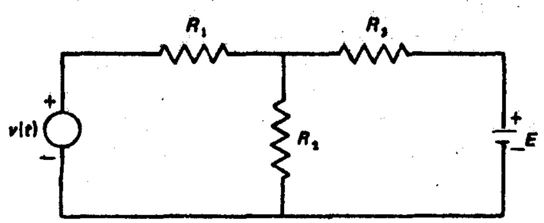

Figure 1.3 A linear network in the usual sense.

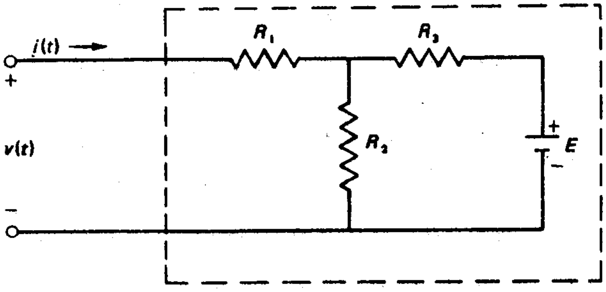

Figure 1.4 A nonlinear one-port network with $E \neq 0$.

initially charged to a voltage $v_C(0+) = V_0$. The terminal relation of the one-port is given by

$$v(t) = Ri(t) + \frac{1}{C}\int_0^t i(x)\,dx + V_0 \tag{1.6}$$

By following Eqs. (1.4), it is easy to confirm that the one-port is portwise nonlinear. Indeed, the presence of any independent sources or any initial conditions on the energy-storing elements in an n-port would render the n-port portwise nonlinear. On the other hand, an n-port network comprised of linear network elements with zero initial conditions and devoid of any independent sources is always portwise linear. For example, in the one-port networks of Figs. 1.4 and 1.6, if the independent source E and the initial voltage V_0 are set to zero, the resulting one-ports become portwise linear.

From the examples discussed above, it is clear that a portwise nonlinear n-port need not contain any nonlinear elements, but the presence of nonlinear elements does not necessarily imply that the n-port is portwise nonlinear. Figure 1.7 is a one-port comprised of two nonlinear resistors connected in series. The nonlinear resistors are characterized by the equations

$$v_\alpha(t) = i_\alpha(t) - i_\alpha^2(t) \tag{1.7a}$$

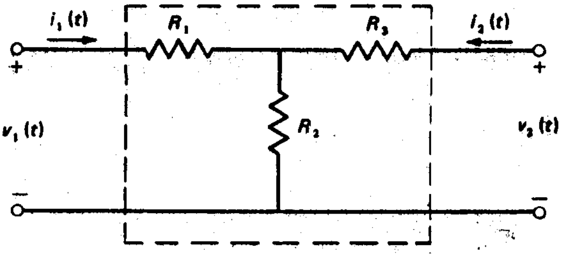

Figure 1.5 A linear two-port network.

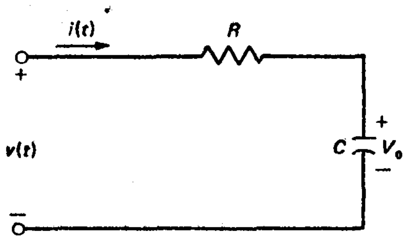

Figure 1.6 A nonlinear one-port network with nonzero initial capacitor voltage.

$$v_\beta(t) = i_\beta^2(t) \tag{1.7b}$$

The port voltage and current are related by the equation

$$v(t) = i(t) \tag{1.8}$$

showing that this one-port is equivalent to a 1-Ω resistor and thus is portwise linear. Suppose that a two-port is formed from this one-port by connecting two wires across one of the resistors as shown in Fig. 1.8. The resulting two-port becomes portwise nonlinear.

We emphasize the difference between the linearity of a network and the portwise linearity of an n-port. Throughout the remainder of this book, we are concerned mainly with portwise linearity. For simplicity, the word portwise will usually be dropped, as also indicated in Definition 1.1, and will be used only for emphasis.

1.2 TIME INVARIANCE AND TIME VARIANCE

A network is said to be *time-invariant* if it contains no time-varying network elements. Otherwise, it is called a *time-varying network*. Like those discussed in the preceding section, if the port behavior of a network is the major concern, the above definition may not be adequate for an n-port. For this reason, we define portwise time-invariance.

Definition 1.2: Portwise time-invariance and time variance An n-port network is said to be *portwise time-invariant* or simply *time-invariant* if, for every real finite constant τ,

$$\mathbf{u}_a(t) = \mathbf{u}_b(t - \tau) \tag{1.9a}$$

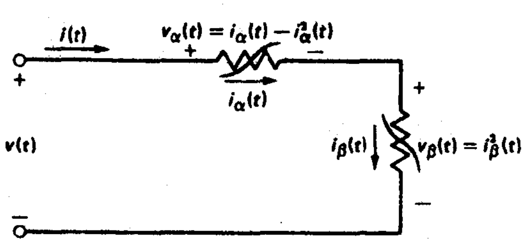

Figure 1.7 A linear one-port network comprised of two nonlinear resistors.