

APPLIED CIRCUIT THEORY

Matrix and Computer Methods

P. R. Aaby

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APPLIED CIRCUIT THEORY: Matrix and Computer Methods

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Preface

During the last ten years the computer has become an indispensable tool of electrical and electronic circuit analysis and design. Every engineer must expect to use a computer at all stages of circuit design, from specification through to manufacture, with the familiarity that was formerly accorded to the slide rule and calculator. One recent trend in the growth of computation is the increasing availability of computers away from computer centres. Minicomputers and microcomputers now provide 32K of memory for less than £1000 (\$2000), and every design laboratory, however small, can run significant analysis programmes at very low cost. Also, the programmes themselves have become more accessible to individuals for adaptation and improvement. Graphic outputs can easily be incorporated, and specialised programmes can be more readily written.

A modern circuit analysis programme running on a large computer achieves accuracy and efficiency by use of sophisticated numerical techniques. This book establishes the circuit theory on which such programmes are founded, and it provides an introduction to recent research in computer-aided circuit design without becoming overburdened with numerical methods. Emphasis throughout the text is on matrix methods and computation. Engineers in industry, and postgraduate students in engineering and physics, will therefore find the book particularly suitable if they have not previously followed this approach to circuit analysis. The book also fills the gap between fundamental circuit theory and computer-aided circuit design with an intermediate treatment, and is therefore also suitable for second and third year undergraduate courses in electronics and electrical engineering in English universities and polytechnics, and equivalent courses elsewhere. Knowledge of matrix algebra and basic circuit theory is assumed, including circuit theorems, sinusoidal steady-state analysis, and the Laplace transform.

Undergraduate courses in circuit theory have undergone a radical change to the computational approach in response to the widespread availability of computers and to the increasing complexity of circuits. One important conse-

quence is that implementation in conjunction with laboratory work can be achieved early in the course. The student quickly becomes concerned with transistors, operational amplifiers, and other active devices both in practice and through circuit theory. This process motivates him with an increased awareness of electronic circuit design, and a familiarization with devices and their circuit models, which is of immense unifying benefit.

The treatment of circuit analysis in this book is predominantly theoretical. To retain adequate coverage of the mass of available material, examples have been restricted to relatively simple circuits. In undergraduate courses it is essential to balance theory with extensive application to practical circuits. Motivation to apply theoretical techniques through laboratory work and design projects rests with the instructor. He is in the best position to provide more extensive and realistic circuit examples which are suited to analysis as well as fitting in with the remainder of the course.

The role of the computer in undergraduate courses has been much debated. The view taken at King's College is that the computer should be used to achieve experience with all possible aspects of circuit behaviour in as wide a range of circuits as possible. Experimentation by computer simulation is encouraged throughout the course, although the underlying objective is the comparison of computed results with design and laboratory measurements. Students are not expected to write analysis programmes, but they do have to write short sub-routines, and they frequently have to determine why analysis does not work. Computing problems are included at the end of each chapter, and students become involved in some of them. Problems vary from straightforward circuit investigations to requests for programmes. The use of time-shared BASIC for large analysis programmes, even if restricted to small circuits when implemented on a minicomputer, has been more than justified by the integration of computation into the course and by enthusiastic student participation.

Five main computer programmes are included in the text: they cover d.c. analysis, a.c. analysis, two-port analysis, transfer function analysis, and random simulation. The programmes may be freely reproduced and used by instructors, and ASCII coded paper tapes are available on request. It is inevitable that some errors remain in the programmes; a regularly updated list of errors and programme improvements will be available from the author in the form of a users' newsletter. Many of the programmes requested in the problems should also become available. More extensively modified and improved versions of these programmes adapted to other computer systems and different implementations of BASIC will be available from Dayton Electronics Ltd, South Way, Newhaven, Sussex, England.

This book has had the benefit of use in manuscript form by engineering students at the University of London King's College. I would like to thank them for their helpful comments and criticism. I would also like to thank Miss Rosemary Ainsworth, Mrs. Margaret Richards and Mrs. Jean Hynes for typing the

text. Finally, I would like to acknowledge my debt to the pioneers of computer-aided circuit analysis and design, whose work I have freely used and adapted to produce a coherent subject for study. Above all else this book rests on their research.

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King's College, London, 1979

