Mechanics of Solids

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Computer Programs

Preface

There are only a few basic principles to be mastered in the study of the subject variously referred to as the mechanics of solids, mechanics of materials or strength of materials. It is the application of these principles to the solution of problems, and the choice of assumptions which must be made, which present the greatest challenge. This text is intended for the first course in mechanics of solids offered to engineering students. It concentrates on developing analysis techniques from basic principles for a range of practical problems which includes simple structures, pressure vessels, beams and shafts. Many worked examples are given. The arrival of computers in general, and personal computers in particular, has revolutionized the way in which engineering problems are solved in practice, and this is being reflected in the way in which subjects such as the mechanics of solids are taught. A distinctive feature of the present book is therefore the inclusion of a number of computer techniques and programs for carrying out the analyses – not merely as appendices, but integrated into the text. The programs will also find many applications in the teaching of design.

It is not intended that the use of computer programs should replace hand calculations in the learning process, but should supplement them, and make it possible for the student to explore more complex and realistic problems of analysis and design. The approach adopted is therefore first to present the underlying theory and the traditional manual methods of solution before introducing computer techniques. The programs, which are coded in FORTRAN 77, are suitable for personal computers, but can also be run on minicomputers or mainframes. Detailed internal and external documentation is provided to aid the understanding of the programs, together with examples of their use. It is intended that students should use them to solve many of the problems which are set at the end of each chapter, and in their design work.

It is assumed that students using this book will have some experience of elementary statics (mechanics of rigid bodies), although Chapter 1 includes a review of the relevant topics. A knowledge of matrix notation for the presentation of linear algebraic equations is also assumed. Some familiarity with the solution of constant coefficient ordinary differential equations (particularly for the buckling problems considered in Chapter 8) is highly desirable, as is experience of partial differentiation and integration if Chapter 10 on more advanced applications is to be studied. Those numerical analysis techniques which are incorporated in the computer programs, particularly for the solution of simultaneous linear algebraic equations and single nonlinear algebraic equations, are described in Appendices. It is assumed that students will have sufficient knowledge of the FORTRAN programming language to be able to read and understand relatively straightforward programs.

There are many ways in which a first course on the mechanics of solids can be presented. The approach adopted here, based on the author's experience of teaching the subject, is to start with types of problems involving uniform stresses. Initially such stresses are *uniaxial*, as *in pin-jointed structures*, progressing to biaxial and even triaxial, but without shearing, as in thin-walled pressure vessels. Statically determinate situations, which require only the consideration of equilibrium conditions for the forces and stresses to be found, are treated before statically indeterminate ones. Problems involving relatively simple variations of stresses are then examined, principally the bending of beams and the torsion of shafts. Finally, an introduction to more complex situations is provided via the analysis of two-dimensional states of stress and strain, failure criteria and the differential equations of equilibrium and compatibility in two dimensions.

In addition to a review of statics, Chapter 1 introduces the concepts of stress and strain in a solid body, the influence of material properties and the principles of the mechanics of solids. These principles are those of equilibrium of forces, compatibility of strains and the stress–strain characteristics of materials, underlying themes which run through the remainder of the book. In Chapter 2, some statically determinate systems are analyzed, in particular pin-jointed structures, thin cylindrical and spherical shells, and flexible cables. A computer program is introduced for the analysis of statically determinate pin-jointed structures, using a simple form of finite element method. Stress–strain relationships for engineering materials are discussed in Chapter 3, and are used to find the deformations of statically determinate systems considered in the previous chapter. Some types of statically indeterminate systems are examined in Chapter 4, notably pin-jointed structures (by finite element computer method), liquid-filled pressure vessels, and problems involving resisted thermal expansion.

Chapters 5 and 6, which form a major part of the book, are concerned with beams and the simple theory of bending. While Chapter 5 deals with shear forces, bending moments and stresses, and the analysis of statically determinate beams (including a finite element computer method), Chapter 6 is concerned with beam deflections, leading to the analysis of statically indeterminate beams (and another finite element method). In Chapter 7, problems of torsion of circular shafts are considered. Following an introduction to problems of instability, Chapter 8 deals with the buckling of struts and columns.

In Chapter 9, attention moves away from problems involving only simple states of mainly uniaxial stresses, towards more complex situations. Transformations of stress and strain components acting on different planes at a point lead to the definition of principal and maximum shear values. A computer method is introduced for analyzing stresses or strains at a point, as is a program for determining the state of strain and stress at a point from strain gage measurements. Criteria for yielding and fracture under complex states of stress are then examined. Finally, Chapter 10 develops the principles of equilibrium and compatibility into the partial differential stress equilibrium and strain compatibility equations for problems involving general one- and two-dimensional variations of stresses and strains. These are applied to beam problems, and serve to demonstrate the levels of approximation involved in the simple theory of bending. In a one-dimensional form suitable for axisymmetric problems they are also applied to rotating disks and thick-walled cylinders used as pressure vessels, and a computer method is introduced for the determination of stresses and strains in compound thick-walled cylinders.

The coverage of topics provided by the text may well be greater than that required for particular courses. For example, not all instructors would wish to deal so fully with pin-jointed structures, although the solution of more realistic problems is so much more practical using computer techniques, and they do provide a natural introduction to finite element methods which students will meet in later courses. Also, Chapter 10, and perhaps parts of Chapter 9, may be more appropriate for a more advanced course. The introduction of computer techniques has meant that some more traditional methods have been omitted. For example, graphical methods for the analysis of pin-jointed structures are not considered. Similarly, the only manual method described for the determination of beam deflections involves integration of the moment–curvature equation: the more graphical moment–area method is not covered. Other topics which have been omitted, but which are only rarely covered to any significant depth in a first course, are energy methods and an introduction to plasticity and the analysis of elastoplastic bending and torsion problems.

Some of the computer techniques are described in the text as finite element methods. This is because they are the kinds of essentially simple techniques which led to the birth of finite element methods. It is useful to describe them in finite element terms, not least because it provides a good preparation for the introduction of more sophisticated finite element methods in later courses.

Many engineering courses have now converted entirely to SI metric units. In the United States, however, US customary (Imperial) units are still widely used, reflecting industrial practice. Consequently, in this book both sets of units are employed. In worked examples, given numerical data and the main calculated results are usually shown first in SI units, followed in parentheses by equivalent values in US customary units. The equivalence is not intended to be exact, and values are normally quoted to only two significant figures. The intention is to provide those readers less familiar with SI units with a better feel for the magnitude of the quantities involved. In many examples, detailed calculations in both sets of units are shown, side by side. The material property data presented in Appendix A are given in both SI and US customary units, together with the appropriate conversion factors.

A substantial number of problems is provided at the end of each chapter, first a set in SI units and then a set in US customary units. Where appropriate, problems are also grouped under topic headings, and within each group they are graded in difficulty. While the elementary problems involve only straightforward application of the methods described in the text, some of the more difficult ones are more open-ended and of the design type. The necessary material properties are in most cases not given in the problems, but are provided in an appendix (Appendix A). Answers to alternate problems are listed at the end of the book, and worked solutions to all problems are contained in a separate instructor's manual.

Magnetic disk copies of all the computer programs can be obtained from the publisher. Programs may be freely copied, used, modified or translated. Although they have been carefully tested, they may contain errors, and I would appreciate being informed of any that are found. While the programs were developed specifically for teaching purposes, they may find applications in the solution of real problems. If they are used for this purpose, I will not be responsible for any errors they may contain.

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List of Symbols

The symbols which are used throughout the book are defined in the following list. In some cases, particular symbols have different meanings in different parts of the book, although this should not cause any serious ambiguity. Other symbols or alternative definitions of the present symbols representing, for example, dimensions of components or constants of integration are introduced within the limited contexts of particular examples or pieces of analysis.

FORTRAN variable names used in the computer programs are defined in the lists associated with the programs.

A	Area (especially cross-sectional area)
$[A]$	Coefficient matrix
b	Breadth
C	Couple
C_1 to C_5	Constants in a finite element shape function
c_1, c_2	Distances from neutral surface to highest and lowest points of a beam cross section
D	Diameter
d	Depth
E	Young's modulus (modulus of elasticity)
e	Normal strain
e	Eccentricity
e_{vol}	Volumetric strain
e_Y	Yield strain
F	Force
$[F]$	Vector of applied external loads
F_r	Body force per unit volume in the r direction
f	Flexibility
$f(\,)$	Function; step or singularity function
$[f]$	Vector of loads on an element
G	Shear modulus (modulus of rigidity)
g	Acceleration due to gravity
H	Horizontal component of force
h	Height
I	Second moment of area of a beam cross section about its neutral axis
i, j	Node numbers
J	Polar second moment of area of a shaft cross section about its axis

K	Stress concentration factor; bulk modulus; radius ratio for a thick-walled cylinder
$[K]$	Overall stiffness matrix
k	Stiffness
$[k]$	Element stiffness matrix
L	Length
L_e	Effective length of the equivalent pin-ended strut
M	Moment; bending moment
m	Element number; modulus ratio
N	Number (of elements, reactions, supports etc. – indicated by appropriate subscripts); moment
n	Buckling parameter defined in equation (8.5)
P	Force
P_e	Euler critical buckling force
p	Pressure; perimeter
Q	Force; first moment of area of the region of a beam cross section above a given distance from the neutral axis
R	Reaction force at a support; radius; radius of curvature of the neutral surface of a beam
r	Radial coordinate in the cylindrical polar system; radius of gyration
S	Elastic section modulus
T	Force in a cable or member of a structure; temperature; torque
$[T]$	Vector of member forces
t	Wall thickness
U, V	Forces in the x and y directions
u, v, w	Displacements in the x, y and z directions
V	Vertical component of force; shear force; volume
$[V]$	Vector of forces and moments
W	Weight
$[W]$	Vector of applied external loads
w	Weight per unit length (of a beam or cable)
X, Y	Global Cartesian coordinates; body forces per unit volume in the x and y directions
x, y, z	Cartesian coordinates
z	Axial coordinate in the cylindrical polar system
α	Angle; coefficient of linear thermal expansion
β	Coefficient of volumetric thermal expansion
γ	Shear strain
Δ	Change of (followed by another symbol)
δ	Small increment of (followed by another symbol)
δ	Displacement; radial interference
$[\delta]$	Vector of displacements
ε	Natural strain
θ	Angular coordinate in the cylindrical polar system; angle
λ	Extension ratio; rotational stiffness
ν	Poisson's ratio

ρ	Density
σ	Normal stress
σ_e	Von Mises equivalent stress
σ_H	Hydrostatic stress
σ_U	Ultimate tensile stress
σ_Y	Yield stress
τ	Shear stress
ϕ	Angle
ω	Angular velocity

Contents

1

Introduction

As engineers we are concerned with solving problems, which may involve not only the engineering sciences but also related subjects such as economics and management science. The ability to solve problems can be gained in two main ways: by practical experience of particular problems and the systematic study of underlying principles. Although both are necessary for the practising engineer, the study of principles leads more rapidly to a genuine understanding, and makes it possible to tackle new problems not previously met. For convenience, the total subject matter of engineering science is usually subdivided into a number of topics, such as solid mechanics, fluid mechanics, heat transfer, properties of materials, and so on, although there are close links between them in terms of the physical principles involved and the methods of analysis employed.

Solid mechanics as a subject is usually further subdivided into the *mechanics of rigid bodies* (sometimes just *mechanics*) and the *mechanics of (deformable) solids*, also known as *mechanics of materials* and *strength of materials*. While the mechanics of rigid bodies is concerned with the static and dynamic behavior under external forces of engineering components and systems which are treated as infinitely strong and undeformable, the mechanics of solids is more concerned with the internal forces and associated changes in geometry of the components involved. Of particular importance are the properties of the materials used, the *strength* of which will determine whether the components fail by breaking in service, and the *stiffness* of which will determine whether the amount of deformation they suffer is acceptable. The subject of mechanics of solids is therefore central to the whole activity of engineering design.

Let us consider the situation illustrated in Fig. 1.1, which shows a person standing on a ladder which has one end on a horizontal surface, and the other resting against a vertical wall. While mechanics (of rigid bodies) would be concerned with whether the ladder will slip, in mechanics of (deformable) solids we are more interested in whether the ladder is strong enough to support its human load, and whether it is stiff enough to make that load feel secure! We must expect the ladder to suffer some deflection, which will be greatest near its center. If it is correctly designed, however, this deflection will be relatively small and *recoverable*. In other words, when the load is removed, the ladder returns to its original straight form. Such behavior is referred to as *elastic* deformation. On the other hand, if it is not well designed, the ladder may suffer large deflections, with possibly gross local deformations at some position along its length, rather like a hinge being formed, and would not return to its original form when it is unloaded. Such behavior, involving large and permanent

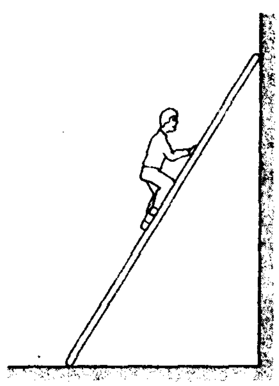

Fig. 1.1.

changes in geometry is referred to as *plastic* deformation. Alternatively, if the material used is *brittle*, the ladder may deflect by a relatively small amount before suffering *fracture* into two pieces. In the context of ladder design, both permanent deformation and fracture must be regarded as modes of *failure*.

Finding the answers to the questions as to whether an engineering system, in this case the ladder, is strong enough and stiff enough involves two main steps. Firstly, we must model the system, and then apply appropriate physical principles to determine the forces and displacements involved. Let us now examine these steps in more detail.

1.1 Modeling of engineering systems

Choosing a suitable model for a system is a matter of making reasonable assumptions in order to simplify the real system far enough to permit it to be analyzed without an excessive amount of labor, but without at the same time simplifying it so far as to make the results of the analysis unreliable for design and other purposes. Successful modeling requires knowledge, experience and good physical insight into the way solid components behave and interact. It is usually the most important, and often the most difficult, part of a solid mechanics analysis.

The importance of knowledge and experience in modeling and analyzing engineering systems is reflected in this book by the many Examples which are described and worked through in detail in the text, together with the Problems provided at the end of each chapter. To some extent, many of these problems are artificial in the sense that the underlying assumptions are implicit, and the data presented in unambiguous forms. In problems encountered in engineering practice, the greatest difficulties often arise in deciding what assumptions are reasonable, and then obtaining sufficient data on which to base a solution. There is little to be gained, however, in attempting problems of the latter type until experience is gained solving problems of the former type.

An example of system modeling is provided by the ladder arrangement shown in Fig. 1.1. The foot of the ladder may well rest, not on a rigid surface, but on soft ground. The rounded end will therefore penetrate the surface, giving a finite area of contact, and a distribution of reaction force acting on the ladder, as shown in Fig. 1.2a. Now, unless we are particularly interested in what happens at the very end of the ladder, which we are not when considering

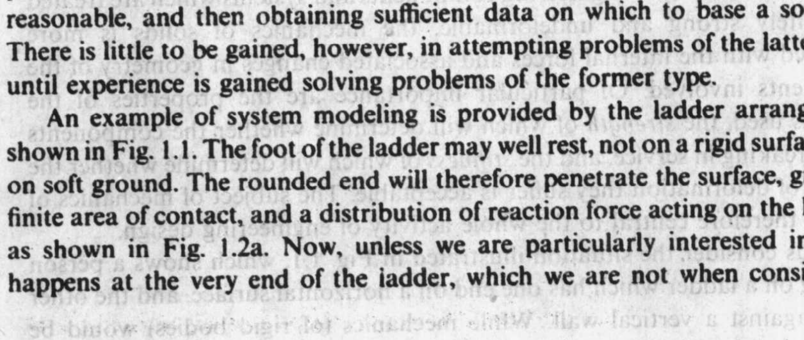

Fig. 1.2.

strength and stiffness of the ladder as a whole, we can model the ladder as a line element of negligible width subject to a single force at its lower end, as shown in Fig. 1.2b. The force is of course the *resultant* of the actual distribution of forces shown in Fig. 1.2a, acting in the appropriate direction, which in general is not along the line of the ladder. Now, instead of defining the total force acting on the foot by the resultant and its direction, we could equally well define the components of this force in two convenient perpendicular directions, such as the horizontal and vertical, along and at right angles to the ground as shown in Fig. 1.2c. In each case two, and only two, pieces of information are required to specify the force vector.

1.2 Review of statics

The types of problems we will be concerned with are either static, involving no motion, or are subject to dynamic loads which do not vary with time, and the physical principles we will need to employ include those of statics. The following notes are intended to summarize those topics in statics which are assumed to be already familiar to the reader.

1.2.1 Some definitions

1 A state of *equilibrium* is a state of no acceleration, in either the translational or rotational senses.
2 A *scalar* is a quantity having only a magnitude. Examples include mass and temperature.
3 A *vector* is a quantity having both magnitude and direction, and satisfies the parallelogram rule of veotor addition. Examples include displacement and force.
4 A *force* is that interaction between bodies which tends to impart an acceleration or to deform. The interaction can occur either through direct contact of the bodies or remotely, as in the case of gravitational attraction, which is responsible for the weight of an object.
5 A *moment* is the product of the magnitude of a force and the (perpendicular) distance of its line of action from a particular point. Moment is also a vector quantity. Indeed, it is the vector product of the force and distance, and its direction is therefore normal to the plane containing the force and the point.
6 A *couple* consists of two forces equal in magnitude but opposite in direction whose lines of action are parallel but not colinear. The couple has the same moment about all points in the plane containing the two forces, equal to the product of the magnitude of either of the forces and the perpendicular distance between them. The resultant of the two forces is zero.

In order to understand the idea of a couple more clearly, consider the plane body shown in Fig. 1.3, which is subject to two forces, each of magnitude F but opposite in direction, acting in the plane of the body and applied to points A and B. The lines of action of these forces are parallel and at a perpendicular distance h apart. The resultant force acting on the body is zero. Let us consider an arbitrary point P in the plane of the body, but not necessarily within the

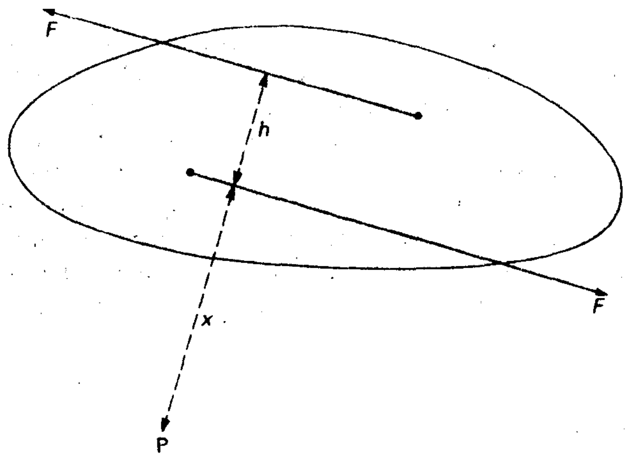

Fig. 1.3.

body itself, this point being at a perpendicular distance x from the nearer of the two forces, as shown. If we calculate moments about point P, the nearer of the two forces has a clockwise moment of magnitude Fx. As a vector, this moment is perpendicular to, and directed into, the plane of the body shown. On the other hand, the force further from P has a counterclockwise moment about it of $F(x+h)$ perpendicular to, and directed out of, the plane shown. The total moment, in the counterclockwise direction, is therefore

$$F(x+h)-Fx=Fh \tag{1.1}$$

This result is independent of the distance x, and therefore of the position of point P.

1.2.2 Laws of motion

Newton's three laws of motion, which are relevant to both dynamics and statics, can be expressed for our purposes as follows.

1 If there is no external force or moment acting on a body, then the body experiences no acceleration.
2 An external force acting on a body produces an acceleration in the direction of the force, the force being equal to the product of the mass of the body and the acceleration.
3 The force exerted by one body B_1 on another B_2 is equal in magnitude and opposite in direction to the force exerted by B_2 on B_1.

The distinction between a force which is external to a body, and one which is internal and acting between different parts of it, is an important one. The reference to external force or moment in the first law really means *resultant* external force or moment: while there may be several external forces and moments acting on a body, provided they have no resultants the body will experience no acceleration. In the context of statics, this means that the body