Mechanics of fluids and transport processes

J.A. Sparenberg

Elements of hydrodynamic propulsion

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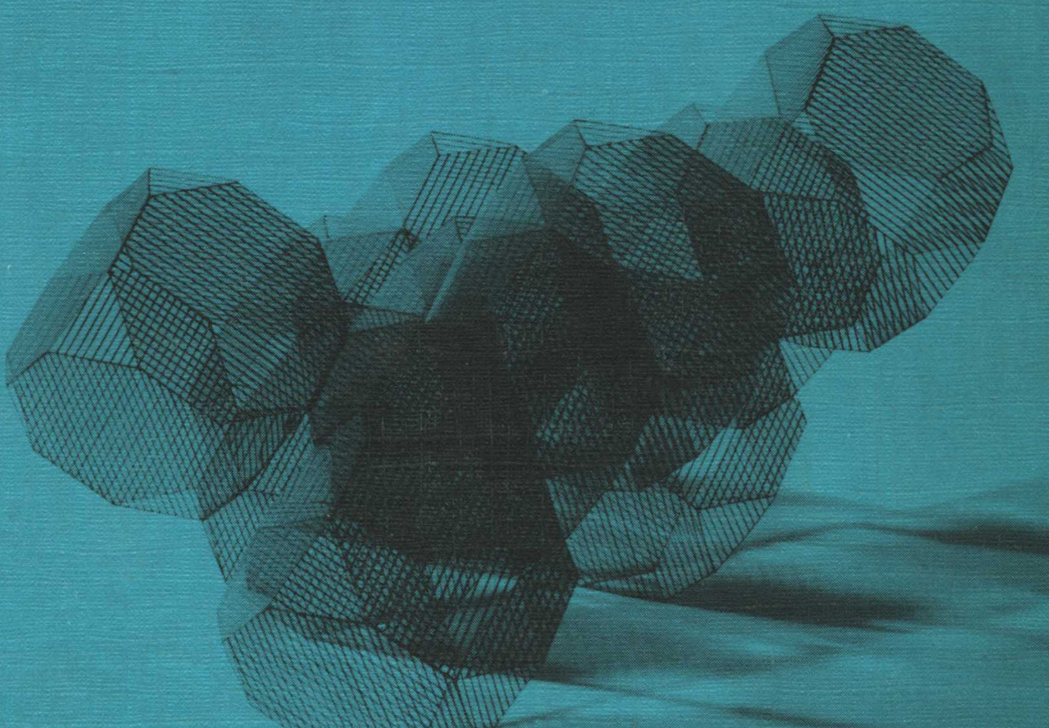

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Elements of hydrodynamic propulsion

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Preface

This is a treatment of a number of aspects of the theory of hydrodynamic propulsion. It has been written with in mind technical propulsion systems generally based on lift producing profiles.

We assume the fluid, which is admitted in conventional hydrodynamics, to be incompressible. Further we assume the occurring Reynolds numbers to be sufficiently high such that the inertia forces dominate by far the viscous forces, therefore we take the fluid to be inviscid. Ofcourse it must be realized that viscosity plays an important part in a number of phenomena displayed in real flows, such as flow separation at the nose of a profile and the entrainment of fluid by a ship's hull. Another approximation which will be used in general is that the problems are linearized. In other words it is assumed that the induced disturbance velocities are sufficiently small, such that their squares can be neglected with respect to these velocities themselves. Hence it is necessary to evaluate the domain of validity of the results with respect to these two a priori assumptions. Anyhow it seems advisable to have first a good understanding of the linearized non-viscous theory before embarking on complicated theories which describe more or less realistic situations. For elaborations of the theory to realistic situations we will refer to current literature.

In low Reynolds number flow, singular external forces and moments are very useful. It is one of the objectives of this book to promote the use of external force fields also in the case of incompressible and inviscid fluids as an expedient to generate velocity fields. Although in most text books external force fields appear in the equations of motion, usually it is assumed that they have an impulsive character or that they are the gradient of a potential function. In the latter case they have lost, in relation to incompressible fluids the ability of inducing velocities, they only change the pressure field. An interesting feature of non conservative external force fields is that they can generate vorticity in an inviscid fluid. By this we have no need in a discussion about the origin of vorticity, to make use of a slight viscosity which afterwards is abandoned again. Using external force fields the concepts of for instance pressure dipole and actuator disk, arise in a natural way from the integration of the equations of motion.

Another objective of the book is to describe a linearized optimization theory for propellers or more generally for systems of lifting surfaces. The theory applies ro rather general types of force actions, for instance to steady and to unsteady propulsion. It is assumed however that the lifting surfaces form angles with the direction of the desired force, which are not small. An exception is as we will show, the calculation of the optimum thrust of the sails of a yacht sailing close to wind. This problem can be reformulated as a problem of energy extraction and in this way it comes under the theory described here.

We mentioned already that in this treatment viscosity has been neglected and that we have to be careful with the interpretation of the results. This especially holds with regard to optimization theory, for this we refer to the introduction to chapter 5.

We also discuss the existence or non existence of optimum propulsion systems for a number of types. We do this mainly for the case of unsteady propulsion. It turns out that in some classes of admitted propellers, optimum propellers do exist and in other classes they do not. In the latter case it does not mean that the admitted class can not contain propellers with a high efficiency. On the contrary, a non-existence proof can be based on the fact that it is possible to construct a minimizing sequence in the considered class of propellers for which the loss of energy per unit of time theoretically tends to zero. However this will occur in general at the cost of wilder and wilder motions so that no acceptable propeller comes out in the limit procedure. Hence the non existence of an optimum propeller means only that we cannot construct an algorithm to find within the considered class a propeller with least energy losses.

For the proof of the existence of optimum propellers it seems that the abstract methods of functional analysis are unavoidable. The reason is that it has to be proved that the lost kinetic energy per unit of time of the propeller is a functional with some desired properties on some given set of motions, such that this functional assumes its minimum at one of the motions of the set.

The choice of the subjects and examples in this book, reflects the field of research of the author and his collaborators, it is not claimed that a complete survey of hydrodynamic propulsion theory is given. For instance slender body propulsion which is of importance in the biological sciences is not treated here. Also cavitation which is often (not always) an undesired phenomenon in propulsion is not considered.

We assume in this monograph the reader to be familiar with a number of basic concepts of hydrodynamics and with their application to wing theory. We mention the velocity potential, the streamfunction, Bernoulli's theorem, the concept of linearization, the law of Biot and Savart, the lifting line and the Kutta condition. Subjects which are fundamental for some types of propulsion, such as the unsteady suction force at the leading edge of a profile are discussed. Also the well-known trailing vorticity of a lifting surface is treated as an illustration of the use of external force fields.

Groningen J.A. SPARENBERG

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Parts of the book have been read by some colleagues, of whom I mention Paul Urbach and Willem Potze, and by some students. My thanks are due to them for tracking down mistakes, and for the remaining ones the author has to be blamed.

Finally I am indebted to Martinus Nijhoff Publishers for offering me the opportunity to write this monograph.

J.A. Sparenberg

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Appendices

1. External force actions

A body moving through a fluid, which we assume to be incompressible and inviscid, will induce velocities and pressures in this fluid. Hence the body will experience forces and moments caused by the integrated action of the pressures on its boundary. Inversely by the law, action equals reaction, the body will exert forces on the fluid. Sometimes these force actions are accompanied by the shedding of vorticity as in the case of a lifting surface of finite span, sometimes there is no vortex shedding as in the case of the accelerated motion of a sphere, where in both cases we assume that no flow separation occurs. In the first part of this chapter we will consider this vorticity shedding of a body more closely.

Next we consider the pressures and velocities induced by an external force field acting directly on the fluid, hence without the intermediary of a body. These considerations are mainly based on a linearized theory. It is discussed that force fields which are conservative are not of much interest in propulsion theory, these fields induce only pressures and no velocities. We calculate the work done per unit of time by an external force field. This gives for instance a possibility to calculate the induced resistance of a lifting surface.

A special case of an external force field is the singular force moving in one way or another through the fluid. The velocity field of a singular force yields the kernel function for a number of problems in lifting surface and actuator surface theory. We will determine this velocity field by means of limit considerations. For a mathematical discussion of the highly singular velocity field the theory of distributions should be used. This however complicates the reasoning to a large extent while it is less easy to recognize the simple physics behind it. It is shown by Urbach [64] that our results agree with those of the theory of distributions.

The vorticity induced by external force fields can be divided into bound vorticity and free vorticity. We will demonstrate that such a denomination is often subject to arbitrariness.

We conclude the chapter with the discussion of the suction force at the leading edge of a profile without thickness. This force is of importance for the calculation of the thrust delivered by a profile carrying out a small amplitude motion.

1.1. Hydrodynamic forces on a moving body

We will discuss here some general results which describe the forces and moments exerted by a fluid on a moving body. Our discussion will be restricted to the derivation of some basic formulas needed in the next section. For a more elaborate treatment of forces on rigid bodies we refer to [3] and [41].

Consider a body B of finite extent moving in an inviscid and incompressible fluid. In this fluid we have a Cartesian coordinate system (x, y, z) with respect to which the fluid at infinity is at rest. During its motion the body is allowed to change its shape and volume. We assume that no vorticity is shed into the fluid. Hence the fluid motion is irrotational and its velocity field $\boldsymbol{v}$ with components v_x, v_y and v_z in the x, y and z direction, can be derived from a potential function $\Phi = \Phi(x, y, z, t)$ at all points of space outside the body

$$\boldsymbol{v} = \text{grad}\,\Phi. \tag{1.1.1}$$

Because the velocity field is free of divergence we have

$$\text{div}\,\boldsymbol{v} = \Delta\Phi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Phi = 0. \tag{1.1.2}$$

Consider around B a control surface $\tilde{H}$ which is coupled to the fluid particles, hence it floats with the fluid. To the fixed amount of fluid in the region $\tilde{\Omega}$ between $\tilde{H}$ and the boundary ∂B of B we can apply the theorem of momentum. This states: the resultant force exerted on an

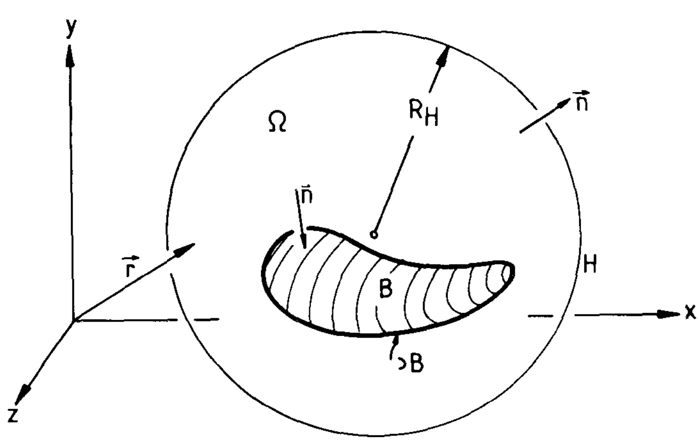

Fig. 1.1.1. Body with control surface H.

amount of fluid equals the change of its momentum per unit of time. For the formulation of the problem however it is more easy to replace $\tilde{H}$ by a surface H fixed in space and to consider the region Ω bounded by H and ∂B. But then we have to add to the rate of change of momentum of the fluid in Ω, the flux of momentum leaving Ω through H and to subtract the incoming flux. For H we take a sphere with sufficiently large radius R_H and with its centre in the neighbourhood of B.

The momentum $\boldsymbol{I} = \boldsymbol{I}(t)$ of the fluid in the region Ω is

$$\boldsymbol{I} = \mu \int_\Omega \operatorname{grad} \Phi \, \mathrm{dVol} = \mu \int_{\partial B + H} \Phi \boldsymbol{n} \, \mathrm{d}S, \tag{1.1.3}$$

where μ is the density of the fluid and the unit normal $\boldsymbol{n} = (n_x, n_y, n_z)$ points out of the region Ω. We want to calculate the force $\boldsymbol{F} = \boldsymbol{F}(\mathrm{t})$ exerted by the fluid on the body B during its motion. We also introduce the force $\boldsymbol{F}_H = \boldsymbol{F}_H(t)$ exerted by the fluid outside H at the fluid inside H. Then we can write the balance of momentum as

$$\boldsymbol{F}_H - \boldsymbol{F} = \mu \frac{\mathrm{d}}{\mathrm{d}t} \int_{\partial B + H} \Phi \boldsymbol{n} \, \mathrm{d}S + \mu \int_H \boldsymbol{v}(\boldsymbol{v} \cdot \boldsymbol{n}) \, \mathrm{d}S, \tag{1.1.4}$$

where the last term is the mentioned momentum flux through H.

The force $\boldsymbol{F}_H$ can be written as

$$\boldsymbol{F}_H = - \int_H p \boldsymbol{n} \, \mathrm{d}S. \tag{1.1.5}$$

Using Bernoulli's equation for unsteady motion

$$p + \tfrac{1}{2}\mu |\boldsymbol{v}|^2 + \mu \frac{\partial \Phi}{\partial t} = p_\infty + \tfrac{1}{2}\mu |\boldsymbol{v}_\infty|^2, \tag{1.1.6}$$

where p_∞ is the pressure at infinity and $\boldsymbol{v}_\infty$ the velocity at infinity, we write (1.1.5) as

$$\boldsymbol{F}_H = \mu \int_H \left\{ \frac{\partial \Phi}{\partial t} + \tfrac{1}{2} |\boldsymbol{v}|^2 \right\} \boldsymbol{n} \, \mathrm{d}S. \tag{1.1.7}$$

In (1.1.7) we used the fact that for any sufficiently smooth closed surface H

$$\int_H \boldsymbol{n} \, \mathrm{d}S = 0. \tag{1.1.8}$$

This vector equality follows from the repeated application of the scalar equality

$$\int_H \{ f_1(y,z)n_x + f_2(x,z)n_y + f_3(x,y)n_z \} \, \mathrm{d}S = 0, \tag{1.1.9}$$

which holds for "arbitrary" functions f_i $(i = 1, 2, 3)$ of the indicated arguments.

Because H is at rest with respect to our coordinate system we have

$$\int_H \frac{\partial \Phi}{\partial t} \boldsymbol{n} \, \mathrm{d}S = \frac{\mathrm{d}}{\mathrm{d}t} \int_H \Phi \boldsymbol{n} \, \mathrm{d}S. \tag{1.1.10}$$

Substitution of (1.1.7) and (1.1.10) into (1.1.4) yields

$$\boldsymbol{F} = -\mu \frac{\mathrm{d}}{\mathrm{d}t} \int_{\partial B} \Phi \boldsymbol{n} \, \mathrm{d}S + \mu \int_H \{ \tfrac{1}{2} |\boldsymbol{v}|^2 \boldsymbol{n} - \boldsymbol{v}(\boldsymbol{v} \cdot \boldsymbol{n}) \} \, \mathrm{d}S. \tag{1.1.11}$$

Next we consider the limit of (1.1.11) when the radius R_H of H tends to infinity. At large distances the velocities induced by B tend to zero as R_H^{-2}. This happens when B changes its volume, otherwise the induced velocities tend to zero more quickly. Anyhow the contribution of the integral over H in (1.1.11) tends to zero for $R_H \to \infty$. This means that

$$\boldsymbol{F} = -\mu \frac{\mathrm{d}}{\mathrm{d}t} \int_{\partial B} \Phi \boldsymbol{n} \, \mathrm{d}S. \tag{1.1.12}$$

In an analogous way we can derive a result for the moment $\boldsymbol{M} = \boldsymbol{M}(t)$, caused by the hydrodynamic pressures at ∂B. This moment will be calculated with respect to the origin O. We apply the theorem: the sum of the moments about O of external forces acting at an amount of fluid equals the change of the moment of momentum about O per unit of time of that amount of fluid.

The moment of momentum about O of the fluid in Ω is

$$\mu \int_\Omega (\boldsymbol{r} * \boldsymbol{v}) \, \mathrm{dVol} = \mu \int_{\partial B + H} \Phi \cdot (\boldsymbol{r} * \boldsymbol{n}) \, \mathrm{d}S, \tag{1.1.13}$$

where $*$ denotes the vector product and the equality follows by partial integration. When we denote by $\boldsymbol{M}_H = \boldsymbol{M}_H(t)$ the moment about O of the hydrodynamic forces at H exerted by the fluid outside H, the just mentioned theorem assumes the form

$$\boldsymbol{M}_H - \boldsymbol{M} = \mu \frac{\mathrm{d}}{\mathrm{d}t} \int_{\partial B + H} \Phi \cdot (\boldsymbol{r} * \boldsymbol{n}) \, \mathrm{d}S + \mu \int_H (\boldsymbol{r} * \boldsymbol{v})(\boldsymbol{v} \cdot \boldsymbol{n}) \, \mathrm{d}S, \tag{1.1.14}$$

where the last term is the moment of momentum flux through the fixed surface H.

The moment $\boldsymbol{M}_H$ can, by using (1.1.6) and (1.1.9), be written as

$$\boldsymbol{M}_H = \mu \int_H \left\{ \frac{\partial \Phi}{\partial t} + \tfrac{1}{2} |\boldsymbol{v}|^2 \right\} (\boldsymbol{r} * \boldsymbol{n}) \, \mathrm{d}S. \tag{1.1.15}$$

Now we substitute (1.1.15) into (1.1.14). Using an analogous formula as (1.1.10) and taking the limit $R_H \to \infty$ we obtain

$$\boldsymbol{M} = -\mu \frac{\mathrm{d}}{\mathrm{d}t} \int_{\partial B} \Phi \cdot (\boldsymbol{r} * \boldsymbol{n}) \, \mathrm{d}S. \tag{1.1.16}$$

Here the integral over the surface H tends to zero because its integrand tends to zero as R_H^{-3} or more quickly when B does not change its volume.

Next we consider the case of an infinitely long cylinder B with generators parallel to the z axis. This cylinder is allowed to move arbitrarily and to change its shape, but such that its generators remain parallel to the z axis. Then the induced fluid flow depends only on the two coordinates x and y.

Again we assume the fluid to be at rest with respect to our coordinate system, at large distances from B. We surround the cylinder B by a circular cylindrical control surface H with radius R_H, fixed in space. Because our consideration will be given for a slab of space of unit width in the z direction, we consider H and the boundary ∂B of B as lines in the (x, y) plane (fig. 1.1.2) and Ω is the two dimensional region bounded by them. An essential difference with the previous three dimensional case is that here Ω is doubly connected.

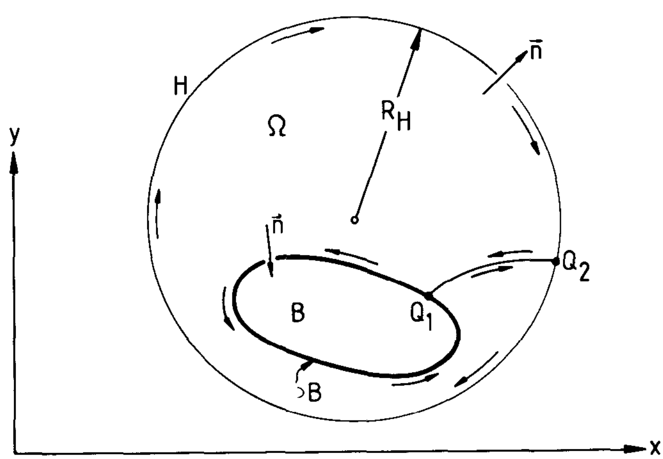

Fig. 1.1.2. Cross section of cylinder B and control surface H.

The momentum $I = (I_x(t), I_y(t))$ of the fluid in the region Ω is

$$I = (I_x, I_y) = \mu \iint_\Omega \left(\frac{\partial \Phi}{\partial x}, \frac{\partial \Phi}{\partial y} \right) \mathrm{d}x \, \mathrm{d}y, \tag{1.1.17}$$

where $\Phi = \Phi(x, y, t)$ is again the velocity potential. Because Ω is doubly connected, the function Φ can be "multivalued", we make Ω simply connected by introducing a cut (Q_1, Q_2) from ∂B, towards H.

By a partial integration we can change the double integrals (1.1.17) into integrals along the line L consisting of H, ∂B and (Q_1, Q_2) in the indicated directions. We find

$$(I_x, I_y) = -\mu \left\{ \int_L \Phi \, \mathrm{d}y, -\int_L \Phi \, \mathrm{d}x \right\} = \mu \int_L (y, -x) \, \mathrm{d}\Phi. \tag{1.1.18}$$

When there is circulation around B the function Φ will assume different values at both sides of (Q_1, Q_2). This circulation has to be independent of time, otherwise vorticity would be shed into the fluid which here just as in the three dimensional case, is assumed not to happen. Because the difference of Φ at both sides of the cut is a constant, the contributions to (1.1.18) from the two sides of (Q_1, Q_2) cancel. When i is the imaginary unit and $\zeta = x + iy$ we can write (1.1.18) as

$$I = (I_x + iI_y) = -i\mu \int_{\partial B + H} \zeta \, \mathrm{d}\Phi. \tag{1.1.19}$$

The resultant hydrodynamic force on ∂B is denoted by $F = (F_x(t) + iF_y(t))$, the resultant force on H by the outside fluid is

$$F_H = -i \int_H p \, \mathrm{d}\zeta. \tag{1.1.20}$$

Now we apply again the theorem of momentum. Then we have as in the previous case, to consider the momentum flux through H. Analogous to the reasoning for the three dimensional case it is easily seen that the contribution of this to the force F tends to zero when the radius R_H tends to infinity. Hence we write

$$F_H - F \approx -i\mu \frac{\mathrm{d}}{\mathrm{d}t} \int_{\partial B + H} \zeta \, \mathrm{d}\Phi, \tag{1.1.21}$$

where the symbol $\approx$ means that the momentum flux through H has been left out of consideration.

Because H is fixed we have

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_H \zeta \, \mathrm{d}\Phi = \int_H \zeta \, \mathrm{d}\frac{\partial \Phi}{\partial t} = -\int_H \frac{\partial \Phi}{\partial t} \, \mathrm{d}\zeta. \tag{1.1.22}$$

The latter equality is based on the following consideration. The potential Φ can change by a certain amount by encircling the body B. This amount however is independent of time because we supposed the circulation around B to be constant, hence $\partial\Phi/\partial t$ assumes the same value after encircling B. From this (1.1.22) follows.

Next we write (1.1.20) by substitution of (1.1.6) and substitute the result together with (1.1.22) into (1.1.21). Then we find for the limit $R_H \to \infty$

$$F = \left(F_x + iF_y\right) = i\mu \frac{\mathrm{d}}{\mathrm{d}t} \int_{\partial B} \zeta \, \mathrm{d}\Phi. \tag{1.1.23}$$

1.2. Force actions and shed vorticity

Consider a body B of finite extent moving through an incompressible and inviscid fluid which is again at rest at infinity with respect to our Cartesian coordinate system (x, y, z). The body B will move with a mean velocity U in the positive x direction and repeats its velocities after each time period τ or after each covered distance

$$b = U\tau, \tag{1.2.1}$$

while also the neighbouring field of flow has the same periodicity. When we assume that no vorticity is shed by B we prove that no mean force can be exerted by B on the fluid.

In this case (1.1.12) is valid. The mean value of $\boldsymbol{F}(t)$ over one period τ of time becomes

$$\frac{1}{\tau}\int_t^{t+\tau} \boldsymbol{F}(t)\,\mathrm{d}t = -\frac{\mu}{\tau}\int_{\partial B} \big[\Phi(x+b, y, z, t+\tau)\boldsymbol{n}(x+b, y, z, t+\tau)$$

$$- \Phi(x, y, z, t)\boldsymbol{n}(x, y, z, t)\big]\,\mathrm{d}S. \tag{1.2.2}$$

The velocities of the fluid at times t and $t+\tau$ are the same for the points (x, y, z) and $(x+b, y, z)$. Hence the difference of the potential at corresponding points and times can only be a constant c, then

$$\frac{1}{\tau}\int_t^{t+\tau} \boldsymbol{F}(t)\,\mathrm{d}t = -\frac{\mu c}{\tau}\int_{\partial B} \boldsymbol{n}\,\mathrm{d}S = 0. \tag{1.2.3}$$

From (1.2.3) we find that a body of finite extent, moving periodically in the way as we described, cannot experience a force with a non zero mean value without shedding vorticity. Inversely, by the principle action equals reaction, such a body cannot exert a mean force on the fluid without leaving behind vorticity. It cannot act as a lift producing wing or a thrust producing propeller. When vorticity is shed periodically the function Φ is not defined in the whole space and the velocities do not tend to zero at infinity in the way as was needed for the derivation of (1.1.12). Hence the foregoing argument does not hold. Because the velocity field belonging to the shed vorticity represents kinetic energy of the fluid, we can state; when a periodically moving body of finite extent inducing a periodic neighbouring field of flow, exerts a mean force on the fluid this has to be accompanied by energy losses.

Next we consider the moment with respect to 0, of the fluid pressures exerted at the body B. Now we use (1.1.16) which is valid when no vorticity is shed. Hence in that case we find for the mean value of the moment around the x axis

$$\begin{aligned}\frac{\boldsymbol{e}_x}{\tau}\cdot\int_0^\tau \boldsymbol{M}(t)\,\mathrm{d}t = -\frac{\mu}{\tau}\boldsymbol{e}_x\cdot\int_{\partial B}\big[&\Phi(x+b,y,z,t+\tau)\\ &\times\{(\boldsymbol{r}(x,y,z,t)+b\boldsymbol{e}_x)*\boldsymbol{n}(x+b,y,z,t+\tau)\}\\ &-\Phi(x,y,z,t)\{\boldsymbol{r}(x,y,z,t)*\boldsymbol{n}(x,y,z,t)\}\big]\,\mathrm{d}S,\end{aligned} \tag{1.2.4}$$

where $\boldsymbol{e}_x$ is the unit vector in the x direction. Because

$$\boldsymbol{n}(x+b,y,z,t+\tau)=\boldsymbol{n}(x,y,z,t) \tag{1.2.5}$$

and again the difference of the potential at corresponding points and times can be only a constant c we find

$$\frac{\boldsymbol{e}_x}{\tau}\cdot\int_0^\tau \boldsymbol{M}\,\mathrm{d}t = -\frac{\mu c}{\tau}\int_{\partial B}(yn_z - zn_y)\,\mathrm{d}S = 0. \tag{1.2.6}$$

The last equality follows from (1.1.9).

From (1.2.6) it follows that a periodically moving body of finite extent which induces a periodic field of flow and which exerts at the fluid a non zero mean moment around a line parallel to its mean direction of motion, has to shed vorticity.

We can also consider moments around lines l_1 and l_2, parallel to the y-axis and z-axis respectively, which translate in the positive x direction with the velocity U. It can be seen, that in the case of no vorticity shedding, the moments around these lines need not to have zero mean values. A simple example is a flat wing of zero thickness of large aspect ratio having constant chordlength. The wing has a nonzero angle of