Advances in Electronics and Electron Physics

EDITED BY
PETER W. HAWKES

Advances in Electronics and Electron Physics

EDITED BY

PETER W. HAWKES

Laboratoire d'Optique Electronique
du Centre National de la Recherche Scientifique
Toulouse, France

VOLUME 59

1982

ACADEMIC PRESS

A Subsidiary of Harcourt Brace Jovanovich, Publishers

New York London

Paris San Diego San Francisco São Paulo Sydney Tokyo Toronto

CONTRIBUTORS TO VOLUME 59

Numbers in parentheses indicate the pages on which the authors' contributions begin.

F. B. Dunning, Department of Space Physics and Astronomy and The Rice Quantum Institute, Rice University, Houston, Texas 77001 (79)

Fred E. Gardiol, Laboratoire d'Electromagnétisme et d'Acoustique, Swiss Federal Institute of Technology, Lausanne, Switzerland (139)

K.-J. Hanszen, Physikalisch-Technische Bundesanstalt, D-3300 Braunschweig, Federal Republic of Germany (1)

K. Hess, Coordinated Science Laboratory and Department of Electrical Engineering, University of Illinois, Urbana, Illinois 61801 (239)

Juan R. Mosig, Laboratoire d'Electromagnétisme et d'Acoustique, Swiss Federal Institute of Technology, Lausanne, Switzerland (139)

R. F. Stebbings, Department of Space Physics and Astronomy and The Rice Quantum Institute, Rice University, Houston, Texas 77001 (79)

ACADEMIC PRESS, INC.
111 Fifth Avenue, New York, New York 10003

United Kingdom Edition published by
ACADEMIC PRESS, INC. (LONDON) LTD.
24/28 Oval Road, London NW1 7DX

Library of Congress Catalog Card Number: 49-7504

ISBN 0-12-014659-2

PRINTED IN THE UNITED STATES OF AMERICA

82 83 84 85 9 8 7 6 5 4 3 2 1

FOREWORD

The first volume of *Advances in Electronics*, as this serial was originally entitled, appeared in 1948, edited by L. Marton, and the editorship has remained a family affair ever since. After the death of L. Marton, his wife Claire continued to edit the publication, and shortly before her death in November 1981 she invited me to collaborate with her and eventually to take over from her.

The publication is stamped with the personalities of its first editors, and the catholicity and timeliness of the subjects covered is impressive evidence that they remained closely in touch with developments in the fields of electronics, electron physics, and electron microscopy; and, indeed, in many related domains. Furthermore, the distinction of this serial was such that many very distinguished authors have found time to write for it.

The formula has thus been amply tried, and, in the main, few changes in the type of coverage provided by Bill and Claire Marton are planned, with one exception. These *Advances* seem a natural home for review articles on various aspects of digital and hybrid image processing and, of course, signal processing in general. We therefore plan to include articles on these subjects in due course. Even this is not strictly a novelty, for some such contributions have already appeared, and one by B. R. Hunt was among the first batch of *Advances* mail to reach me.

The contributions in the present volume were all commissioned by Dr. or Mrs. Marton, who would, I am sure, wish me to thank all the authors most warmly for their efforts. Two further volumes will follow shortly, containing other manuscripts that had accumulated during Claire Marton's last illness or that have arrived subsequently.

Suggestions for topics that should be covered and offers of articles, even if highly tentative or preliminary, will always be very welcome. As usual, a list of reviews planned for future volumes is given below.

Critical Reviews:

Atomic Frequency Standards	C. Audouin
Electron Scattering and Nuclear Structure	G. A. Peterson
Large Molecules in Space	M. and G. Winnewisser
The Impact of Integrated Electronics in Medicine	J. D. Meindl
Electron Storage Rings	D. Trines
Radiation Damage in Semiconductors	N. D. Wilsey and J. W. Corbett
Visualization of Single Heavy Atoms with the Electron Microscope	J. S. Wall

Light Valve Technology	J. Grinberg
Electrical Structure of the Middle Atmosphere	L. C. Hale
Microwave Superconducting Electronics	R. Adde
Diagnosis and Therapy Using Microwaves	M. Gautherie and A. Priou
Computer Microscopy Image Analysis of Biological Tissues Seen in the Light Microscope	E. M. Glaser
Low-Energy Atomic Beam Spectroscopy	E. M. Hörl and E. Semerad
History of Photoemission	W. E. Spicer
Power Switching Transistors	P. L. Hower
Radiation Technology	L. S. Birks
Diffraction of Neutral Atoms and Molecules from Crystalline Surfaces	G. Boato and P. Cantini
Auger Spectroscopy	M. Cailler, J. P. Hanachaud, and D. Roptin
Digital Image Processing and Analysis	B. R. Hunt
Infrared Detector Arrays	D. Long and W. Scott
Energy Levels in Gallium Arsenide	A. G. Milnes
Polarized Electrons in Solid-State Physics	H. C. Siegmann, M. Erbudak, M. Landolt, and F. Meier
The Technical Development of the Shortwave Radio	E. Sivowitch
Chemical Trends of Deep Traps in Semiconductors	P. Vogl
Potential Calculation in Hall Plates	G. DeMey
Gamma-Ray Internal Conversion	O. Dragoun
CW Beam Annealing Process and Application for Superconducting Alloy Fabrication	J. F. Gibbons
Polarized Ion Sources	H. F. Glavish
Ultrasensitive Detection	K. H. Purser
The Interactions of Measurement Principles, Interfaces, and Microcomputers in Intelligent Instruments	W. G. Wolber
Fine-Line Pattern Definition and Etching for VLSI	Roy A. Colclaser
Recent Trends in Photomultipliers for Nuclear Physics	J. P. Boutet, J. Nussli, and D. Vallat
Waveguide and Coaxial Probes for Nondestructive Testing of Materials	F. E. Gardiol
The Measurement of Core Electron Energy Levels	R. N. Lee and C. Anderson
Millimeter Radar	Robert D. Hayes
Recent Advances in the Theory of Surface Electronic Structure	Henry Krakauer
Long-Life High-Current-Density Cathodes	Robert T. Longo
Microwaves in Semiconductor Electronics	J. L. Allen
Applications of Quadrupole Mass Spectrometers	I. Berecz, S. Bohatka, and G. Langer
Advances in Materials for Thick-Film Hybrid Microcircuits	J. Sergent
Guided-Wave Circuit Technology	M. K. Barnoski
Fast-Wave Tube Devices	J. M. Baird
Spin Effects in Electron–Atom Collision Processes	H. Keinpoppen
Recent Advances in and Basic Studies of Photoemitters	H. Timan
Thermal and Electrothermal Instabilities in Semiconductors	M. P. Shaw and Y. Yildirim

High-Resolution Spectroscopy of Interstellar Molecules	G. Winnewisser
Solid State Imaging Devices	E. H. Snow
Structure of Intermetallic and Interstitial Compounds	A. C. Switendick
Smart Sensors	W. G. Wolber

Supplementary Volumes:

Microwave Field-Effect Transistors	J. Frey
Magnetic Reconnection	P. J. Baum and A. Bratenahl

P. W. HAWKES

CONTENTS

Aspects of High-Field Transport in Semiconductor Heterolayers and Semiconductor Devices

K. HESS

ADVANCES IN ELECTRONICS AND ELECTRON PHYSICS, VOL. 59

Holography in Electron Microscopy

K.-J. HANSZEN

Physikalisch-Technische Bundesanstalt
Braunschweig, Federal Republic of Germany

ISBN 0-12-014659-2

I. Introduction

The prehistory of holography is closely connected with electron microscopy and with structural research on crystal lattices. In this connection, the experiments of Boersch (*7*, *10*), who attempted to reconstruct the lattice image from the intensity distribution of its diffraction pattern, must be mentioned. In the experiments of Bragg (*13*, *14*) and Buerger (*17*), the reconstruction results were improved by prior knowledge of the phases in the diffraction pattern.

In the first article of Gabor (*25*), who later called the new method "holography" (*26*), the actual purpose was to invent an electron optical device able to produce strongly magnified images, the aberrations of which could be eliminated afterward by light optical processing. According to his proposals, the shadow electron microscope of Boersch (*8*, *9*) could be used for the electron optical step. The optics of this system is identical with that of a modern scanning transmission electron microscope, using a fixed and slightly defocused electron probe. The first experiments of Haine and Dyson (*30*) soon revealed, however, that electron holograms can be taken in a conventional electron microscope by applying large defocusings (see Fig. 1). The hologram is, in this case, a Fresnel or Fraunhofer diffraction image of the object.

It was Gabor's intention to achieve a perfect transfer of the phase as well as the amplitude of the wave function existing in the object plane of the electron microscope into the image plane of the light optical reconstruction setup. In the first experiments of Haine and Mulvey (*31*), this project did not lead to the success expected. Before the reasons for this failure could become evident, however, our understanding of the electron microscopical object and of the image processing had to be deepened and further experimental knowledge had to be acquired. First, it had to be established to what extent the electron optical object could be classified as either an amplitude or phase object, and how the information on the amplitude and phase parts is transferred into the wave function existing in the hologram plane. This problem can be conveniently solved by means of the Fourier optical transfer theory

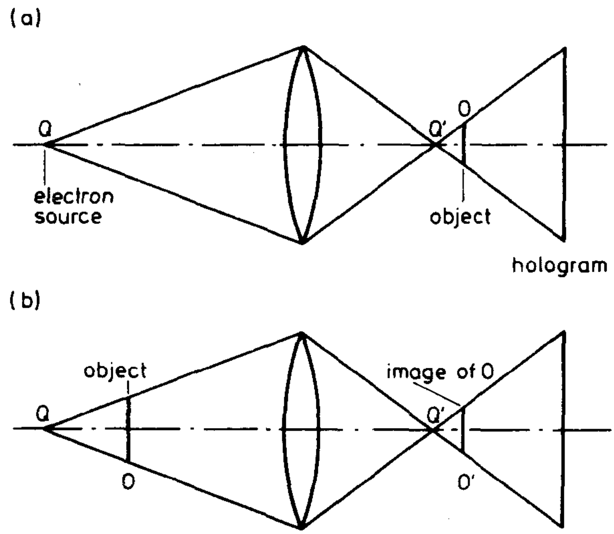

FIG. 1. General device for producing in-line holograms: (a) the projection method of Gabor (*25*) and (b) the transmission method of Haine and Dyson (*30*). In the transmission method, the Gaussian image O′ is in the same position as the object O in the projection method. If the objective lens has no aberrations, the equivalence of both methods is obvious. If aberrations are present and O′ approaches Q', this equivalence is only approximate [cf. Ref. *53* in connection with systematic descriptions of the STEM, e.g., by Zeitler and Thomson (*91*, *113*); see also Refs. *27* and *42*].

[see the work of Hanszen (*37*) and Ade (*2*, *3*, *5*)]. A survey of this theory will be given in Section II.

It also had to be considered under what circumstances the reconstruction results of Gabor's method could be expected to be reliable. This problem was solved by Thompson (*90*), who introduced the concept of Fraunhofer holography, and by Tonomura *et al.* (*94, 95*), who realized this concept in electron microscopy. A detailed discussion of this method has been given by Hanszen (*36*). Sections III and IV deal with these topics.

In the course of these considerations, it has become clear that the range of application of Gabor's method (in the following referred to as "in-line holography") is restricted either to pure weak amplitude objects or to pure weak phase objects. The restriction to pure amplitude or phase objects can be abandoned in the "single-sideband holography" suggested by Lohmann (*73*). With the application of special measures, the amplitude and phase components of electron microscopical objects can be separately reconstructed [see Hoppe (*60, 61*)]. The extension of this method to strong objects, however, is not possible. These problems are discussed in Section V.

The reconstruction of strong objects is possible only by using a reference wave according to the suggestions of Leith and Upatnieks (*69*, *70*). After preliminary experiments of Möllenstedt and Wahl (*81*), Tomita *et al.* (*92*, *93*) succeeded in applying this method (called "out-line holography" or "off-axis holography") in electron microscopy. Recent off-axis holographic attempts by Tonomura (*97*, *98*) were aimed at solving Gabor's original problem of eliminating the lens aberrations by holography.

In principle, holography is a three-step rather than a two-step process: The photographic process, being a stage between the electron and light optical step, plays an important part in determining the quality of the reconstructed image. Due to the nonlinearity of this process, in-line holography is inappropriate for reliable reconstruction of strong objects and appropriate only for weak objects when certain conditions concerning the photographic characteristics are fulfilled [cf. Gabor (*25*); Hanszen *et al.* (*55*)]. In off-axis holography, the influence of photographic nonlinearities is much less detrimental than in the above case. In a recent investigation of the whole problem, it was shown by Ade (*4*, *6a*) that some difficulties remain in off-axis holography, particularly if the compensation of lens aberrations is envisaged.

Even if we assume that the object wave could be perfectly reconstructed in the recording plane of the light optical step, we are still faced with the problem of how to detect the amplitude and phase distribution separately in this plane; to be more precise: to detect one unaffected by the other. The problem of phase detection, first suggested by Cohen (*18*) and experimentally attacked by Wahl (*106*, *107*), with work on it being continued by Tonomura (*96*) and Hanszen *et al.* (*45*, *55*), can be solved by the simultaneous reconstruction of two identical holograms inserted into the branches of a light optical interferometer.

All the problems connected with off-axis holography are discussed in Section VI.

Hitherto, object and reference waves in the electron microscope were created by splitting the illumination wave, using an electrostatic biprism. But in close analogy to the current light optical reconstruction methods, object and reference waves can also be generated by means of a scattering foil. Also the interferometric method employing amplitude splitting by crystal diffraction [Marton (*74*)] instead of wave front splitting [Möllenstedt and Düker (*80*)] was recently brought to a successful conclusion by Matteucci *et al.* (*75*, *76*). Section VII deals with recent investigations in this field.

In accordance with the editor's conception, complete coverage of the literature was not attempted in this survey. More extensive lists of references can be found in the summary articles by Wade (*105*) on electron holography, by Hawkes (*59*) on coherence in electron optics, and by Missiroli *et al.* (*79*) on electron interferometry.

II. Fourier Optical Treatment of Electron Microscopical Holography, Explained by the Example of In-Line Holography[1]

A. General Case

The object is described by an object function F, depending only on the position vector $\mathbf{x} = (x, y)$:

$$F(\mathbf{x}) = A(\mathbf{x}) \exp[i\Phi(\mathbf{x})] \tag{1}$$

where $A(\mathbf{x})$ is the object amplitude and $\Phi(\mathbf{x})$ the object phase. This object is illuminated by a wave $\psi_Q(\mathbf{x})$ which is almost homogeneous. For the wave function in the object plane, we can write

$$\psi(\mathbf{x}) = \psi_Q(\mathbf{x}) A(\mathbf{x}) \exp[i\Phi(\mathbf{x})] \tag{2}$$

In the case of three-dimensional objects, $\psi(\mathbf{x})$ is understood to be the wave function in the exit plane of the object. The product representation of Eq. (2) is not applicable for strong objects, such as crystals [see, e.g., (*82*)]. In this case, the object function depends not only on the object coordinates, but also on the illumination angle, φ_Q, which means that $F = F(\mathbf{x}, \varphi_Q)$.

When a plane illumination wave is used, impinging on the object at a distinct angle φ_Q, the wave function behind the object does not permit us to draw unequivocal conclusions concerning the three-dimensional structure of the object. The resolution of the crystal structure in three dimensions therefore requires more extended theoretical and experimental means than those described in this report.

In the case of isoplanatic imaging, the diffraction pattern (object spectrum) is the Fourier transform of the object wave function:

$$\tilde{\psi}(\mathbf{R}) = \iint_{-\infty}^{+\infty} \psi(\mathbf{x}) \exp(-2\pi i \mathbf{R} \cdot \mathbf{x})\, d\mathbf{x} \tag{3}$$

where $R = (R_x^2 + R_y^2)^{1/2} = |\alpha/\lambda|$ is the spatial frequency, α the diffraction angle, and λ the electron wavelength. The influence of the lens aberrations can be described by the pupil function

$$\tilde{F}_P(\mathbf{R}) = \tilde{F}_M(\mathbf{R}) \exp[2\pi i \mathscr{W}(\mathbf{R})] \tag{4a}$$

where $\mathscr{W}$ is the reduced wave aberration (see below),

$$\tilde{F}_M(\mathbf{R}) = \begin{cases} 1 & \text{for } |\mathbf{R}| \leq R_e \\ 0 & \text{otherwise} \end{cases} \tag{4b}$$

[1] Notation similar to that of Hanszen (*37*).

and R_e reduced is the radius of the aperture hole. The image spectrum is the product of Eqs. (3) and (4):

$$\tilde{\psi}_H(\mathbf{R}) = \tilde{\psi}(\mathbf{R})\tilde{F}_P(\mathbf{R}) \tag{5}$$

In the simplest case, the hologram can be identified with an aberrated image, the spectrum of which is characterized by the last equation. Thus, the hologram wave function and hologram intensity can be expressed as

$$\psi_H(\mathbf{x}_H) = \iint_{-\infty}^{+\infty} \tilde{\psi}_H(\mathbf{R}) \exp\left[2\pi i \frac{\mathbf{R}}{m} \cdot \mathbf{x}_H\right] d\mathbf{R}$$

$$=_{\mathrm{df}} \psi_{QH}(\mathbf{x}_H) A'(\mathbf{x}_H) \exp[i\Phi'(\mathbf{x}_H)] \tag{6}$$

$$I_H(\mathbf{x}_H) = \psi_H(\mathbf{x}_H)\psi_H^*(\mathbf{x}_H) \tag{7}$$

where $\mathbf{x}_H$ is the position vector in the hologram plane, and $m = R/R_H$ is the scaling factor for the related object and hologram frequencies R and R_H. The intensity $I_H(\mathbf{x}_H)$ is the physical property to be recorded photographically. In the simple case of small values of the electron microscopical defocus,[2] the reduced wave aberration can be written as

$$\mathscr{W}(\mathbf{R}) = \frac{C_S\lambda^3}{4} R^4 + \frac{\Delta z\lambda}{2} R^2 \tag{8}$$

where C_S is the spherical aberration constant and Δz is the object defocus. This expression is sufficient for a first discussion of the transfer theoretical concept of holography. Due to the lens aberrations, A' in Eq. (6) is not equal to A and Φ' is not equal to Φ. A phase structure, for example, may be present in the image of a pure amplitude object $F(\mathbf{x}) = A(\mathbf{x})$, or an amplitude structure in the image of a pure phase object $F(\mathbf{x}) = \exp[i\Phi(\mathbf{x})]$. More precise specifications are given in the next section.

B. Transfer Theory for Weak Objects

For the local distributions of the object wave function and object intensity, we can write[3]

$$\begin{aligned}\psi(\mathbf{x}) &= \psi_Q[1 + f(\mathbf{x})] \\ &= \psi_Q[1 + a(\mathbf{x}) + ib(\mathbf{x})]\end{aligned} \tag{9a}$$

$$I(\mathbf{x}) = |\psi_Q|^2[1 + 2a(\mathbf{x}) + \cdots] \tag{9b}$$

[2] For large values of the defocus (cf. *3*, *38*, *42*).

[3] The inaccurate normalization, leading to $I(\mathbf{x}) > |\psi_Q|^2$, does not give rise to problems in the following considerations.

where ψ_Q is a constant (axial illumination); f is complex, $|f| \ll 1$; a, b are real, $|a|$, $|b| \ll 1$; $|\psi_Q|^2$ is the intensity of the illuminating wave. The Fourier representation of the object function is

$$F(\mathbf{x}) = 1 + \iint_{-\infty}^{+\infty} [\tilde{a}(\mathbf{R}) + i\tilde{b}(\mathbf{R})] \exp(2\pi i \mathbf{R} \cdot \mathbf{x})\, d\mathbf{R} \tag{10}$$

with

$$\tilde{a}(\mathbf{R}) = \tilde{a}^*(-\mathbf{R}); \qquad \tilde{b}(\mathbf{R}) = \tilde{b}^*(-\mathbf{R}) \tag{11}$$

and the object spectrum is

$$\tilde{\psi}(\mathbf{R}) = \psi_Q[\delta(\mathbf{R}) + \tilde{a}(\mathbf{R}) + i\tilde{b}(\mathbf{R})] \tag{12}$$

where δ is the delta function. Using Eqs. (6) and (7) for the wave function and intensity in the hologram plane, we derive the following expressions:

$$\psi_H(\mathbf{x}_H) = \psi_{QH}\left\{1 + \iint_{-\infty}^{+\infty} \tilde{F}_M(\mathbf{R}) \exp[2\pi i \mathscr{W}(\mathbf{R})][\tilde{a}(\mathbf{R}) + i\tilde{b}(\mathbf{R})] \times \exp\left(2\pi i \frac{\mathbf{R}}{m} \cdot \mathbf{x}_H\right) d\mathbf{R}\right\}$$

$$= _{\mathrm{df}}\, \psi_{QH}[1 + f'(\mathbf{x}_H)] \tag{13a}$$

$$I_H(\mathbf{x}_H) = |\psi_{QH}|^2 \left\{1 + \iint_{(\mathrm{pupil})} [\mathscr{D}(\mathbf{R})\tilde{a}(\mathbf{R}) + \mathscr{B}(\mathbf{R})\tilde{b}(\mathbf{R})] \times \exp\left(2\pi i \frac{\mathbf{R}}{m} \cdot \mathbf{x}_H\right) d\mathbf{R}\right\} \tag{13b}$$

where

$$\mathscr{D}(\mathbf{R}) = \exp[2\pi i \mathscr{W}(\mathbf{R})] + \exp[-2\pi i \mathscr{W}(\mathbf{R})] = 2\cos[2\pi \mathscr{W}(\mathbf{R})] \tag{14a}$$

is the contrast transfer function for the amplitude component of the object, and

$$\mathscr{B}(\mathbf{R}) = i\{\exp[2\pi i \mathscr{W}(\mathbf{R})] - \exp[-2\pi i \mathscr{W}(\mathbf{R})]\} = -2\sin[2\pi \mathscr{W}(\mathbf{R})] \tag{14b}$$

is the contrast transfer function for the phase component of the object.

Referring to Eqs. (13) and (14), the following can be stated:

(*i*) When objects with both components ("mixed objects") are imaged, not only the amplitude component but also the phase component is partially transferred into the image intensity. Unambiguous determination of both components from one single hologram is not possible.

(*ii*) Each spatial frequency $\mathbf{R}$ of the object is transferred into the image intensity with its proper transfer factor (14).

The isoplanatic condition, necessary in the Fourier formalism given above, does not constitute an essential restriction for the description of holography by transfer functions. Being aware of this, we can include all third-order aberrations [see Ade (*5*)]. Starting from the fact that the images of the elementary gratings formed by the spatial frequencies $\pm\mathbf{R}$ in the object are distorted in the off-axial region (see Fig. 1 in Ref. *3*) and considering the optical distance (point characteristic) S^* between the spectral points in the virtual entrance pupil and the hologram plane (see Fig. 23 in Ref. *50*), the Fourier integral (6) must be replaced by the Fourier-like expression

$$\psi_H(\mathbf{x}_H) = |\psi_{QH}(\mathbf{x}_H)| \int\int_{-\infty}^{+\infty} \tilde{F}(\mathbf{R})\tilde{F}_M(\mathbf{R}) \times \exp\left\{\frac{2\pi i}{\lambda}\left[\frac{z_O - z_Q}{2}(\lambda\mathbf{R})^2 - S^*(\mathbf{R};\mathbf{x}_H)\right]\right\} d\mathbf{R} \tag{6a}$$

[see Eq. (24) in Ref. *3*]. Here $z_0 - z_Q$ is the separation between electron source and object. According to Eq. (43) in Ref. *3*, this equation can be rewritten as

$$\psi_H(\mathbf{x}_H) = \psi_{QH}(\mathbf{x}_H) \int\int_{-\infty}^{+\infty} \tilde{F}(\mathbf{R})\tilde{k}(\mathbf{R};\mathbf{x}_H) \exp\left(2\pi i \frac{\mathbf{R}}{m}\cdot\mathbf{x}_H\right) d\mathbf{R} \tag{6b}$$

where the illumination wave ψ_{QH} in the hologram differs from the corresponding wave in the object mainly by phase terms of fourth order in $\mathbf{x}_H$ and the new pupil function $\tilde{k}(\mathbf{R};\mathbf{x}_H)$ differs from the earlier function by additional phase terms of first to fourth order in $\mathbf{x}_H$ which are associated with the Seidel aberrations [see Eqs. (44) and (45) in Ref. *3*]. With this knowledge, the information content of the hologram can be described by the following transfer functions:

$$\mathscr{D}(\mathbf{R};\mathbf{x}_H) = \tilde{k}(\mathbf{R};\mathbf{x}_H) + \tilde{k}^*(-\mathbf{R};\mathbf{x}_H) \tag{15a}$$

$$\mathscr{B}(\mathbf{R};\mathbf{x}_H) = i[\tilde{k}(\mathbf{R};\mathbf{x}_H) - \tilde{k}^*(-\mathbf{R};\mathbf{x}_H)] \tag{15b}$$

[see Eq. (52) in Ref. *3*]. By the use of these functions instead of Eqs. (14), the procedures described in this article can also be applied to nonisoplanatic imaging.

The hologram, as described above, is an electron micrograph suffering from spherical aberration and defocusing. Since the transfer functions given by Eqs. (14) possess zeros, we must be aware that the total frequency spectrum of the object is not contained in the hologram and that the object function can by no means be perfectly transferred by holography into the reconstructed image. In special cases the losses in the spectrum do not cause severe disturbances. This question will be discussed in Section II,E.

C. The Photographic Process

In the reconstruction step (L), the hologram is used as a diffracting object in a coherent light beam. On account of various photochemical processes during exposure, development, and fixing, the hologram exhibits an amplitude and phase structure. Thus, the intensity of a coherent incident light wave is modulated by a factor $T_I(\mathbf{x}_H)$ and consequently, the wave function is modulated by a factor $T(\mathbf{x}_H) = \sqrt{T_I(\mathbf{x}_H)}$. In addition, a phase modulation $\varphi_L(\mathbf{x}_H)$ is often present as a secondary photographic effect (*32*, *89*). The same information concerning both the object amplitude $A(\mathbf{x})$ and the object phase $\Phi(\mathbf{x})$ is similarly stored in $T_I(\mathbf{x})$ and $\varphi_L(\mathbf{x}_H)$.

The reconstruction results strongly depend on the functional relationships $T(A;\Phi)$ or $\varphi_L(A;\Phi)$. Some important cases will be described in Sections III,B, IV,C, and VI,E.

Case (a): Processing of the amplitude structure of the hologram (see Ref. *26*). In the case of weak objects, a slope of $\gamma = -2$ of the characteristic curve $-\log T_I$ versus $\log E$ can be achieved by a proper control of the photographic process. If the exposure interval $E_{max} - E_{min}$ is narrow enough, $T(\mathbf{x}_H)$ will then be directly proportional to $E(\mathbf{x}_H) = I_H(\mathbf{x}_H)t$ in this interval, i.e., $T = cE = CI_H$, where t is the exposure time, and c and C are constants. If, furthermore, the phase structure of the hologram is suppressed by an immersion liquid such as glycerine—in this case the hologram acts as a pure weak amplitude object—the reconstruction process can be described simply in terms of the transfer function $\mathscr{D}_L(\mathbf{R})$. Details are given below.

Case (b): Processing of the phase structure of the hologram. If the amplitude structure of the plate is destroyed by means of a bleaching process, a pure phase hologram is obtained. If its phase modulation is weak, the reconstruction can similarly be described by the transfer function $\mathscr{B}_L(\mathbf{R})$. A detailed discussion is given by Hanszen (*36*).

D. Transfer Theory of the Reconstruction Step[4]

The reconstruction step (L) can be treated analytically in the same way as the electron microscopical step (E). Under the conditions specified for

[4] A pictorial presentation is given by Hanszen (*40*).