# Modern Topics in Electromagnetics and Antennas

PPL Conference Publication 13

# MODERN TOPICS IN ELECTROMAGNETICS AND ANTENNAS

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# FOREWORD

During the week beginning August 30, 1976, eight lecturers and seventy-five participants assembled at the Technical University of Eindhoven for what turned out to be one of the most successful short courses ever organised on Electromagnetics and Antennas. The course, which originated in Urbana, Illinois in 1970, has previously been offered in Copenhagen, Naples, Trondheim, and at a number of locations in the United States. On each of these occasions the lecturers carefully prepared a set of lecture notes for distribution to the participants at the beginning of the course. Two of these were later edited by Raj Mittra and published as texts in 1973 and 1975. Both of these texts have been well-received in the electromagnetics community throughout the world.

The course has evolved with each offering ever since its inception in 1970. The contents of the 1976 Eindhoven course were not available in any of the previous publications; as a result, the organisers of the Eindhoven course believed that the publication of these lecture notes would be a welcome addition to the previous two texts.

Though the organisers has originally planned to thoroughly edit the notes from the point of view of unifying the notation, format, etc., this thought was later abandoned on the advice of the publisher for the sake of expediency of publication time. In addition, the cost-saving photo-offset method of printing was opted for, once again to save time and costs. Also, the original order of the various chapters was retained in order to avoid retyping the equation numbers, etc., that a reorganisation of the chapters would have required. Hopefully, however, the reader will not find it too difficult to shift from one chapter to another, each of which is essentially self-contained. The book may, in fact, be regarded as somewhat like the proceedings of a special symposium in which the contributions from a number of different authors are collectively presented in a special volume. Nevertheless, a guide to the chapters, given below, may be useful to a reader going through the volume for the first time.

Chapter 1 by Van Bladel is an exposition of the low-frequency asymptotic technique for solving scattering and coupling problems in the long wavelength region. Next, the reader may wish to turn to Chapter 6 by de Hoop in which the general theoretical aspects of the integral equation methods and their numerical solutions are discussed. He may follow this up with a look at

Chapter 3 by Davies in which the numerical techniques for solving electromagnetic problems are developed in more detail. Leaving this topic, the reader may now move on to the high-frequency regime by going to Chapter 4 which provides him with the fundamentals of ray techniques for electromagnetics. Once he has digested this information, at least by familiarising himself with the basic concepts on ray methods, he may go on to Chapter 5 by Bach which emphasises the practical application of Geometrical Theory of Diffraction (GTD) in considerable detail. Having whetted his appetite on the low- and high-frequency methods, he may then wish to turn to Chapter 7 which presents an overview of methods for combining the integral equation technique with asymptotic solutions and the application of Galerkin's method in the transform domain. The latter topic was also touched on in Chapter 6 by de Hoop and the reader may wish to return to that chapter for additional details.

The remaining chapters, viz., 2 and 8, are somewhat independent in their own right. Chapter 2 by Clarricoats discusses the subject of hybrid-mode feeds for reflector antennas, a topic that is of considerable current interest in the area of satellite communication antennas. Though numerical methods are employed for the solution of propagation characteristics of uniform and tapered corrugated waveguides, the emphasis here is on practical applications of these structures. Finally, Chapter 8 by Unger deals with the topic of the "waveguides of the future", viz., planar and fiber waveguides for optical communication, and presents a comprehensive review of analysis and performance of these two types of dielectric waveguides. Ray methods developed in Chapter 4 are frequently used for analysing these inhomogeneous waveguides with attendant simplification in analysis and gain of physical insight into the mechanism of operation of these waveguides.

Typically, the short courses on electromagnetics and antennas cover a diverse range of topics and the Eindhoven course was no exception. By its very nature, this book deviates in theme and structure from the standard texts in electromagnetic theory or antennas. The editors sincerely hope, however, that the wealth of information in this book, which has been put together by eight leaders in the field, will be challenging and estimulating to the electromagnetics community, and that this book will be reviewed with the same generous interest and enthusiasm as the previous volumes.

E.J. Maanders and R. Mittra Directors,
Eindhoven 1976 Summer School on Electromagnetics
and Antennas

Contents

# 1. Low-frequency asymptotic techniques

By J. Van Bladel University of Ghent, Belgium

## INTRODUCTION

In the present chapter, we assume the maximum dimensions L of the objects of interest (current-carrying volumes, scatterers...) to be small with respect to the free-space wavelength $\lambda$ (Fig. 1.1). The factor kL, where L is the diameter of the smallest sphere containing all points of the object, is therefore a small parameter. Low-frequency techniques are essentially perturbation techniques, based on a power-expansion of the form

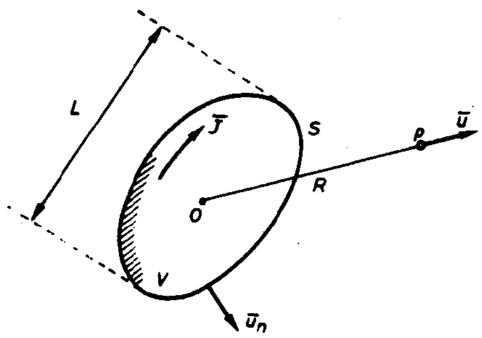


*Fig. 1.1*
*Current-carrying volume*

$$\phi=\phi_0+kL\phi_1+(kL)^2\phi_2+\dots \qquad (k=\frac{2\pi}{\lambda}) \qquad (1.1)$$

An equivalent expansion, of more frequent use, is

$$\phi=\phi_0+jk\phi_1+(jk)^2\phi_2+\dots \qquad (1.2)$$

The perturbation technique proceeds by inserting (1.2) in the relevant differential or integral equations, and by subsequently identifying terms of the same order in jk on both sides of the equations. The mathematical level of the method is therefore rather elementary, except for some very difficult questions of convergence. The practical implications, however, are considerable, and form the justification for the present review. Our survey is not restricted to electromagnetic topics, but will also encompass some selected acoustic problems. Acoustic situations are often directly relevant to their electromagnetic counterpart. They have, in addition, the advantage of floodlighting the essential points of a method, without noise interference from mathematical details.

# ACOUSTIC SCATTERING

## 1.1 Scattering by a soft body. The zero-th order term

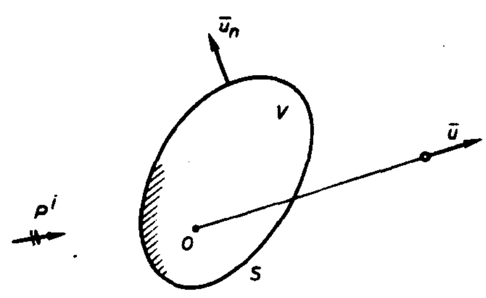


*Fig. 1.2*

*Acoustic scatterer in an incident wave*

Fig. 1.2 shows a soft body immersed in an incident pressure-wave $P^i$. The boundary condition $P = 0$ can be rewritten as

$$P^s = -P^i \qquad \text{on } S \tag{1.3}$$

The unknown scattered pressure is expanded in a series

$$P^s = P_0^s + jkP_1^s + (jk)^2 P_2^s + \dots \tag{1.4}$$

Separate (iterated) equations for the $P_n^s$, can be obtained from the integral representation [1]

$$P^s(\bar{r}) = \iint_S \left[ G(\bar{r}|\bar{r}') \frac{\partial P^s}{\partial n'} - P^s \frac{\partial G(\bar{r}|\bar{r}')}{\partial n'} \right] dS' \qquad (\bar{r} \text{ outside } S) \tag{1.5}$$

where $G(\bar{r}|\bar{r}')$ is the Green's function for free-space, viz.

$$G(\bar{r}|\bar{r}') = -\frac{1}{4\pi} \frac{e^{-jk|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|} \tag{1.6}$$

Equation (1.5) implies that $P^s$ can be calculated once $P^s$ and $\frac{\partial P^s}{\partial n}$ are given on the surface (in fact, one of these values suffices to determine the field uniquely). In the present case, $P^s$ is known through (1.3), but $\frac{\partial P_s}{\partial n}$ is not given explicitly. A formula such as (1.5) might therefore be branded as useless. It turns out, however, that significant results obtain from inserting (1.4) in (1.5), together with a corresponding expansion for $e^{-jk|\bar{r}-\bar{r}'|}$. For distances $|\bar{r}-\bar{r}'|$ small with respect to $\lambda$ (i.e. in the static region), identification of terms of zero-th order in jk yields

$$P_0^s(\bar{r}) = -\frac{1}{4\pi} \iint_S \frac{1}{|\bar{r}-\bar{r}'|} \frac{\partial P_0^s}{\partial n'} dS' - \frac{1}{4\pi} \iint_S P_0^i(\bar{r}') \frac{\partial}{\partial n'} \left( \frac{1}{|\bar{r}-\bar{r}'|} \right) dS' \tag{1.7}$$

If the body is immersed in a plane wave $e^{-jk\bar{u}_i \cdot \bar{r}}$, $P_0^i$ is equal to one, and the second term disappears. The first term, however, survives. Setting $\bar{r}$ on S(whereupon the left hand member takes the value $P_0^s = -P_0^i = -1$) yields an integral equation for $\frac{\partial P_0^s}{\partial n}$, which is precisely of the type satisfied by the surface-charge density $\rho_S$ on the metallized surface S carried to unit potential. The determination of $\frac{\partial P_0^s}{\partial n}$ is seen to reduce to the solution of an electrostatic problem. This is not surprising, as propagation effects require distances of the order of $\lambda$ before their full power can be felt. The static nature of the low-frequency approximation was recognized years ago by Lord Rayleigh. Present-day efforts center on the determination of higher-order correction terms in (1.4), with a view toward extending the use of the low-frequency approximations to the lower part of the resonance region (where L becomes of the order of $\lambda$).

## 1.2 Scattering by a soft body. Higher-order terms

To determine the various $P_n$, it is first possible to express the problem in the form of a sequence of (iterated) integral equations [2]. The "mother" equation is obtained by writing a relationship similar to (1.5) for the incident field, viz.

$$0=\iint_S\left[G(\bar{r}|\bar{r}')\frac{\partial P^i}{\partial n'} - P^i\,\frac{\partial G(\bar{r}|\bar{r}')}{\partial n'}\right]dS'=0 \qquad \bar{r} \text{ on } S \qquad (1.8)$$

and adding it to (1.5) to yield

$$\iint_S G(\bar{r}|\bar{r}')\left[\frac{\partial P^s}{\partial n'} + \frac{\partial P^i}{\partial n'}\right] dS'=-P^i \qquad \bar{r} \text{ on } S \qquad (1.9)$$

This equation generates the iterated set

$$\frac{1}{4\pi}\iint_S \frac{1}{|\bar{r}-\bar{r}'|}\left(\frac{\partial P_0^s}{\partial n'} + \frac{\partial P_0^i}{\partial n'}\right)dS'=P_0^i$$

$$\frac{1}{4\pi}\iint_S \frac{1}{|\bar{r}-\bar{r}'|}\left(\frac{\partial P_1^s}{\partial n'} + \frac{\partial P_1^i}{\partial n'}\right)dS'=P_1^i+L_0 \qquad (1.10)$$

In the second equation, $L_0$ is a characteristic length of the body, given by

$$L_0=\frac{C}{4\pi\varepsilon_0} \qquad (1.11)$$

where C is the capacity of the metallized body. For a sphere, for example, $L_0$ is equal to $\underline{a}$, the radius.

Equation (1.9) is a singular integral equation of the first kind, to be satisfied by the normal derivative of the total pressure $P=P^i+P^s$. It is also possible [ 4] to obtain an integral equation of the second kind for $\frac{\partial P}{\partial n}$. Thus,

$$\frac{1}{2}\frac{\partial P(\bar{r})}{\partial n} - \lim_{\sigma\to 0}\iint_{S-\sigma}\frac{\partial G(\bar{r}|\bar{r}')}{\partial n}\frac{\partial P}{\partial n'}dS' = \frac{\partial P^i(\bar{r})}{\partial n} \qquad \text{for } \bar{r} \text{ on } S \tag{1.12}$$

where $\sigma$ is a small area containing point $\bar{r}$.

Having determined the expansion terms in jk for $\frac{\partial P^s}{\partial n}$ on S, we are now in a position to calculate $P^s$ everywhere through use of (1.5). Thus,

$$P^s(\bar{r},k)=\iint_S G(\bar{r}|\bar{r}')\left(\frac{\partial P_0^s}{\partial n'} + jk\frac{\partial P_1^s}{\partial n'} + \ldots\right)dS'$$

$$-\iint_S (P_0^i+jkP_1^i+\ldots)\frac{\partial G(\bar{r}|\bar{r}')}{\partial n'}dS' \tag{1.13}$$

This expression is suitable for the evaluation of $P^s$ in the radiation zone. If the incident field is a plane wave, for example, one finds

$$\lim_{R\to\infty} P^s=-L_0\frac{e^{-jkR}}{R} - jkL_0^2\frac{e^{-jkR}}{R} + \text{terms in } k^2 \tag{1.14}$$

The first two orders clearly represent omnidirectional patterns. They show that the (low-frequency) far-field does not reveal anything about the shape of the scatterer; it merely gives information on a bulk property, namely $L_0$. Higher frequencies are necessary to better "feel" the shape of the object, through non-uniform phase-illumination of its various parts. This, in turn, results in direction-sensitive interference, and a fine structure in the pattern. Equivalently, the detailed structure of the scatterer cannot be revealed unless higher multipole modes in (1.14) are excited through use of higher frequencies. Analog remarks can be made for the electromagnetic field.

Eq. (1.14) shows that the <u>first order</u> field at large distances depends only on a <u>zero-th order</u> property, namely $L_0$. This result was arrived at by making use of a basic reciprocity property, viz. [ 1]

$$\iint_S \phi_1\frac{\partial\phi_2}{\partial n}dS=\iint_S\phi_2\frac{\partial\phi_1}{\partial n}dS \tag{1.15}$$

valid when $\phi_1$ and $\phi_2$ are solutions of Laplace's equation and are regular at infinity. It follows that the dominant term in the scattering cross--section is also a function of $L_0$ alone. More precisely,

$$\sigma = 4\pi L_0^2 + \text{terms in } k^2 \tag{1.16}$$

The curve of $\sigma$ as a function of kL is seen to start as a constant augmented by a parabola of the second degree.

The value of $L_0$ satisfies various inequalities, which can serve to establish upper and lower bounds for this quantity. For example [3]

$$L_0 > \sqrt[3]{\frac{3}{4\pi}} \sqrt[3]{V} \tag{1.17}$$

where V is the volume of the scatterer. For all convex bodies, in addition,

$$\frac{S^2}{12\pi V} > L_0 \tag{1.18}$$

where S is the area of the boundary surface. The equality sign corresponds to the sphere. Senior has calculated $L_0$ for rotationally symmetric bodies (e.g. the spheroid), and discovers that the variation of $\left(\frac{L_0}{\ell + w}\right)$ as a function of $(\ell/w)$ is very similar for all shapes which he investigated ($\ell$=axial length, w=width).

The scattering problem for the soft body is, fundamentally, an exterior Dirichlet problem for the wave-equation. Kleinman [4] has described a method by which the solution is explicitly expressed in terms of the Green's function for the corresponding potential problem. The resulting integral equation can be solved iteratively.

The exterior problem can also be tackled by inserting expansion (1.4) in the relevant differential equation, viz. $\nabla^2 P^s + k^2 P^s = 0$. The procedure, which is often termed Stevenson's method, yields

$$\begin{aligned} &\nabla^2 P_0^s = 0 \qquad && P_0^s = -P_0^s \quad \text{on } S \\ &\nabla^2 P_1^s = 0 && P_1^s = -P_1^i \quad \text{on } S \\ &\nabla^2 P_2^s = P_0^s && P_2^s = -P_2^i \quad \text{on } S \end{aligned} \tag{1.19}$$

To solve these equations, it is necessary to know the behavior of $P^s$ in the distant static field (i.e. at distances large with respect to L, but small with respect to $\lambda$). Eq.(1.7) implies that $P_0^s$ is regular there. A similar formula, written for $P_1^s$, shows that this quantity approaches $L_0$ at large distances, but that $(P_1^s-L_0)$ is regular [ 1] .

## 1.3 Low-frequency scattering by hard bodies

The configuration of interest is shown in Fig. 1.2. The boundary condition for a hard body is $\frac{\partial P}{\partial n} = 0$ or, equivalently,

$$\frac{\partial P^s}{\partial n} = -\frac{\partial P^i}{\partial n} \quad \text{on } S \tag{1.20}$$

The analysis proceeds much as in the case of a soft body. In an incident plane wave, for example, the zero-th order term $P_0^s$ is absent, and the series for $P^s$ starts with the first order term. Thus,

$$P^s=jkP_1^s+(jk)^2P_2^s+\ldots \tag{1.21}$$

The reason for the absence of $P_0^s$ is evident : $P_0^i$ is constant in the vicinity of S, hence satisfies the boundary condition on its own, without the need for an additional (scattered) field. The requirements on $P_1^s$ are :

$$\begin{aligned} &\nabla^2 P_1^s = 0 \\ &\frac{\partial P_1^s}{\partial n} = \bar{u}_i\cdot\bar{u}_n \quad \text{on } S \\ &P_1^s \quad \text{regular at large distances} \end{aligned} \tag{1.22}$$

The solution can be expressed as the superposition of a single- and a double-layer potential [ 1] . Each direction of incidence has its own $P_1^s$, but it suffices to solve the problem for three independent direction x, y, z to obtain $P_1^s$ for arbitrary $\bar{u}_i$ (Fig.1.3). Consider for example the potential generated in the vicinity of a magnetic conductor (boundary condition $\frac{\partial\psi}{\partial n}=0$), by an incident field $\bar{u}_x=-\text{grad}\psi_x^i$. The additional potential due to the presence of the scatterer satisfies

$$\begin{aligned} &\nabla^2 \psi_x^s = 0 \\ &\frac{\partial \psi_x^s}{\partial n} = \bar{u}_n\cdot\bar{u}_x \quad \text{on } S \end{aligned} \tag{1.23}$$

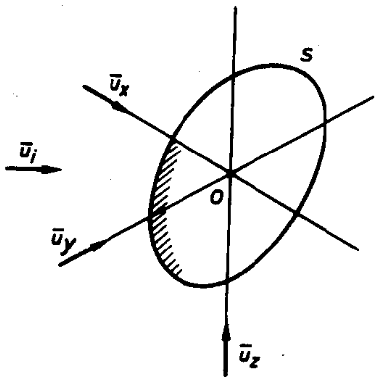


*Fig. 1.3*
*Scatterer in three orthogonal incident fields*

Analog equations can be written for the y- and z-directions. Once they are solved, the sought potential for arbitrary $\bar{u}_i$ follows as

$$P_i^s = \psi_x^s(\bar{u}_i \cdot \bar{u}_x) + \psi_y^s(\bar{u}_i \cdot \bar{u}_y) + \psi_z^s(\bar{u}_i \cdot \bar{u}_z)$$

$$= \bar{u}_i \cdot (\psi_x^s \bar{u}_x + \psi_y^s \bar{u}_y + \psi_z^s \bar{u}_z) = \bar{u}_i \cdot \bar{\psi}^s \tag{1.24}$$

The vector $\bar{\psi}^s$ characterizes the shape of the body, and is independent of the direction of incidence.

For a sphere of radius $\underline{a}$, for example,

$$\bar{\psi}^s = \frac{a}{2}\bar{u}_R \qquad \text{on } S \tag{1.25}$$

where $\bar{u}_R$ is a unit vector along the radius. It is to be noticed that $\bar{\psi}^s$ is a function of x, y, z, hence that it varies along S.

Turning now to $P_2^s$, we notice that this function satisfies [ 1]

$$\nabla^2 P_2^s = 0$$

$$\frac{\partial P_2^s}{\partial n} = -(\bar{u}_i \cdot \bar{u}_n)(\bar{r} \cdot \bar{u}_i) \tag{1.26}$$

from which can be deduced that the $\bar{u}_i$ dependence is of the form

$$P_2^s = \bar{u}_i \cdot \bar{\bar{\mu}} \cdot \bar{u}_i \tag{1.27}$$

where the dyadic $\bar{\bar{\mu}}$ characterizes the shape of the body. The far-field of the scatterer turns out to be

$$\lim_{R\to\infty} P^s = -\frac{k^2}{4\pi} V \frac{e^{-jkR}}{R} + \frac{k^2}{4\pi}\frac{e^{-jkR}}{R}\bar{u} \cdot [V\bar{u}_i - \iint_S (\bar{u}_i \cdot \bar{\psi}^s)\bar{u}_n \, dS] \tag{1.28}$$

$$+ \text{ terms in } k^3$$

It is seen that $P^s$ consists of a combination of a monopole- and a dipole term. The latter can be put in the form

$$\bar{u}.[\, V\bar{u}_i - \iint_S (\bar{u}_i . \bar{\psi}^s) \bar{u}_n dS] = \bar{u}_i . \bar{\bar{\pi}}_m . \bar{u} \tag{1.29}$$

where the dyadic $\bar{\bar{\pi}}_m$ is given by

$$\bar{\bar{\pi}}_m = V\, \bar{\bar{I}} - \iint \bar{\psi}^s \bar{u}_n dS = V\, \bar{\bar{I}} + \bar{\bar{W}} \ldots \tag{1.30}$$

For a sphere, for example, $\bar{\bar{\pi}}_m = 2\pi a^3 \bar{\bar{I}}$ ($\bar{\bar{I}}$ is the identity dyadic). The (symmetric) dyadic $\bar{\bar{\pi}}_m$ can also be written as

$$\bar{\bar{\pi}}_m = -\iint \bar{\psi} \bar{u}_n dS \tag{1.31}$$

with

$$\bar{\psi} = (\psi_x^s - x)\bar{u}_x + (\psi_y^s - y)\bar{u}_y + (\psi_z^s - z)\bar{u}_z = \psi_x \bar{u}_x + \psi_y \bar{u}_y + \psi_z \bar{u}_z) \tag{1.32}$$

Clearly, $\psi_x$ is the total potential which appears on the scatterer when the latter is immersed in an incident field $\bar{u}_x$.

Higher order terms for $P^s$ can be found, either by solving the differential equations with the help of Stevenson's method, or by considering the integral equations [ 2]

$$\frac{1}{2} P(\bar{r}) + \lim_{\sigma \to 0} \iint_{S-\sigma} \frac{\partial G(\bar{r}|\bar{r}')}{\partial n'} P(\bar{r}') dS' = P^i(\bar{r}) \tag{1.33}$$

$$\lim_{\sigma \to 0} \frac{\partial}{\partial n} \iint_{S-\sigma} \frac{\partial G(\bar{r}|\bar{r}')}{\partial n'} P(\bar{r}') dS' = \frac{\partial P^i(\bar{r})}{\partial n} \tag{1.34}$$

These can be solved iteratively. It is to be noticed that the hard-body problem is a Neumann problem. Its solution can be expressed in terms of the static Green's function relative to the Neumann boundary condition (i.e. $\frac{\partial P}{\partial n} = 0$ on S) [ 5] . Some numerical results are available [ 6][ 7] . In a more recent method, the solution is obtained by iteration of a boundary integral equation [ 8][ 9] . A pair of coupled surface integral equations is necessary to solve the scattering problem for a body which is neither soft nor hard, but penetrable [ 10] .

The "magnetic polarizability" dyadic $\bar{\bar{\pi}}_m$ has been studied extensively for bodies of revolution, in particular because of its relevance to electromagnetic problems (see Sec.1.11). Senior et al. have used (1.33) to obtain numerical results for several shapes, some of these good approximations to the profile of a rocket. For a body of revolution, the