MODERN CONTROL THEORY

Second Edition

William L. Brogan, Ph.D.

MODERN CONTROL THEORY

Second Edition

William L. Brogan, Ph.D.
Professor of Electrical Engineering
University of Nebraska

PRENTICE-HALL, INC.
Englewood Cliffs, New Jersey 07632

Library of Congress Cataloging in Publication Data

Brogan, William L. (date)
Modern control theory.

Bibliography: p.
Includes index.
1. Control theory. I. Title.
QA402.3.B76 1985 629.8′312 84-24818
ISBN 0-13-590316-5

Editorial/production supervision: *Raeia Maes*
Manufacturing buyer: *Anthony Caruso*

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-590316-5 01

Prentice-Hall International, Inc., *London*
Prentice-Hall of Australia Pty., Limited, *Sydney*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamerica, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Prentice-Hall of Southeast Asia Pte. Ltd., *Singapore*
Whitehall Books Limited, *Wellington, New Zealand*

CONTENTS

3

FUNDAMENTALS OF MATRIX ALGEBRA / 70

4

VECTORS AND LINEAR VECTOR SPACES / 92

5
SIMULTANEOUS LINEAR ALGEBRAIC EQUATIONS / 118

6
TRANSFORMATIONS / 152

7
EIGENVALUES AND EIGENVECTORS / 173

8

FUNCTIONS OF SQUARE MATRICES AND THE CAYLEY-HAMILTON THEOREM / 201

9

STATE VARIABLES AND THE STATE SPACE DESCRIPTION OF DYNAMIC SYSTEMS / 225

10

ANALYSIS OF THE STATE EQUATIONS FOR CONTINUOUS-TIME LINEAR SYSTEMS / 256

14

STABILITY FOR LINEAR AND NONLINEAR SYSTEMS / 345

15

THE RELATIONSHIP BETWEEN STATE VARIABLE AND TRANSFER FUNCTION DESCRIPTIONS OF SYSTEMS / 367

16

DESIGN OF LINEAR FEEDBACK CONTROL SYSTEMS / 397

Chapter 1

BACKGROUND AND PREVIEW

1.1 INTRODUCTION

Control theory is often regarded as a branch of the general, and somewhat more abstract, subject of systems theory [117].* The boundaries between these disciplines are often unclear, so a brief section is included to delineate the point of view of this book.

In order to put control theory into practice, a bridge must be built between the real world and the mathematical theory. This bridge is the process of modeling, and a summary review of modeling is included in this chapter [15, 109].

Control theory can be approached from a number of directions. The first systematic method of dealing with what is now called control theory began to emerge in the 1930s. Transfer functions and frequency domain techniques were predominant in these "classical" approaches to control theory. Starting in the late 1950s and early 1960s a time-domain approach using state variable descriptions came into prominence. This is what this book refers to as "modern" control theory. At the present time there is a blurring of the boundaries and a merging of these methods, along with new developments, into a unified theory. However, distinctions still exist, including the types of systems that can be dealt with efficiently, the forms of the system models, the methods of analysis, and even the philosophies that underlie the methods of design and analysis. Some of these distinctions are discussed in this chapter. A very brief look at the basic parts of classical theory will be reviewed in the next chapter. After that, the great majority of this text will be aimed at the state variable method. Bridges between the two will be built at several points throughout the book as seems appropriate.

*Reference citations are given numerically in the text in brackets. The references are in a single section at the end of the book.

1.2 SYSTEMS, SYSTEMS THEORY, AND CONTROL THEORY

According to the *Encyclopedia Americana*, a system is "... an aggregation or assemblage of things so combined by nature or man as to form an integral and complex whole ...." Mathematical systems theory is the study of the interactions and behavior of such an assemblage of "things" when subjected to certain conditions or inputs. The abstract nature of systems theory is due to the fact that it is concerned with mathematical properties rather than the physical form of the constituent parts.

Control theory is more often concerned with physical applications. A control system is considered to be any system which exists for the purpose of regulating or controlling the flow of energy, information, money, or other quantities in some desired fashion. In more general terms, a control system is an interconnection of many components or functional units in such a way as to produce a desired result. In this book control theory is assumed to encompass all questions related to design and analysis of control systems.

Figure 1.1 is a general representation of an *open-loop* control system. The input or control $\mathbf{u}(t)$ is selected based on the goals for the system and all available a priori knowledge about the system. The input is in no way influenced by the output of the system, represented by $\mathbf{y}(t)$. If unexpected disturbances act upon an open-loop system, or if its behavior is not completely understood, then the output will not behave precisely as expected.

Another general class of control systems is the *closed-loop* or *feedback* control system, as illustrated in Fig. 1.2. In the closed-loop system, the control $\mathbf{u}(t)$ is modified in some way by information about the behavior of the system output. A feedback system is often better able to cope with unexpected disturbances and uncertainties about the system's dynamic behavior. However, it need not be true that closed-loop control is always superior to open-loop control. When the measured outputs have errors which are sufficiently large, and when unexpected disturbances are relatively unimportant, closed-loop control can have a performance which is inferior to open-loop control.

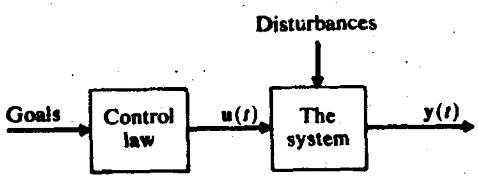


Figure 1.1 An open-loop control system.

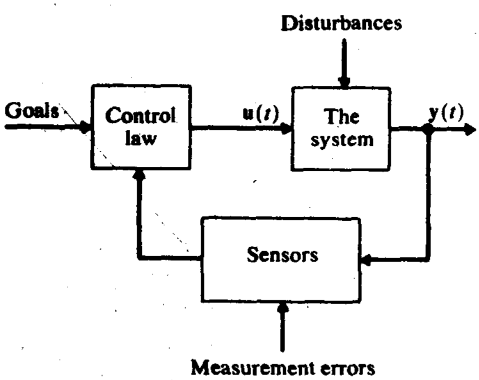


Figure 1.2 A closed-loop control system.

Example 1.1

A man's goal is to provide as much financial security as possible for his retirement years. He decides to have an extra $300.00 per month deducted from his paycheck and deposited in a tax sheltered annuity. His "input" each month is $u(t)$ and the system output $y(t)$ is the accrued value in his account. Since $u(t)$ is in no way affected by the current economic climate or by $y(t)$, this is an open-loop system. ■

Example 1.2

Another man has the same goal of achieving financial security. He decides to directly control his investments in the stock market. His input $u(t)$ at any given time is influenced by his perception of the market conditions, how well he has done so far, and so forth. This is a feedback or closed-loop system. ■

Example 1.3

A typical industrial control system involves components from several engineering disciplines. The automatic control of a machine shown in Fig. 1.3 illustrates this. In this example, the desired time history of the carriage motion is patterned into the shape of the can. As the cam-follower rises and falls, the potentiometer pick-off voltage is proportional to the desired carriage position. This signal is compared with the actual position, as sensed by another potentiometer. This difference, perhaps modified by a tachometer-generated rate signal, gives rise to an error signal at the output of the differential amplifier. The power level of this signal is usually low and must be amplified by a second amplifier before it can be used for corrective action by an electric motor or a servo valve and a hydraulic motor or some other prime mover. The prime mover output would usually be modified by a precise gear train, a lead screw, a chain and sprocket, or some other mechanism. Clearly, mechanical, electrical, electronic, and hydraulic components play important roles in such a system. ■

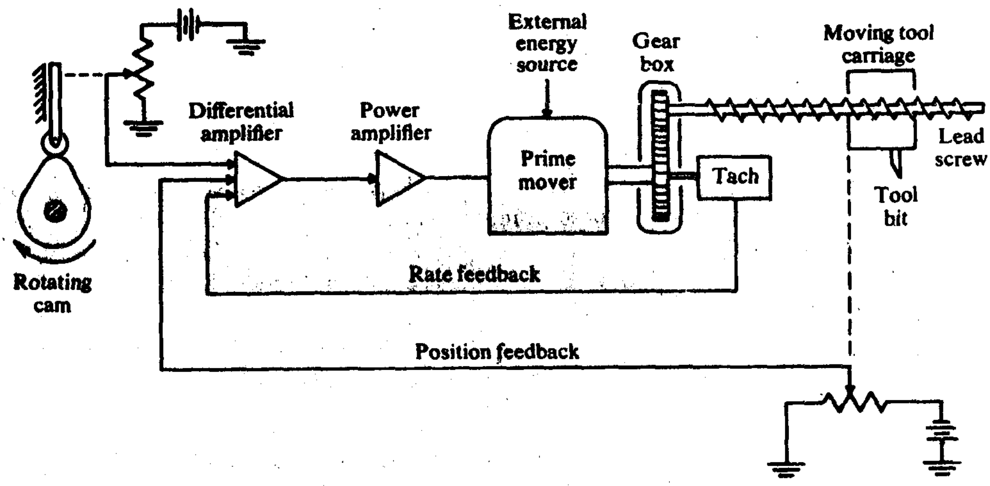


Figure 1.3

Example 1.4

The same ultimate purpose of controlling a machine tool could be approached somewhat differently using a small computer in the loop. The continuous-time or analog signals for position and velocity must still be controlled. Measurements of these quantities would probably be made directly in the digital domain using some sort of optical pulse counting circuitry. If analog measurements are made, then an analog-to-digital (A/D) conversion is necessary. The desired position and velocity data would be available to the computer in numeric form. The digital measurements would be compared and the differences would constitute inputs into a corrective control algorithm. At the output of the computer a digital-to-analog (D/A) conversion could be performed to obtain the control inputs to the same prime mover as in Example 1.3. Alternately, a stepper motor may be selected because it can be directly driven by a series of pulses from the computer. Figure 1.4 shows a typical control system with a computer in the loop. ■

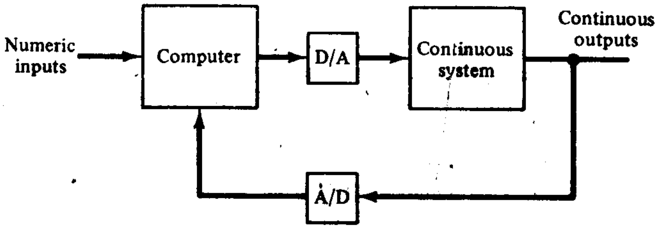


Figure 1.4

1.3 MODELING

Engineers and scientists are frequently confronted with the task of analyzing problems in the real world, synthesizing solutions to these problems, or developing theories to explain them. One of the first steps in any such task is the development of a mathematical model of the phenomenon being studied. This model must not be oversimplified, or conclusions drawn from it will not be valid in the real world. The model should not be so complex as to complicate unnecessarily the analysis.

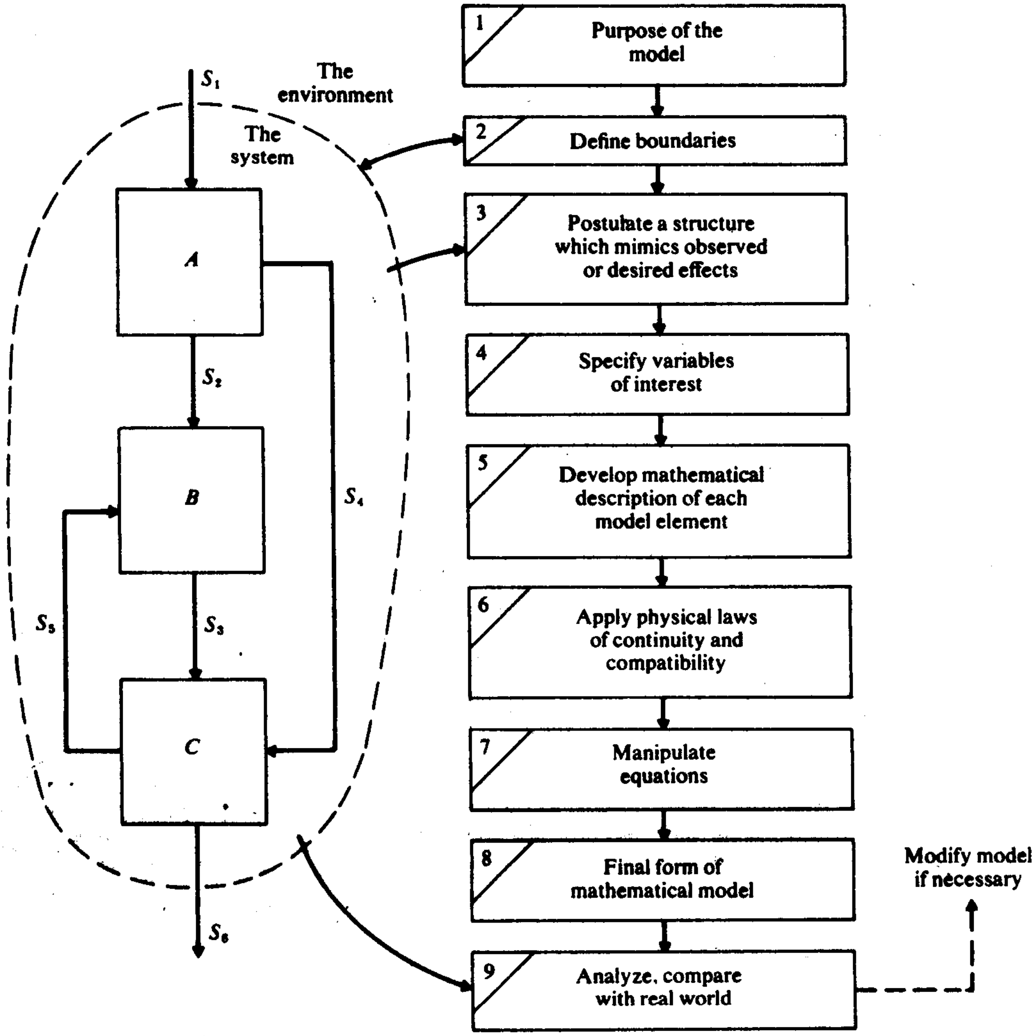
System models can be developed by two distinct methods. *Analytical modeling* consists of a systematic application of basic physical laws to system components and the interconnection of these components. *Experimental modeling*, or modeling by synthesis, is the selection of mathematical relationships which seem to fit observed input-output data. Analytical modeling is emphasized here. Some aspects of the other approach are presented in Chapters 5, 11, and 13 (least-squares data fitting).

An outline of the analytical approach to modeling is presented in Fig. 1.5. The steps in this outline are discussed in the following paragraphs.

1. *The intended purposes of the model must be clearly specified.* There is no single model of a complicated system which is appropriate for all purposes. If the purpose is a detailed study of an individual machine tool, the model would be very

different from one used to study the dynamics of work flow through an entire factory.

2. *The system boundary is a real or imagined separation of the part of the real world under study, called the system, and the rest of the real world, referred to as the environment.* The system boundary must enclose all components or subsystems of primary interest, such as subsystems A, B, and C in Fig. 1.5a.



(a) A portion of the real world

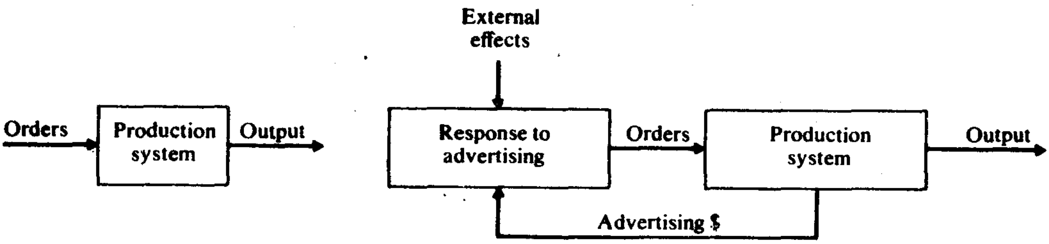
(b) Steps in modeling

Figure 1.5 Modeling considerations.

A second requirement on the selection of the boundary is that all causative actions or effects (called signals) crossing the boundary be more or less one-way interactions. The environment can affect the system, and this is represented by the input signal S_1. The system output, represented by the signal S_6, should not affect the environment, at least not to the extent that it would modify S_1. If there is no interest in subsystem A, then a boundary enclosing B and C, and with inputs S_2 and S_4, could be used. Subsystem C should not be selected as an isolated system because one of its outputs S_5 modifies its input S_3 through subsystem B. The requirement is that all inputs are known, or can be assumed known for the purpose of the study, or can be controlled independently of the internal status of the system.

Example 1.5

The purpose of the models of Fig. 1.6 is to study the flow of work and information within a production system due to an input rate of orders. These orders could be an input from the environment, as in Model I. If the purpose is to study the effects of an advertising campaign, then orders are determined, at least in part, by a major system variable. In this case the rate of orders should be an internal variable, as in Model II. ■



(*a*) Model I (*b*) Model II

Figure 1.6

3. *All physical systems, whether they are of an electrical, mechanical, fluid, or thermal nature, have mechanisms for storing, dissipating, or transferring energy, or transforming energy from one form to another.* The third step in modeling is one of reducing the actual system to an interconnection of simple, idealized elements which preserve the character of these operations on the various kinds of energy. An electric circuit diagram illustrates such an idealization, with ideal sources representing inputs. In mechanical systems, idealized connections of point masses, springs, and dashpots are often used. In thermal or fluid systems, and to a certain extent in economic, political, and social systems, similar idealizations are possible. This process is referred to as *physical modeling*. The level of detail required depends on the type of information expected from the model.
4. *If the physical model is properly selected, it will exhibit the same major characteristics as the real system.* In order to proceed with development of a mathematical model, variables must be assigned to all attributes of interest. If a quantity of interest does not yet exist and thus cannot be labeled, a modification will be

required in Step 3 so as to include it. The classification of system types is discussed in the next section. This book deals mainly with *deterministic lumped-parameter systems*. In all lumped-parameter systems there are basically just two types of variables. They are *through variables* (sometimes called path variables or rate variables) and *across variables* (sometimes called point variables or level variables). Through variables flow through two-terminal elements and have the same value at both terminals. Examples are electric current, force or torque, heat flow rate, fluid flow rate, and rate of work flow through a production element. Across variables have different values at the two terminals of a device. Examples are voltage, velocity, temperature, pressure, and inventory level.

5. *Each two-terminal element in the idealized physical model will have one through and one across variable associated with it.* Multiterminal devices such as transformers or controlled sources will have more. In every device, mathematical relationships will exist between the two types of variables. These relationships, called *elemental equations*, must be specified for each element in the model. This step could uncover additional variables that need to be introduced. This would mean a modification of Step 4. Common examples of elemental equations are the current-voltage relationships for resistors, capacitors, and inductors. The form of these relations may be algebraic, differential, or integral expressions, linear or nonlinear, constant or time-varying.
6. *After a system has been reduced to an interconnection of idealized elements, with known elemental equations, equations must be developed to describe the interconnection effects.* Regardless of the physical type of the system, there are just two types of physical laws that are needed for this purpose. The first is a statement of *conservation* or *continuity* of the through variables at each node where two or more elements connect. Examples of this basic law are Kirchhoff's node equations, D'Alembert's version of Newton's second law, conservation of mass in fluid flow problems, and heat balance equations. The second major law is a *compatibility* condition relating across variables. Kirchhoff's voltage law around any closed loop is but one example. Similar laws regarding relative velocities, pressure drops, and temperature drops must also hold. Both of these laws yield linear equations in through or across variables, regardless of whether the elemental equations are linear or nonlinear. This fact is responsible for the name given to linear graphs, an extremely useful tool in applying these two laws.

Example 1.6

Consider the system with six elements, including a source v_0, shown in Fig. 1.7. Each element is represented as a branch of the linear graph, and the interconnection points are nodes. Each node is identified by an across variable v_i, and each branch has a through variable, called f_i, with the arrow establishing the sign convention for positive flow. Let

b(number of branches) $= 6$
s(number of sources) $= 1$
n(number of nodes) $= 4$