

Finite element modelling of composite materials and structures

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D Hitchings and C Soutis**



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The Centre for Composite Materials at Imperial College has for many years organised a series of training courses on various aspects of composite materials. In 1996, recognising the increasing interest in structural analysis, a new course on 'Finite Element Analysis of Composites' was launched. The course was given jointly by the Centre and the Department of Aeronautics, and supported by the Engineering & Physical Sciences Research Council (EPSRC) as an MSc level module.

This book is based on the lecture notes prepared for the above course. The emphasis throughout is on long fibre-reinforced polymer (FRP) matrix composites, although any general analysis would be applicable to other forms of composite. The book starts with a review of the basic behaviour of FRP. Then, the fundamentals of finite element (FE) analysis are rehearsed. Following this, special issues relating to the applications of FE to FRP are discussed. Finally, a number of particular situations, such as holes and free edges, are presented, with FE results set alongside classical analysis and experimental data.

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Part I

Overview and review of composite materials

F. L. MATTHEWS

This part serves merely to 'set the scene' for the remainder of the book and also to revise the fundamentals of composite materials. The basic nature of fibre-reinforced plastic and its constituents is reviewed. Following this, two-dimensional stress-strain analysis is covered, leading to laminated plate theory. Finally some limitations of the latter are discussed.

1.1 Composite materials

1.1.1 General

The group of materials known as ‘composites’ is extremely large, although its boundaries depend on definition. Basically, we can consider a composite as any material that is a combination of two or more distinct constituents. This definition would encompass bricks, concrete, wood, bone, as well as modern synthetic composites such as fibre-reinforced plastics (FRP). The latter have become increasingly important over the past 50 years, and are now the first choice for fabricating structures where low weight in combination with high strength and stiffness are required. Such materials are sometimes referred to as ‘high-performance’ composites, and would often be composed of carbon fibres and epoxy resin.

Of course, composites can also be made by combining fibres with a metal or a ceramic matrix, but at the moment these have a very small market share and would be considered as specialist materials.

1.1.2 Properties and applications

Density, stiffness (modulus) and strength are the properties that initially come to mind when thinking of FRP, and these would certainly be the design drivers for materials’ selection for transport applications such as aircraft, motor vehicles and trains. However, this is a very narrow view of the potential of such materials, and they often score over metals and other conventional materials because of other mechanical, physical or chemical properties.

For example, FRPs are extremely corrosion-resistant and are, in consequence, used for chemical plant, water transport and storage, and flue gas desulphurisation plant. They have interesting electromagnetic properties and because of this glass fibre-reinforced resin is used to construct mine

counter measures naval vessels, the requirement being a non-magnetic material. As another illustration, carbon fibre-reinforced epoxy is used in medical applications because it is transparent to X-rays. Thermal properties can also be important; such materials have a low conductivity, making them useful for fire protection, and can have a zero coefficient of expansion, hence making possible the construction of temperature-stable components.

A key issue is that of cost. Although the constituents of FRP are relatively expensive, compared with conventional materials, the final component may well be less costly than one fabricated from metal, say. This is due, in part, to having a more integrated construction (lower 'parts count'), but depends crucially on the level of automation in the manufacturing process.

1.1.3 Production methods

The important factor about FRP is that, unlike metals, the material is made at the same time as the component. While this gives an increased freedom to the design process, it does mean that designers/analysts must pay close attention to the fabrication of the component or structure. The same constituents if processed by one method, could produce a composite with some properties modified compared with those produced by another method. Also, it is important to realise that the designer may call for a particular configuration that cannot actually be fabricated; for example, it is not possible to maintain a constant fibre angle and wall thickness along the length of a tapered cylinder produced by filament winding. Good communication at all levels throughout the whole design/fabrication/evaluation process is, therefore, essential.

1.2 Structural analysis

1.2.1 Classical analysis

The use of classical (continuum) methods of stress analysis has developed over many decades to give techniques that can be applied satisfactorily to a vast range of situations. Such analyses are based on the application of the equations of equilibrium and compatibility, together with the stress-strain relations for the material, to produce governing equations which must be solved to obtain displacements and stresses. Usually, assumptions must be made before a solution can be effected. So, for example, problems are considered as one- or two-dimensional, as when considering beams and plates, respectively. Often we take the material to be isotropic, but many analyses also exist for anisotropic materials.

As we move away from simple situations, say from a plain rectangular plate to one containing a cut-out, the governing equations become increasingly complicated and require ever-more sophisticated mathematical techniques to solve them. Classical methods are limited to simple geometries and 'real' structural features, e.g. the details of attachment of a stringer to a skin panel, cannot be analysed. In such cases we have to resort to finite element methods.

1.2.2 Finite element analysis

Finite element (FE) analysis is merely an alternative approach to solving the governing equations of a structural problem. Hence, FE and classical methods will produce identical results for the same problem, provided the former method is correctly applied.

The method consists of imagining the structure to be composed of discrete parts (i.e. finite elements), which are then assembled in such a way as to represent the distortion of the structure under the specified loads. Each element has an assumed displacement field, and part of the skill of applying the method is in selecting appropriate elements of the correct size and distributions (the FE 'mesh').

The FE method was initially developed for isotropic materials and the majority of elements available (the 'library') in any software package would be for such materials. To apply the technique to composites requires different element formulations that adequately represent their anisotropic, or orthotropic, stiffness and strength, as well as the laminated form of construction often used.

The main purpose of this book is to discuss the special issues that must be considered when using the FE method to analyse composite materials and structures. One of the key factors is the low through-thickness stiffness and strength; the former results in the need to adapt elements used for plates, and the latter results in the need to model delamination. Also some features of composite construction, such as filament winding, cannot easily be represented (if at all) by some FE packages. Following a review of composites and the FE method, the application of the method to composites is discussed in detail. The particular issues are then illustrated via a number of examples taken from particular situations.

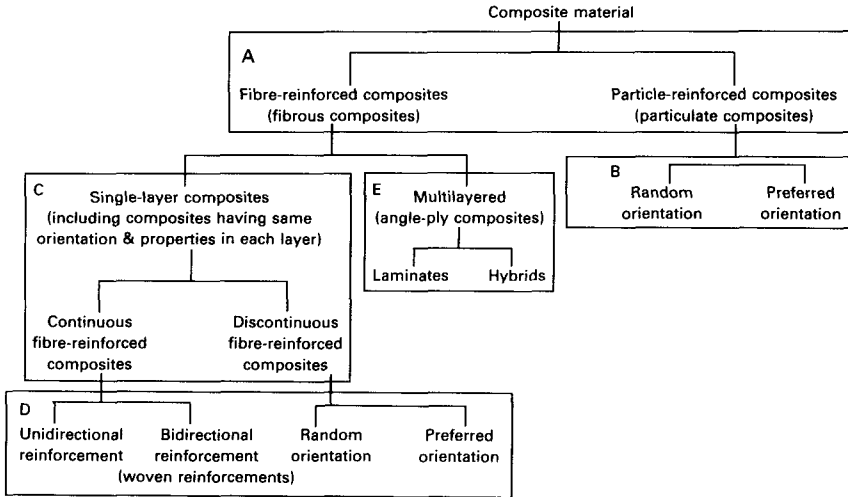
2.1 Basic characteristics

2.1.1 Definitions and classification

A composite is a mixture of two or more distinct constituents or phases. In addition three other criteria are normally satisfied before we call a material a composite. Firstly, both constituents have to be present in reasonable proportions. Secondly, the constituent phases should have distinctly different properties, such that the composite's properties are noticeably different from the properties of the constituents. Lastly, a synthetic composite is usually produced by deliberately mixing and combining the constituents by various means.

We know that composites have two (or more) chemically distinct phases on a microscopic scale, separated by a distinct interface, and it is important to be able to specify these constituents. The constituent that is continuous and is often, but not always, present in the greater quantity in the composite is termed the matrix. The normal view is that it is the properties of the matrix that are improved upon when incorporating another constituent to produce a composite. A composite may have a ceramic, metallic or polymeric matrix. The mechanical properties of these three classes of material differ considerably. As a generalisation, polymers have low strengths and Young's moduli, ceramics are strong, stiff and brittle, and metals have intermediate strengths and moduli, together with good ductilities, i.e. they are not brittle. Because of their economic importance, the emphasis in this text will be on polymer matrix composites (PMCs).

The second constituent is known to as the *reinforcing phase*, or *reinforcement*, as it enhances or reinforces the mechanical properties of the matrix. In most cases the reinforcement is harder, stronger and stiffer than the matrix, although there are some exceptions; for example, ductile metal reinforcement in a ceramic matrix and rubberlike reinforcement in a brittle polymer matrix. At least one of the dimensions of the reinforcement is



2.1 Classification of composite materials.

small, say less than 500µm and sometimes only of the order of a micro-metre. The geometry of the reinforcing phase is one of the major parameters in determining the effectiveness of the reinforcement; in other words, the mechanical properties of composites are a function of the shape and dimensions of the reinforcement. We usually describe the reinforcement as being either fibrous or particulate. Figure 2.1 represents a commonly employed classification scheme for composite materials which utilizes this designation for the reinforcement (Fig. 2.1, block A).

Particulate reinforcements have dimensions that are approximately equal in all directions. The shape of the reinforcing particles may be spherical, cubic, platelet or any regular or irregular geometry. The arrangement of the particulate reinforcement may be random or with a preferred orientation, and this characteristic is also used as a part of the classification scheme (block B). In the majority of particulate-reinforced composites the orientation of the particles is considered, for practical purposes, to be random.

A fibrous reinforcement is characterized by its length being much greater than its cross-section dimensions. However, the ratio of length to a cross-section dimension, known as the aspect ratio, can vary considerably. In single-layer composites long fibres with high aspect ratios give what are called continuous fibre-reinforced composites, whereas discontinuous fibre composites are fabricated using short fibres of low aspect ratio (block C). The orientation of the discontinuous fibres may be random or preferred. The frequently encountered preferred orientation in the case of a continuous fibre composite is termed unidirectional and the corresponding random