

IUTAM SYMPOSIUM GOTHENBURG 1970

CREEP IN STRUCTURES

1970

EDITOR

JAN HULT

**INTERNATIONAL UNION OF THEORETICAL
AND APPLIED MECHANICS**

CREEP IN STRUCTURES

1970

**SYMPOSIUM GOTHENBURG (SWEDEN)
AUGUST 17—21, 1970**

**EDITOR
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WITH 171 FIGURES

SPRINGER-VERLAG BERLIN HEIDELBERG NEW YORK 1972

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ISBN 3-540-05601-7 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-05601-7 Springer-Verlag New York Heidelberg Berlin

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Library of Congress Catalog Card Number: 75-182441

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Preface

The first IUTAM Symposium on Creep in Structures was held in Stanford, Cal. 1960 (Proceedings, ed. N. J. HOFF, Springer-Verlag 1962). That meeting reflected an intense research activity in the field of structural creep, and many important results were presented.

In spite of this progress design rules for high temperature equipment are still often inadequate. Design against creep deformation and creep rupture is all too often based only on experience and intuition. With technological processes moving into ever higher temperature regions the effects of creep must be given steadily increasing attention. The IUTAM Bureau in 1967 decided to arrange a Second Symposium on Creep in Structures, to be held in Gothenburg under the chairmanship of F. K. G. ODQVIST.

This meeting was to take stock of the progress made in the ten years since the first symposium. Notable advances in the establishment of a sound phenomenological theory, refined experimental techniques and several new methods of structural analysis of shells in the creep range gave impetus to this second symposium. Problems of rupture and stability under complex loads had been studied and many new results in these fields were presented and discussed.

Attendance was strictly limited to persons active in the field covered by the symposium. Financial support was generously provided by IUTAM, Chalmers University, the Swedish Board for Technical Development and four Swedish industrial companies (ASEA, Sandvikens Jernverk, Saab-Scania, Volvo).

The local arrangements of the symposium were in the hands of a committee (chairman J. HULT, secretary B. SUNDSTRÖM) which gratefully acknowledges all the devoted assistance given by the entire staff of the Solid Mechanics Division at Chalmers University.

The discussions during the technical sessions as well as in between them were vivid and fruitful. Concepts like Skeletal Points and Reference Stress gave impulses to debate and even to poem writing, as witnessed by the following opus by R. G. ANDERSON

Macavity's Stress

(with posthumous apologies to T. S. ELIOT)

The Reference Stress is a Helluva stress
But less of a stress than a maximum stress
And more of a stress than a minimum stress.

It seems to be positive, used by men,
Helps cut out algebra, gets rid of "n"
Appears in papers by Alec¹ and Fred¹,
Keeps some of the Delegates sleepless in bed.

And soon no doubt will make the news
As another information explosion in Applied Mechanics Refuse.

I want to express my gratitude to Dr. SUNDSTRÖM and to my secretary Miss INGRID WENNESJÖ for rapid and careful work in preparing this edition of the Proceedings, and to Springer-Verlag for undertaking the printing and publishing of the volume.

Finally I thank the Scientific Committee and all participants for making the symposium a good one. I am therefore sure that this volume will prove to be of good use among researchers as well as designers.

Gothenburg, November 1971

Jan Hult

¹ The Author could be referring to A. C. MACKENZIE and F. A. LECKIE.

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- Session 1:* Opening
(chairman: HULT)
GRALÉN, NIORDSON, ODQVIST, FINNIE
- Session 2:* Experimental analysis
(chairman: ŻYCKOWSKI)
HENDERSON/SNEDDEN, BESSELING/LAMBERMONT
- Session 3:* Experimental analysis
(chairman: JAHSMAN)
ALDÉN/ROHLIN, FINNIE/ABO EL ATA
- Session 4:* Composites
(chairman: LEMAITRE)
MILEIKO
- Summary:* HOFF
- City's reception

Tuesday, August 18

- Session 5:* Phenomenological theory
(chairman: SETH)
SOBOTKA, LECKIE
- Session 6:* Phenomenological theory
(chairman: STORÅKERS)
PONTER, MARRIOTT
- Sightseeing
- Session 7:* Phenomenological theory
(chairman: NASH)
RABOTNOV, SETH
- Session 8:* Phenomenological theory
(chairman: SPENCE)
MAZET
- Summary:* ŻYCKOWSKI

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- Session 9:* Thick-walled shells
(chairman: PONTER)
FAIRBAIRN/MACKIE, LEMAITRE
- Session 10:* Thick-walled shells
(chairman: KNETS)
JOHNSSON
- Summary:* CALLADINE
- Excursion*
- Banquet at Älvsborgs Castle*

Thursday, August 20

- Session 11:* Thin-walled shells
(chairman: PIECHNIK)
SPENCE, CALLADINE
- Session 12:* Thin-walled shells
(chairman: LECKIE)
ANDERSON, KUBA, PENNY
- Session 13:* Rupture
(chairman: HENDERSON)
WILLIAMS, PIECHNIK/CHRZANOWSKI
- Session 14:* Rupture
(chairman: LAMBERMONT)
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- Summary:* MARRIOTT

Friday, August 21

- Session 15:* Stability
(chairman: MAZET)
STORÅKERS, DISTEFANO/SACKMAN
- Session 16:* Stability
(chairman: PENNY)
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- Session 17:* Stability
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SAMUELSON, LEVI/HOFF
- Session 18:* Stability
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- Summary:* BESSELING
- Closure:* RABOTNOV

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Opening Address

By

F. I. Niordson

Secretary-General of IUTAM, Copenhagen, Denmark

Mr. Chairman! Ladies and Gentlemen!

First, may I extend the thanks of the International Union, which I am representing here today, to the President of this University, Professor NILS GRALÉN for housing the Symposium on the premises of his University and for his warm and hearty words of welcome.

The official opening of the Conference this morning marks the end of a long and tedious period of preparation, a period which has in fact lasted some two years. Having had the opportunity to follow the actions and the able and careful planning by the Scientific Committee under the chairmanship of Professor ODQVIST and by the Local Organizing Committee under the chairmanship of Professor HULT, I have seen, and I can assure you, that no effort has been spared to make this symposium a success.

The Bureau of the Union is fully aware of the strain and the burden which has been carried by the Host Institution in organizing this Symposium and gratefully recognizes the devoted work done by Professor HULT and his collaborators. We hope and believe that they will find their work worthwhile through the source of inspiration, which this conference will certainly become for the scientists and the young students at the Division of Solid Mechanics of the Chalmers University of Technology.

Now, Ladies and Gentlemen, on behalf of the International Union of Theoretical and Applied Mechanics, I have the honor and the great pleasure to declare this "Second Symposium on Creep in Structures" officially opened. May the scientific results obtained during the lectures and in the discussions be an ample reward for the efforts of all the participants.

Thank you Mr. Chairman.

From Stanford 1960 to Gothenburg 1970

By

F. K. G. Odqvist

Royal Institute of Technology, Stockholm, Sweden

Ladies and Gentlemen!

It is a great honor and a pleasure for me to greet you all a hearty welcome to this Second IUTAM Symposium on Creep in Structures.

The corresponding First Symposium took place at Stanford in 1960 [1] under the chairmanship of NICHOLAS HOFF. At that time the phenomenological theory of creep mechanics was about 50 years old, if one takes as point of departure the publication of the fundamental researches of the British physicist DA COSTA ANDRADE [2], who in 1910 introduced the terminology still in use. The concepts primary, secondary and tertiary creep in the case of uniaxial creep tests with constant load or stress are due to him, see Fig. 1.

It was, however, only about 20 years later that further important steps were taken, as a consequence of the demands of technology. At that time, the working temperature of structural parts of prime

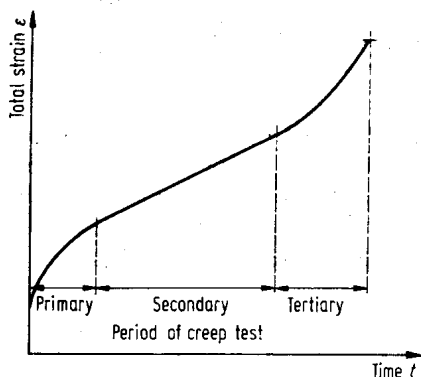


Fig.1. Creep curve for constant temperature and constant stress

movers, heat engines etc. had risen to a level when creep becomes of importance for designers and mechanical engineers. This called for better knowledge of creep phenomena. The next step was taken by F. H. NORTON [3] of MIT, Cambridge, Mass., who discovered the exponential law

$$\dot{\epsilon} \equiv d\epsilon/dt = k\sigma^n,$$

which applies to many metals.

Round 1930, R. W. BAILEY [4] of the Metropolitan Vickers Co had shown that creep deformation of structural metals—just as in time-independent plasticity—takes place under constant volume, and that a superimposed hydrostatic pressure does not influence creep deformation.

From these facts and the assumption of isotropy, the present speaker [5] in 1934 deduced constitutive relations for secondary creep under triaxial stress

$$\dot{\epsilon}_{ij} \equiv d\epsilon_{ij}/dt = f(\sigma_e) s_{ij},$$

where $\sigma_e^2 = (3/2) s_{ij}s_{ij}$ and, for conformity with Norton's law

$$f(\sigma_e) = (3/2) k\sigma_e^{n-1}.$$

The equations have the same form as von Mises' equation for time-independent plasticity, but time now appears as an essential variable. In creep theory, the normality condition is preserved from time-independent plasticity, but is now referred to the surfaces $\sigma_e = \text{constant}$, where σ_e is the second invariant of the stress deviation tensor. The tensorial creep rate $\dot{\epsilon}_{ij}$ is proportional to a function $f(\sigma_e)$, to some extent arbitrary, but may be fully determined if the two material constants of Norton's law are known.

In a doubly logarithmic plot of stress σ against creep rate v (and correspondingly σ_e against v_e in the triaxial case) Norton's law corresponds to a straight line with an inclination determined by the exponent n . This straight line will seldom fit experiments outside a range of three decades with respect to v . It is customary to use a representation with a broken set of straight lines, for example two lines as in Fig. 2, if a relationship valid over a larger range is looked for. Many other analytic representations of the function $f(\sigma_e)$ have been used and will make their appearance in the papers of this Symposium—in all probability.

For some materials, and particularly so in certain stress and temperature ranges, it is necessary to take into account primary creep. This was for instance the case when heat resistant alloys were developed during the 1930s and 1940s.

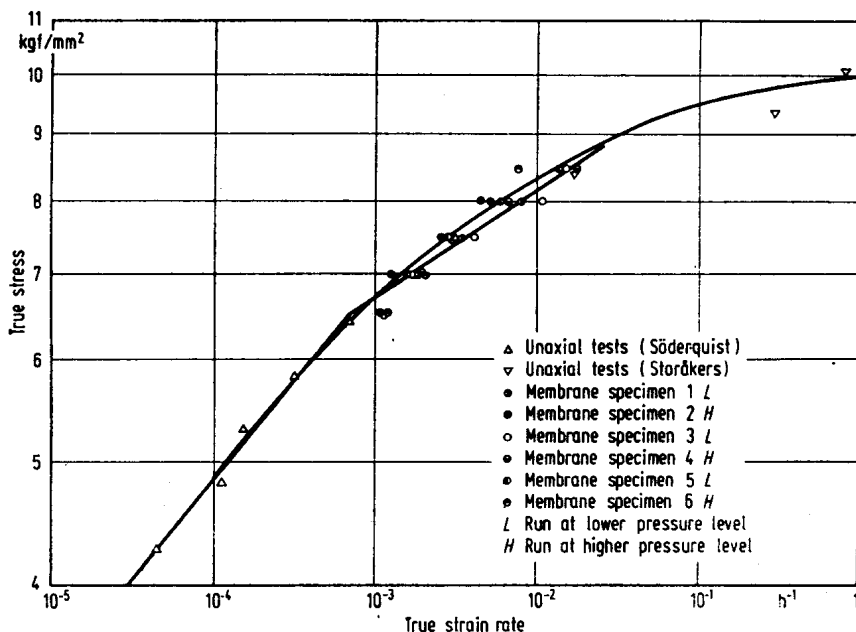


Fig. 2. Creep rate vs. stress for uniaxial and biaxial tests with Mg alloy ZRE 1

The first serious attempt towards a theory of primary creep was due to A. NADAI [6] and his group at the Westinghouse Electric Co in 1938. He suggested an "equation of state for strain-hardening". A corresponding equation for "time-hardening" was discarded for lack of agreement with experiments, for instance by I. FINNIE, but will make its reappearance at this Symposium, if I am not mistaken.

There are some more details which could be mentioned here but I shall content myself by stating, that the First IUTAM Symposium on Creep in Structures in 1960 was devoted mainly to applications of analytic methods to problems where the previously mentioned physical facts and constitutive equations derived from them were used. The fact that the equations of creep mechanics are strongly non-linear (mainly due to the non-linearity of Norton's law) puts formidable resistance to the development of exact analytic methods. It is not surprising that by this time the possibilities are almost at an end.

Very complete lists of known solutions are due to I. FINNIE [7] (1960) and to YU. N. RABOTNOV [8] (1966/1969). These authors point to the possibilities of numerical solutions. It is true that the development of digital computers has greatly helped to further our possibilities to find the consequences of nonlinear theories and to apply them to

real engineering structures. We shall see some such results also in this Symposium. The high value of reasonable physical and geometrical assumptions made by intuition, may for example be seen in the paper by C. R. CALLADINE, to be presented here.

The theories of tertiary creep and creep rupture have, almost exclusively, been developed after 1960. It is true that L. M. KACHANOV published his first researches in 1958, but they became known in the West only several years later [9]. Still within the limits of phenomenological theories, the Soviet-Russian schools of KACHANOV and RABOTNOV have introduced the concept of material deterioration. In its simplest form, it may be characterized by two material constants derived from an ordinary creep rupture curve, i.e. a relationship between applied stress and time to rupture.

Fig. 3a shows a typical creep rupture curve in doubly logarithmic plot. Notice the downward bend at higher life times. This renders the phenomenon of creep rupture an insidious character from the engineers' point of view. This was known long before any phenomenological theory of creep rupture had been developed, and was first attributed to material faults.

Fig. 3b shows the relationship between initial stress σ_0 and life-time t^* , as derived from the phenomenological theories. t_H^* represents a theory by HOFF [10] (1953), where ductile creep rupture only is taken into account and considered as a problem of instability. Further

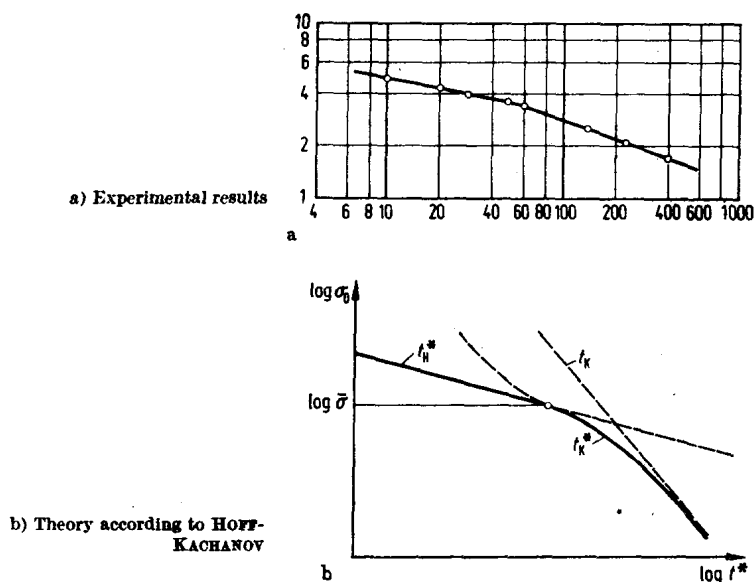


Fig. 3. Creep rupture times in doubly logarithmic plot