THE INSTRUMENT MANUAL

THE

INSTRUMENT MANUAL

FOURTH EDITION

UNITED TRADE PRESS LTD
9 GOUGH SQUARE, FLEET STREET, LONDON, E.C.4

1st Edition published 1949
2nd Edition published 1953
3rd Edition published 1960
4th Edition published 1971

Price £8·50 post free.

Printed for the Proprietors, United Trade Press Ltd., by Fisher, Knight & Co. Ltd., St. Albans, Herts., and published by United Trade Press Ltd. at 9, Gough Square, Fleet Street London, E.C.4.

INSTRUMENT MANUAL

INTRODUCTION

LET us commence (in a somewhat unorthodox fashion perhaps) with *two* quotations:

"The pace of British instrument development is such that on each of the sections in this work a text-book could be written, and in many cases this has been done. It is manifestly impossible to include the equivalent of twenty five text-books in one volume, and, as the editing proceeded, it became apparent that the task was becoming one of deciding more what was to be left out rather than what was to be included". (Introduction to third edition). "On all fronts progress has seemed to take on a change in pace which threatens to become overpowering and which has already proved to have overtaken many and left even the best of us breathless". (J. E. Samson, President of the Institute of Measurement and Control in his new year message for 1970, *Measurement and Control* Vol. 3, No. I. Jan. 1970, p5). The situation is admirably expressed by these two paragraphs. The "overpowering pace" has resulted in a flood of information in articles in the technical press, in papers read at meetings of learned institutes and institutions and in symposia and conference proceedings. On the theoretical side of automatic control alone, the number of text books published over the last 10 years has reached a hundred or two. In the first quotation above, a reference is made to the fact that a text book, in many cases, had been written on the subject of each section. There is a further development in this direction. Text books have not only been compiled on the main subject of a section, but also on the individual techniques handled in that section. One can quote a typical situation here. Section 21, in the present edition, deals with Non-destructive Testing. There are a large number of volumes devoted to this field as a whole, but at the same time a large number describe individual methods such as radiological and ultrasonic testing. The outpouring of data is significant from another aspect. An information service established solely for measurement and control instrumentation and components reached well over 3,000 items in its first year of operation.

The problem is obvious. How does one attempt to record in one volume of 500–600 pages the whole activities of the measurement and control field? The answer must lie in concise accounts of the principles of the older but well-established techniques and of the newer ones which have certainly come to stay, with some typical applications in design and uses in the field. Even so, one is on the inevitable razor's edge with a decidedly uncertain balance. One thing, however, is certain, fringe subjects in the light of modern trends must disappear. Accordingly the following sections have been eliminated from this edition.

Sections Eliminated

Engineering Precision Instruments and Gauges
Electrical Measuring Instruments
Aeronautical Instruments
Miscellaneous Electronic Instruments
Surveying Instruments
Microscopy
Meteorological Instruments

New sections that have been added.

New Sections

Fluid Logic
Techniques for Measuring Surface Temperature
Chromatography
Moisture Content of Liquids and Solids
S.I. Units
Machine Tool Control

Most of the remaining sections have been extensively revised and recast.

Long and detailed discussions were held with various authorities in the computer control field. Without exception they advised that the subject of direct digital control could not be satisfactorily covered in the space of a section of the Instrument Manual, however large. This advice was emphasized by the publication *Progress in Direct Digital Control* by the Instrument Society of America which includes articles and papers by a number of workers in D.D.C. and deals with most aspects of the field. To have emulated this in the Instrument Manual would have required about 300 pages! Rather than have a presentation inadequate in its treatment it was reluctantly decided to omit D.D.C. from the present edition. with the thought that the subject might appear in a companion volume devoted solely to control.

S.I. Units

These also have presented a problem. Although their adoption by the instrument industry of this country is established, no directive (at the time of going to press) has been issued on the subject of preferred ranges. Our present scales do not lead always to useful equivalents in S.I. Units. This is particularly evident in temperature measurement if one tries to produce Kelvins from even scales in degrees Celsius (0 to 100° C for example). Again, this publication is used in countries which have not yet adopted the S.I. system and are likely for a few years, at least, to retain the present system of units. We are in a transient period and for this edition it seemed preferable to include a small section on S.I. units with equivalents and some discussion on problems associated with their use. In addition, whereever possible in the text of the various sections equivalent ranges in S.I. units have been included. Where these are not included, it means that the corresponding S.I. unit range has yet to be decided.

Acknowledgements

One must thank all the manufacturers who have supplied information. This is suitably acknowledged in the text. Any omission is accidental and not intentional.

J. T. M.

General Editor
J. T. Miller, B.Sc., F.Inst.P., Hon.M.Inst.M.C.

CONTRIBUTORS TO FOURTH EDITION

Section	*Name of Section*	*Author*
1.	Automatic Control	M. Healey M.Sc., C. Eng., M.I.E.E.
4.	Measurement and Control of Viscosity (from 3rd Edition)	R. McKennell
5.	Measurement of Fluid Flow	A. Linford B.Sc. C.Eng. M.I.C.E., A.M.I.W.E.
7.	Measurement and Control of Moisture Content of Air and Gases	A. Pande, M.Sc., F.Inst.P.
8.	Techniques for Measuring Surface Temperature	G. G. Watson, C. Eng., M.I.Mech.E., M.I.E.E.
12.	Fluidics	A. Linford, B.Sc C.Eng. M.I.C.E., A.M.I.W.E.
19.	Telemetry	E. J. Whiteman, C.Eng., A.M.I. Mech.E, A.F.R.Ae.s. M.I.E.E.
22.	Numerical Control	C. J. Charnley, B.Sc.

CONTENTS

1. AUTOMATIC CONTROL

CONTENTS OF SECTION

1. Introduction

THE practice of automatic control is long standing, one of the earliest examples being the Watt governor in 1788. Theory, lagging behind practice, was put on a firmer basis only a century ago by Maxwell with his paper on dynamic stability, but it was not until 1932 that the major break-through came. In that year, Nyquist introduced his paper on frequency response analysis of feedback amplifiers, followed in 1934 by Hazen's paper applying Nyquist's work to servomechanisms. World War II created the opportunity for fast progression and Bode, Weiner and others rapidly introduced new and practical techniques.

Rather unfortunately, the control of processes and the control of mechanisms became separate problems, due entirely to the difference in time constants involved. Servo-mechanisms and feedback amplifiers were amenable to solution by frequency response methods while process controls were not since the frequencies involved (cycles/minute) could not be generated by commercial equipment. This same discrepancy of time constants led the process control into the pneumatic field while servomechanisms could use the much easier electronic techniques.

Thus, an undesirable gulf appeared between the techniques, practices and terminology of process and mechanical control systems. Modern electronics have brought a change in the state, however, in the following ways :

(i) Integrated electronics allow the use of cheap amplifiers to achieve long time constants, making available the non-interacting controller which proved generally unattainable with pneumatics.* This was achieved at an increase in cost so the process controller is indeed fortunate in being able to choose between electronic (more accurate) or pneumatic (reliable, cheap) controllers to suit his purpose.

(ii) Very low frequency oscillators using digital generating techniques are commercially available at low cost.

(iii) The digital computer has made possible much larger scale, more complex forms of control, moving all the time nearer to *optimum* control.

Thus, the two fields of process control and mechanical control are again better approached from a common front. This means, in many cases, adoption of new terminology and so this chapter begins with some definitions, with the old terminology shown in brackets.

An Automatic Control System is a control system which tends to maintain a prescribed relationship of one system variable to another by comparing functions of these variables and using the difference as a means of control. (Fig. 1).

The Controlled Variable (controlled condition or output), C, is that quantity or condition of the system which is directly measured and controlled.

The Reference Input (set value or index value), R, is a signal established as a standard to dictate the desired or set value of C.

The Primary Feedback Signal, B, is the measured value of C which is compared with R to obtain the actuating signal.

* One exception was the Williamson controller of Negretti & Zambra Ltd.

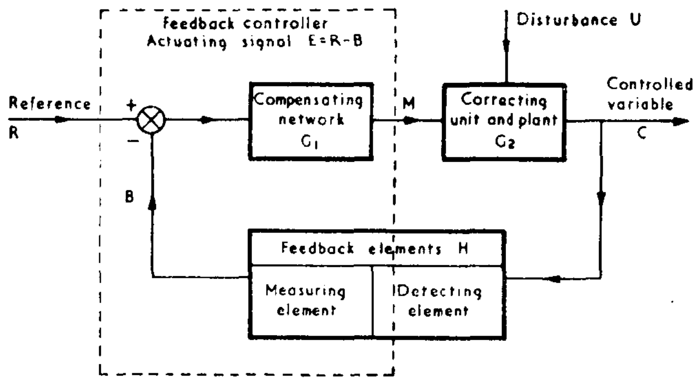

Fig. 1. Schematic block diagram of a basic closed loop automatic control system

The Actuating Signal (error signal, deviation), E, is R minus B. Deviation is also designated by θ.

The Manipulated Variable (correcting conditions), M, is the quantity or condition, derived from E, which the controller applies to the controlled system.

The Feedback Controller (controller) is a device which accepts the reference, R, and the detected value of C and produces a signal, M, dependent upon some functions of $E=R-B$.

The Summing Point is a symbol used to indicate that part of the controller which sums signals such as R and B. The polarity of the summation is indicated by plus or minus signs and arrows are used to show inputs and outputs.

The Feedback Elements (measuring unit, pick-off, transducer or monitor), H, comprise the portion of the feedback control system which establishes the relation between B and C. This is often split in process controls into a *detecting element* and a *measuring element,* since it is physically convenient to have the detecting element in the controller.

Compensating Network (controlling unit, control elements), G_1 is that part of the controller which establishes the relationship between M and E, e.g. proportional plus derivative action.

Controlled System (plant), G_2, is the body, process or plant, a particular quantity or condition of which is to be controlled.

Correcting Unit is that part of the plant which adjusts the physical quantity of the plant to affect adjustment of C.

A Disturbance, U, is a signal other than R which tends to affect C.

2. The System

Many systems are encountered, of varying degrees of complexity, in which one or more parameters require careful control. Typical examples are the control of temperature, pressure, or flow in the process field ; position, velocity or torque in the mechanical field ; and voltage amplification ratio in the electrical field. The examples in the preceding list are very diverse in physical nature but are similar in more superficial ways, i.e. they are all prone to vary due to external or internal "disturbances" and it is likely that their value will have to be changed to meet some new requirement.

The parameter of the system which it is required to control, e.g. the temperature, velocity, etc., is called THE CONTROLLED VARIABLE, C.

The basic elements of the system are called the PLANT, e.g. the heater and liquid bath, motor and gearing, etc., so that C is the controlled variable of the plant. The term "output" has been commonly used for C, but this is misleading, since for a simple hot water supply the output is the water drawn off, the controlled variable is the *temperature* of the water.

If it is to be possible to alter, and eventually to control, the controlled variable, then the plant must be capable of variation and some form of CORRECTING UNIT is required to manipulate the plant, e.g. in a flow control system, a throttle valve is the correcting unit, the setting of which (possibly indirectly controlled by an electric motor or a pneumatic motor) governs the flow.

It is important that the power required to adjust the correcting unit is small so that a low power input signal can control a large power in the plant itself.

The signal fed to the correcting unit is processed by a CONTROLLER which is "told" the required value of the controlled variable (DESIRED VALUE) and converts this into an appropriate form to suit the correcting unit. The controller could be no more complex than a calibrated dial on a valve but for automatic systems will be required to perform more complex duties and is either a pneumatic or an electronic device.

3. The Feedback Principle

Broadly speaking, all systems having a variable quantity which is to be controlled can be put into one of two categories : (a) unmonitored and (b) monitored as shown in Fig. 2.

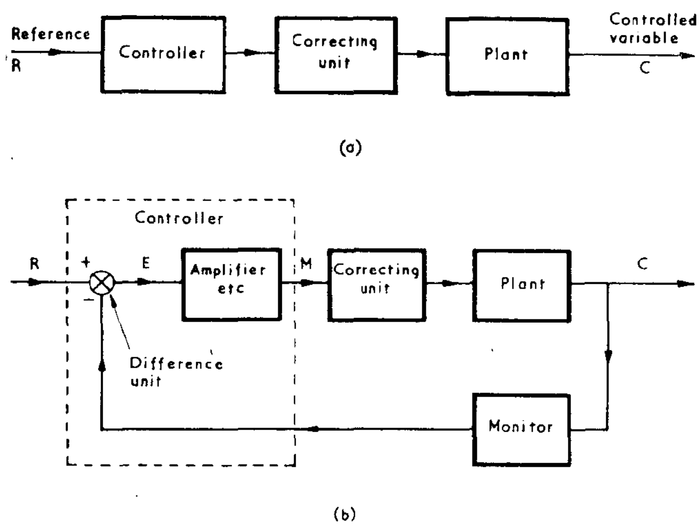

Fig. 2 (a) Unmonitored (open loop) system. (b) Monitored (closed loop) system

(a) Unmonitored or Open Loop System

The REFERENCE, R, is fed direct to the controller at a value calculated to give the correct value of C. One could consider the previous example of flow control, where the controlled variable, C, is the flow and the control is a fixed valve opening governed by the reference pressure, R, fed to the operating cylinder. Instead, to bring out the general nature of the control problem, consider a water heating system (C=temperature) where a fixed voltage is applied to an immersion heater, the temperature rising to such a value that the natural heat losses (radiation, conduction, etc.) equal the heat input. Such a system is inaccurate, since any change in the *status quo* will affect the temperature, e.g. change in ambient temperature and drawing off water from the tank.

(b) Monitored or Closed Loop System

In a closed loop system the signal fed to the correcting unit, termed the **ACTUATING SIGNAL, E,** is the *difference* between the reference and the monitored or measured value of the controlled variable, the differencing process being performed by the controller.* This means that when C is at the value set by R, the signal from the monitor, B, equals R and $E=R-B$ is zero. No further action takes place. Whenever B is different from R, indicating that C is incorrect, a finite actuating signal is developed of such a polarity as to produce action from the controller and plant tending to move C towards its set value. This is termed **NEGATIVE FEEDBACK,** positive feedback would mean that with $B \neq R$, the finite value of E causes action so as to make C further in error, thus losing all control; such devices are used only for oscillators and are therefore of no consequence here.

but it will certainly reduce the effect. The system is said to have REGULATION against disturbances.

(ii) *Controllability.* The correcting unit in practice may control the flow of large amounts of power into the plant, but it is operated by a low power signal, E. Thus, since E *controls*, and not provides the work effort, it is easy to change R, at low power levels, to control indirectly the controlled variable at high power levels. This is particularly significant in large complex processes, leading to easier interconnection of parts of the system.

4. Some Applications of Closed Loop Control

Control systems applications are split into categories as shown in Fig. 3.

The process industries encompass most categories. The overall control of a process, say a distillation column by varying the inlet flow, temperature and reflux, may well employ subsidiary controlled systems for, say, the temperature control. Again the control of large scale processes will involve remote control of valves with increasing use of motorized valves with position control.

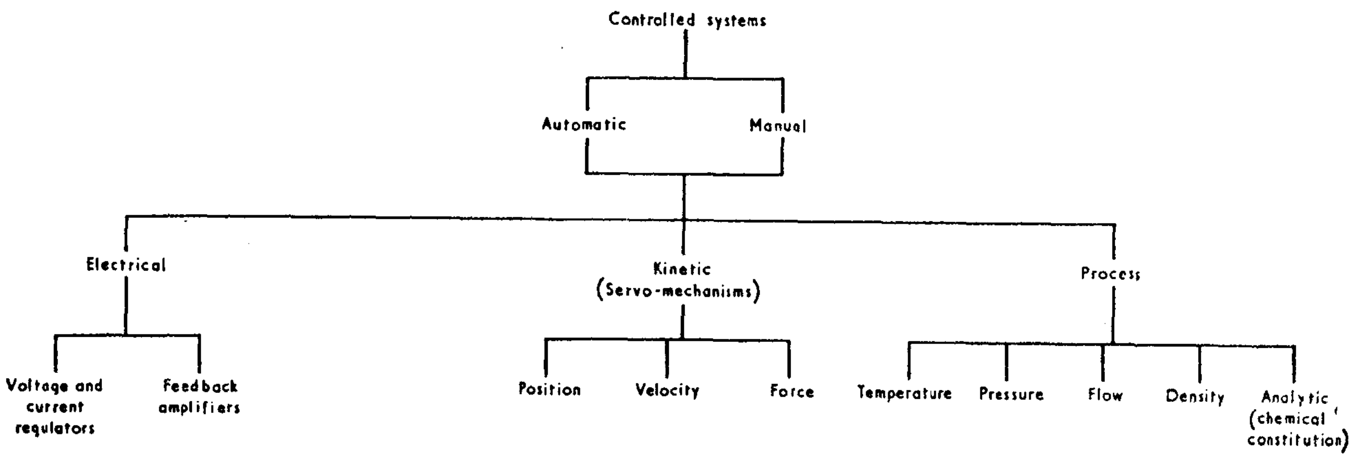

Fig. 3. Classes of control systems

Closed loop, negative feedback systems can also be very descriptively defined as "error actuated".

The closed loop system has two distinctly desirable features:

(i) *Regulation.* Consider C to be at a correct value corresponding to a particular value of R, so that E is zero. Some uncontrolled "disturbance" on or in the system now causes C to change, for example a change in viscosity of fluid being pumped in a system controlling the flow rate. Immediately the controller produces an actuating signal causing a correcting action from the plant pushing C back to its set value. In practice it will not completely correct for the disturbance as will be shown later

* The more sophisticated controller is required to feed the correcting unit with E plus some functions of E, e.g. the derivative and integral of E, in order to improve the dynamic performance.

5. Automatic and Manual Control

A closed loop system in which the control, monitoring and differencing are performed by mechanical, electric or pneumatic devices is said to be AUTOMATICALLY CONTROLLED. If the correcting action is performed by a man, the system is said to be MANUALLY CONTROLLED.

History lends a perfect illustrative example in the speed control of steam engines. Prior to 1788 steam engine speed control was either an open loop system, set at roughly the correct speed and left; or else it was manually controlled by a man reading a speed indicating meter and increasing or decreasing the steam flow accordingly to correct for fall or rise in speed. Here the speed indicator was the monitor, the desired speed the reference, the man's eyes and brain the controller and the man's hands and the steam supply control valve the correcting unit; the engine itself, of course, being the plant. Then came James Watt and the engine governor, which is an automatic system. The engine speed is measured by the "lift" of a rotating ball working against a spring, the initial tension ("set point") of which is the reference, determined by the desired speed. As the rotating balls rise or fall they close or open the steam control valve via a mechanical linkage in such a sense as to compensate for the speed change. This is shown in Fig. 4.

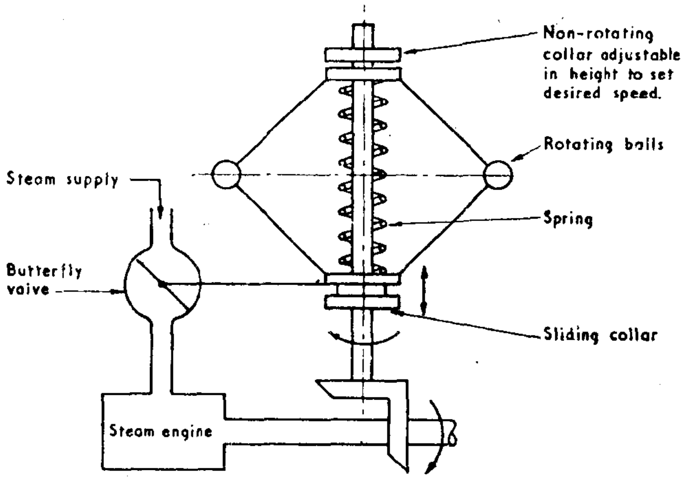

Fig. 4. Steam engine governor

Two other examples of automatic control systems are temperature control and water level control in cisterns.

The temperature control system is normally fitted with a heating element which is switched on when the temperature is below the set value and switched off when the set value is reached. Water level control is achieved by measuring the water height with a floating ball which controls the flow of inlet water to an extent dependent upon the fall in level; for a small fall in level the ball valve will only be "cracked" open giving a small inflow; for a larger fall in level the valve will open further causing a faster inflow to give greater correcting action.

6. Modes of Automatic Control

The preceding three examples demonstrate the variety in the mode of operation of automatic control systems.

The first choice is between CONTINUOUS or DISCONTINUOUS control. The simplest example of continuous control is PROPORTIONAL control where the controller action is proportional to the actuating signal; the speed control and level control are examples of this. The temperature control is, however, discontinuous, being simply TWO STEP or ON–OFF control.

A second categorization is necessary to define whether or not the system is capable of absorbing as well as supplying power. Both the temperature and level controls are incapable of reverse action, e.g. if some other source (say, adiabatic heating in a process) causes the temperature to rise, then the heater will be switched off by the thermostat but there will be no correcting action. To correct for the rise in temperature a power absorption unit must be switched on (a refrigerator). The resultant system would still be discontinuous but would be a TWO-STEP ACTION WITH DEAD ZONE, i.e. low temperature—heater switched on; high temperature—refrigeration on; close to set value—both off. Such an arrangement would be too expensive and unnecessary for most systems, e.g. room heating in England, where natural heat losses would keep the temperature too low.

The water level control is also capable of supplying but not absorbing power, i.e. if some external source overfills the tank, the ball valve is shut and there is no corrective action to bring the level down.

The speed control goes some way towards full control since any attempt of the load to overrun, requiring the engine to brake, will shut the steam supply off, thus applying the friction, etc., load of the engine to the output. This, however, may not be sufficient, as in a lift control where lowering the lift tends to drive the engine and may require additional braking effort. More comprehensive systems provide full driving/braking effort, either drawing power from, or regenerating back to, the supply.

A final defining mode of control is whether or not the reference can be changed. Systems like the water level control are sometimes called REGULATORS because only one desired level can be controlled (regulated); the other two systems will accept a change in desired value.

In general, continuous control is more accurate and more expensive than discontinuous control.

7. Recording or Blind Control

It is common on process controls to attach a recorder, usually a pen recorder, to the monitored output so that the degree of control can be observed. Plants not fitted with recorders are said to be BLIND controlled.

8. Mathematical Models of System

Other sections of the Manual are devoted to describing in detail the controllers, processes and monitoring/recording instruments commonly encountered. The topic of servomechanisms is covered in various texts (for example, references 1 and 4). The more theoretical outlook following is based on the acceptance by the reader that, say, flow can be controlled by feeding an electrical or pneumatic signal to a valve. Whether the valve is pneumatically or electrically operated is of no consequence in this analysis, provided the defining flow/pressure or flow/current equations can be formulated, so that the valve can be represented by a "black box" with an input (current or pressure) and a related output (flow). Similarly the temperature of a fluid can be related to the voltage across the heater and so on. It must be made clear that such mathematical models are invariably approximate and not exact replicas of the system. Always in making mathematical models of systems, some approximations have to be made, e.g. viscosity of a fluid stays constant; stirring of a fluid results in homogeneous distribution; pressure drop across an orifice is *exactly* proportional to the square root of the flow and so on. More complex models can often be derived to improve accuracy, but the line must be drawn somewhere. LINEARITY is a term coined to indicate that one function can be related to another by equations, the coefficients of which are constants, e.g. $V=IR$ represents a linear system since the coefficient, R, relating the variables I and V is a constant. Such equations as $P=k\sqrt{Q}$ represents a nonlinear system. There are other forms of nonlinearities such as saturating systems; systems with dead zones, hysteresis and back-lash; and two-step systems.

The definition of linearity can be extended to systems involving energy storage (capacitance, inertia, etc.) ; in the next section it is shown that such systems can be represented by differential equations, all coefficients of which are constants for a linear system.

9. Dynamic Characteristics

It is most important to realise that it is not sufficient just to describe the steady state performance of a system. It is probable that undesirable "transient" effects can occur during the time taken for the system to settle to a new value after a disturbance or a change in the reference. It is a feature of closed loop systems that transients can possibly increase in amplitude and not settle to a new steady value at all ; this condition is known as INSTABILITY (Hunting). Because of the possibility of such undesirable transients it is necessary to analyse systems as accurately as possible *before* constructing the system in order to indicate any design modifications needed.

The behaviour of a system can be represented by a mathematical equation, usually a differential one. Having established this equation (or set of equations) it is possible to analyse the system by solving the equation. This leads to an indication of design techniques by suggesting modifications to the equations which must in turn be incorporated as hardware into the system, usually modifying the controller characteristics to be other than simply proportional to the actuating signal. The principle of some modifications, e.g. derivative and integral control, described later, can be worked out intuitively, but a proper, quantitative assessment is best achieved from mathematical analysis.

All processes involve some form of time delay in the transmission of information which will appreciably affect the performance of a closed loop system. These time delays are incorporated in the differential equations representing the system. It is most instructive to investigate the simpler systems first and to calculate their characteristics.

9.1 First Order Lag

A first order lag is also called an exponential or transfer lag. A system involving one energy storing element results in a first order equation (the *order* of the equation or system is the highest derivative involved).

Ex. 1

A flywheel of inertia J lb-ft.-sec² and viscous friction coefficient F lb-ft./rad./sec. is initially at rest. A torque T_K lb-ft. is suddenly applied at t=0 ; find the resulting speed variation with time.

Applied torque = acceleration + viscous torque

$$\therefore T_K = J\frac{d\omega}{dt} + F\omega \qquad (1)$$

where ω = speed in rad./sec. so that the acceleration is the first derivative of ω. The applied torque could be any function of time for this equation to apply ; here it is given as a constant for $t > 0$, which is a particular case.

$$\text{Re-arranging (1),}\quad \frac{d\omega}{T_K - F\omega} = \frac{1}{J}dt$$

$$\text{integrating}\quad -\frac{1}{F}\log_e(T_K - F\omega) = \frac{1}{J}t + C$$

Putting ω = 0 when t = 0

$$C = -\frac{1}{F}\log_e T_K$$

$$\therefore \log_e\frac{T_K - F\omega}{T_K} = -\frac{F}{J}t$$

$$\therefore T_K - F\omega = T_K e^{-\frac{F}{J}t}$$

or

$$\omega = \frac{T_K}{F}\left(1 - e^{-\frac{t}{T}}\right) \qquad (2)$$

where $T = \frac{J}{F}$ secs is called the TIME CONSTANT.

If t = T

$$\omega = \frac{T_K}{F}(1 - e^{-1}) = 0{\cdot}63\frac{T_K}{F}\ \text{rads/sec.} \qquad (3)$$

Now T_K/F is the maximum speed, which is a steady value, achieved when there is no further acceleration and the applied torque is all absorbed in viscous friction. The time taken to reach ω_{max} is theoretically infinite so that the time constant is a good way of distinguishing between one system and another. It is the time taken to reach 63% of a step change. It is shown in Fig. 5.

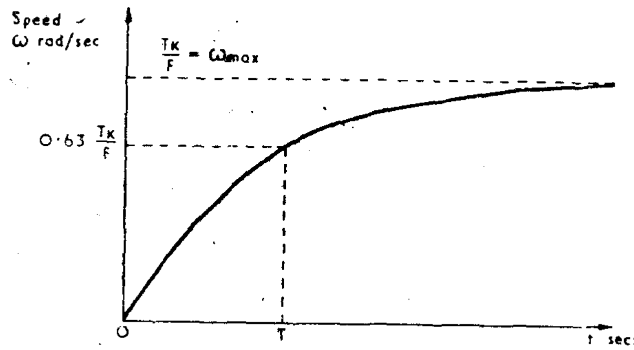

Fig. 5. Speed response of a flywheel to a step torque applied at t=0

Many other physically different systems result in similar differential equations to the above, e.g. for the resistance–capacitance (R–C) network of Fig. 6 the equation is

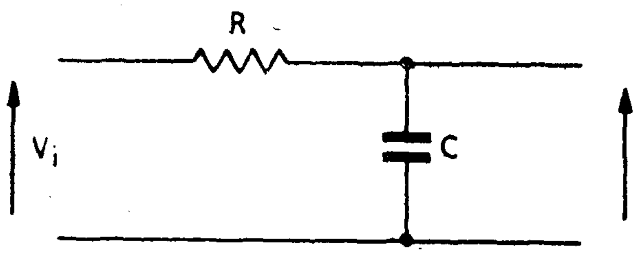

Fig. 6. A simple RC network.

$$v_i = iR + \frac{1}{C}\int i\,dt \qquad (4)$$

and

$$\frac{1}{C}\int i\,dt = v_o \quad \text{or } i = C\frac{dv_o}{dt}$$

$$\therefore v_i = CR\frac{dv_o}{dt} + v_o \qquad (5)$$

given that $v_o = 0$ when t=0.

If v_1 is a step voltage, i.e. a voltage V suddenly applied at $t=0$, this is a similar equation to the previous example, so that

$$v_0 = V\left(1 - e^{-\frac{t}{T}}\right) \text{ where } T = CR \text{ secs.}$$

The term exponential lag for these first order terms can be seen to derive from the exponential form of the output in response to a step change in input.

It must be appreciated that a step input is not likely to be encountered in a real system but it is an excellent method of indicating the expected performance of a system, or the relative performance of alternative systems.

9.2 Effect of a Closed Loop on System Response

Consider the simple example of an electrical power amplifier feeding a heating coil, controlling the temperature of a fluid, shown schematically in Fig. 7.

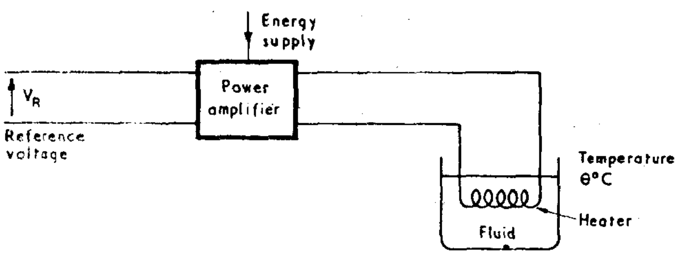

Fig. 7. An open loop heating system

The power amplifier can take many physical forms, e.g. a motor-generator set or a thyristor amplifier. It may be treated to a first approximation as developing power into the coil proportional to v_R. (Power is proportional to V^2 so that the linear amplifier must be a non-linear *voltage* amplifier. Proportional power amplification is assumed to simplify the following calculation.)

Thus if

$$\text{heat input} = K_1 v_R \text{ BTU/sec.}$$
$$\text{and heat lost} = K_2\,\theta \text{ BTU/sec.}$$
$$\text{and heat stored} = K_3\,\dot{\theta} \text{ BTU/sec.}$$
$$\text{then } K_1 V_R = K_2\theta + K_3\dot{\theta} \qquad (7)$$

As previously we treat the response (change in θ) to a change in v_R equal to a suddenly applied voltage V at $t=0$. Then as shown in section 8.1

$$\theta = \frac{K_1}{K_2} V \left(1 - e^{-\frac{t}{T}}\right) \text{ where } T = \frac{K_3}{K_2} \text{secs} \qquad (8)$$

Now consider a similar system with closed loop control. The temperature is monitored and a voltage proportional to θ is subtracted from the reference voltage, the difference

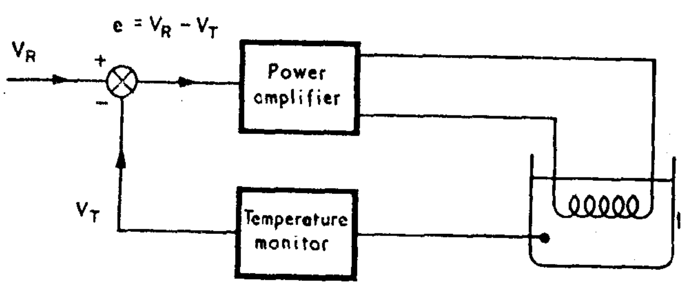

Fig. 8. A closed loop heating system

(the actuating signal) being fed to the amplifier (Fig. 8).

Now if the voltage from the monitor, $v_T = K_T\,\theta$

$$\text{then} \quad e = v_R - K_T\,\theta \qquad (9)$$
$$\text{and} \quad K_1 e = K_2\theta + K_3\dot{\theta} \qquad (10)$$
$$\text{so that} \quad K_1(v_R - K_T\theta) = K_2\theta + K_3\dot{\theta}$$
$$\therefore K_1 v_R = (K_2 + K_1K_T)\,\theta + K_3\dot{\theta} \qquad (11)$$

Again, for comparison with the open loop example, put $v_R = V$ for $t > 0$, then

$$\theta = \frac{K_1}{K_2 + K_1K_T} V\left(1 - e^{-\frac{t}{T_c}}\right) \qquad (12)$$

$$\text{where} \quad T_c = \frac{K_3}{K_2 + K_1K_T}$$

Compare now the response of the open loop system, equation (8), with that of the closed loop system, equation (12).

The constant relating the steady temperature to the reference voltage is smaller (appreciably smaller for a practical case when we make $K_1K_T \gg K_2$) for the closed loop so that for a required temperature the reference voltage must be bigger for a closed loop system.

More important, the time constant is smaller for the closed loop (again much smaller for $K_1K_T \gg K_2$). In other words, the response is much quicker.

This advantage of quicker response plus the other advantages of regulation and controlability associated with closed loop systems make the reduction in "sensitivity" easy to tolerate!

The speed of response and other features can be improved by increasing the OPEN LOOP GAIN K_1K_T/K_2, i.e. the change in monitor output/unit change in actuating signal if the loop were inoperative. It will be later shown that with more complex systems with 3 or more delay elements the increase in open loop gain can be overdone and the response, as well as becoming initially sharper, becomes oscillatory causing instability.

9.3 Finite Time Lag (Distance—Velocity Lag).

In processes involving transportation of material from one point to another, with substantially no change in conditions, the time of transportation results in a change at point A being correctly detected at point B, but a finite time later, i.e. heat is applied to a moving column of liquid at point A, changing the temperature of the liquid as shown in Fig. 9(a). For physical reasons the temperature monitor is placed downstream at B. Assuming that the liquid temperature has not changed in moving from A to B, the signal from the temperature monitor will be as shown in Fig. 9(b).

Mathematically,

$$\text{if} \quad \theta_A = f(t) \qquad (13)$$
$$\theta_B = f(t-T) \text{ where } T = \text{time delay}$$

Analysis of closed loop systems involving finite time delays is difficult and is not mentioned until later in the section.

Systems in which the state changes during transport, i.e. heat loss along the pipe causes the temperature of the fluid at A to fall before it reaches B are even more complex.

9.4 A Short Hand Method of Expressing Time Constants in Equations.

From the example in Section 9.2 the differential equation was

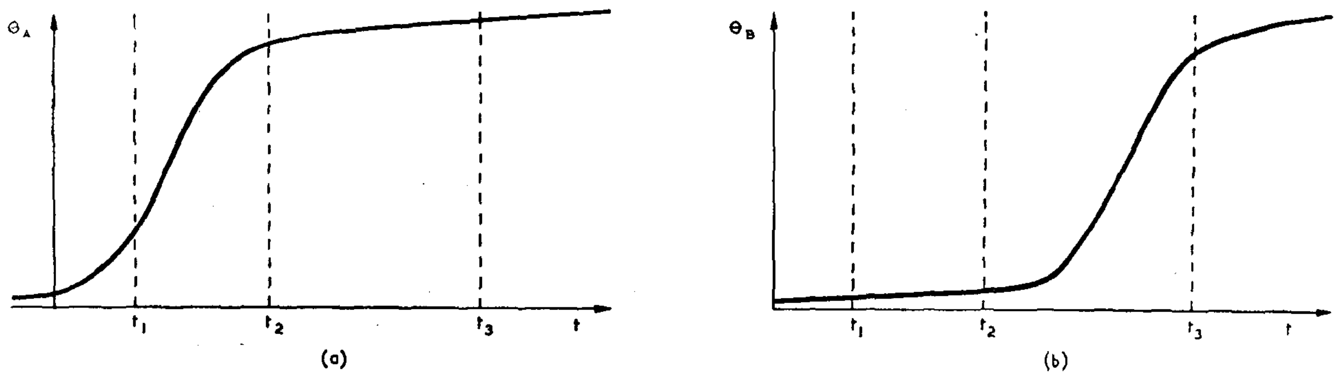

Fig. 9. An example of finite time delay

$$K_1 v_R = K_2\theta + K_3 \frac{d\theta}{dt} \qquad (14)$$

If we could write $s\theta$ for $d\theta/dt$ where s was an operation which transformed the differentiation function into a simple multiplication then the differential equation would become an algebraic equation much easier to manipulate. To be fair, we must stress that we have changed the original equation somewhat and to do this we indicate that the variable θ is now some algebraic function of s instead of a real time variable related by differentials by writing $\Theta(s)$ instead of θ; similarly we write $V_R(s)$ instead of v_R. In fact all variables which are functions of time (this is all we can consider physically) are denoted by small letters and the equivalent "transformed" functions by capital letters and the bracketed s.

Thus we write equation (14) as

$$K_1 V_R(s) = K_2 \Theta(s) + K_3 s\, \Theta(s) \qquad (15)$$

In general we write $s^n\,\Theta(s)$ in place of $\frac{d^n\theta}{dt^n}$

Equation (15) can be easily simplified

$$K_1 V_R(s) = (K_2 + sK_3)\, \Theta(s) \qquad (16)$$

$$\therefore \Theta(s) = \frac{K_1}{K_2 + sK_3} \cdot V_R(s)$$

$$= \frac{K_1}{K_2} \cdot \frac{1}{1 + sT} V_R(s) \qquad (17)$$

$$\text{where } T = \frac{K_3}{K_2}$$

Thus this system can be said to achieve a temperature of K_1/K_2 °C/volt with a time constant T. Conversely, given the steady state relation and time constant, an equation similar to (17) can be written down.

This is a brief introduction to the *Laplace Transform* method of solving differential equations. It is a comprehensive technique but here used only as a form of short hand. Other texts use the symbol D instead of s, but fail to stress that equation variables are not the same as before the substitution.

For a given value of v_R (as a function of time—say, $v_R = V$, a constant applied at $t=0$ as in the previous example) there is an equivalent value of $V_R(s)$ which substituting in equation (17) would give an equivalent value of $\Theta(s)$. This function must also have an equivalent value of θ which is the desired answer.

9.5 Transfer Functions and Block Diagrams

By using the transform method just outlined the system differential equation can be expressed as an equivalent algebraic equation. The most significant feature of this is that the output variable ($\Theta(s)$ in the previous example) can be expressed as a function of the input variable ($V_R(s)$). Note from equation (14) that it is impossible to express an equation $\theta = ?$ unless a particular value is assigned to v_R and the equation solved. A change in v_R will require a complete new solution.

However, from equation (17),

$$\Theta(s) = \frac{K_1}{K_2} \cdot \frac{1}{1 + sT} \cdot V_R(s) \qquad (18)$$

Making use of this simple algebraic equation we define a TRANSFER FUNCTION as

$$\text{T.F.} = \frac{\text{Transform of the output}}{\text{Transform of the input}} \qquad (19)$$

For the above system

$$\text{T.F. is } \frac{\Theta(s)}{V_R(s)} = \frac{K_1}{K_2} \cdot \frac{1}{1 + sT} \qquad (20)$$

A BLOCK DIAGRAM is a diagram showing the schematic layout of the system, but also relating the variable at each point of the system by allocating a transfer function to each block as in Fig. 10.

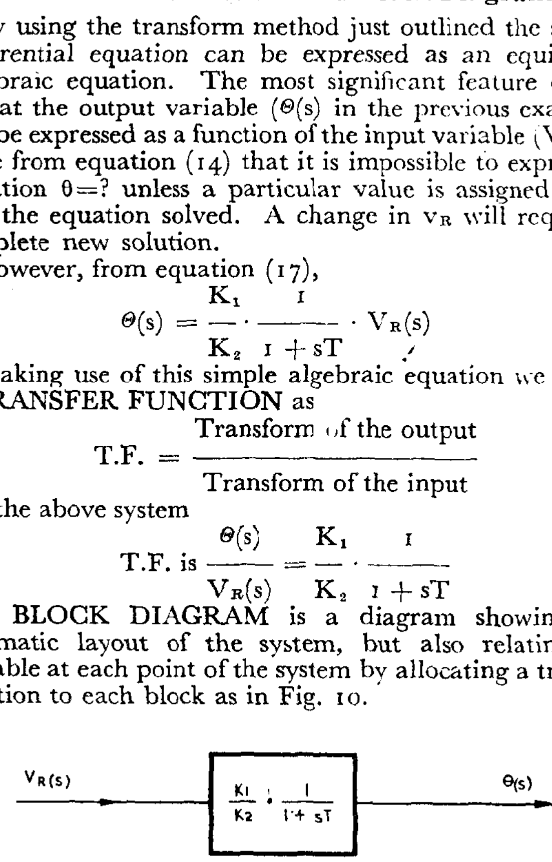

Fig. 10. Simple block diagram illustrating transfer function

This technique can be used to analyse closed loops as well. The circular symbol is used for a SUMMING POINT. The arrows indicate direction of signal flow and the polarity by plus or minus signs. This is a convenient symbol, not necessarily a physical part of the system. (Fig. 11)

The transfer function G(s) is used to denote the feed-forward part of the loop and H(s) the feed-back part. The open loop transfer function is G(s) H(s). The closed loop transfer function is C(s)/R(s) which is calculated thus:

$$C(s) = G(s)\,E(s)$$

$$E(s) = R(s) - B(s) = R(s) - H(s) \,.\, C(s)$$

$$\therefore C(s) = G(s)\,[R(s) - H(s)\,C(s)]$$

$$\therefore \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)} \qquad (21)$$

This is a most important equation. From it we can see that if $G(s)\ H(s) \gg 1$ (this is essential for an effective closed loop system so that there is no point in considering otherwise)

$$\frac{C(s)}{R(s)} \simeq \frac{G(s)}{G(s)\ H(s)} = \frac{1}{H(s)} \qquad (22)$$

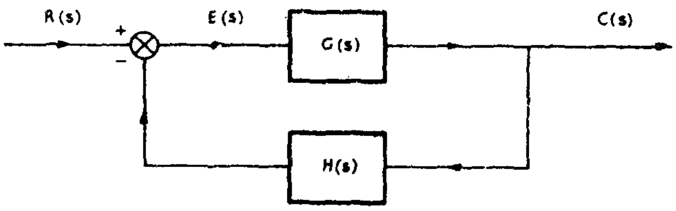

Fig. 11. Basic block diagram for a closed loop system

This summarizes the previous "physical" explanation of closed loop control in that the controlled variable and reference are dominantly related only by H(s), thus being relatively unaffected by changes and lags in the plant, G(s).

9.6 Second Order Systems

Practical systems will always involve more than one element of lag. The simplest example is the cascading of first order lags while more complex examples occur in closed loops.

In the previous temperature control system it was assumed that heat output from the heating coil was directly proportional to v_R.

However, the coil will have inductance so that a time constant is involved in relating the power output to v_R. Thus two time constants will be involved in relating θ to v_R. In a practical system other time constants will also be present in the monitors, etc.

Some typical values of time constants are :

(i) *Process Lags*
Very variable—can range from 30 minutes for chemical reactions to minutes for heating and flow controls to seconds for mechanical inertia-friction constants and milliseconds for electrical heating coils. It is of interest to note that electronic amplifiers involve important time constants measurable in microseconds ! (Some of these may well be finite time lags.)

(ii) *Monitor Lags*
Typical values are quoted as :
Mercury in steel thermometer in gas at 50 ft./sec.—70 secs.
Sheathed resistance thermometer in water at 5 ft./sec.—20 secs.
Float operated manometer 3–30 secs.
Force balance systems 0·05 sec.

(iii) *Transmission Lags*
(transmission of signals to a distant point)
Negligible for electrical systems and pneumatic systems up to 200 ft.

(iv) *Controller Lags*
Usually negligible.

(v) *Correcting Unit Lags*
Lags are introduced of 1–10 secs. due to the inertia of moving parts (if present) and of similar order in pneumatic units due to pipe resistance/cylinder capacity time constants.

Consider a system such as the temperature control already discussed but with allowance for the power amplifier time constant T_2. Call the plant time constant $K_3/K_2 = T_1$ (see Section 9.2) so that the system can be represented by the block diagram of Fig. 12.

The second block comes from the equation

$$\text{Heat input, } U = \text{heat lost} + \text{heat stored} = K_2\,\theta + K_3\,\dot{\theta}$$

$\therefore$ transforming

$$U(s) = K_2\,\theta(s) + K_3 s\,\theta(s) = K_2\,(1 + s\,T_1)\,\theta(s)$$

$$\therefore \frac{\theta(s)}{U(s)} = \frac{1}{K_2(1 + sT_1)} \qquad (23)$$

From Fig. 12 we can write

$$\theta(s) = \frac{K_1}{1 + sT_2} \cdot \frac{1}{K_2(1 + sT_1)} \cdot V_R(s)$$

$$= \frac{K_1}{K_2} \cdot \frac{1}{(1 + sT_1)(1 + sT_2)} V_R(s) \qquad (24)$$

$$\therefore \theta(s) = \frac{K_1}{K_2} \cdot \frac{1}{(1 + s\,(T_1 + T_2) + s^2 T_1 T_2)} \cdot V_R(s) \qquad (25)$$

$$\therefore \frac{K_1}{K_2} V_R(s) = \theta(s) + (T_1 + T_2)\,s\,\theta(s) + T_1 T_2\,s^2\,\theta(s) \qquad (26)$$

The differential equation equivalent to this is

$$\frac{K_1}{K_2} V_R = \theta + (T_1 + T_2)\,\dot{\theta} + T_1 T_2\,\ddot{\theta} \qquad (27)$$

which is a second order equation.

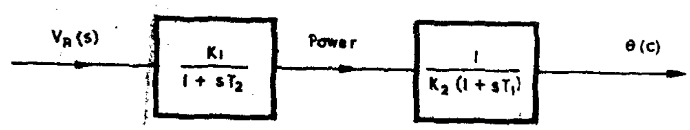

Fig. 12. Block diagram of an open loop temperature control system

The equation (27) arose when two separate first order lags were cascaded. Second order terms, however, exist when two time constants are interdependent, i.e. the cascading of a second block affects the first in which case a simple block diagram such as Fig. 12 cannot be drawn.

From Fig. 13(a) the steady state output is $v_o = v_i$ and the time constant is $C_1R_1 = T_1$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{1}{1 + sT_1} \qquad (28)$$

For Fig. 13(b) the addition of C_2 and R_2 affect the relation between v and v_i. C_2 and R_2 are a network with input voltage v and output v_o similar to equation (28), so that

$$\frac{V_o(s)}{V(s)} = \frac{1}{1 + sT_2} \qquad (T_2 = C_2R_2)$$

However, $\frac{V(s)}{V_i(s)}$ is not simply $\frac{1}{1 + sT_1}$ due to the "loading"

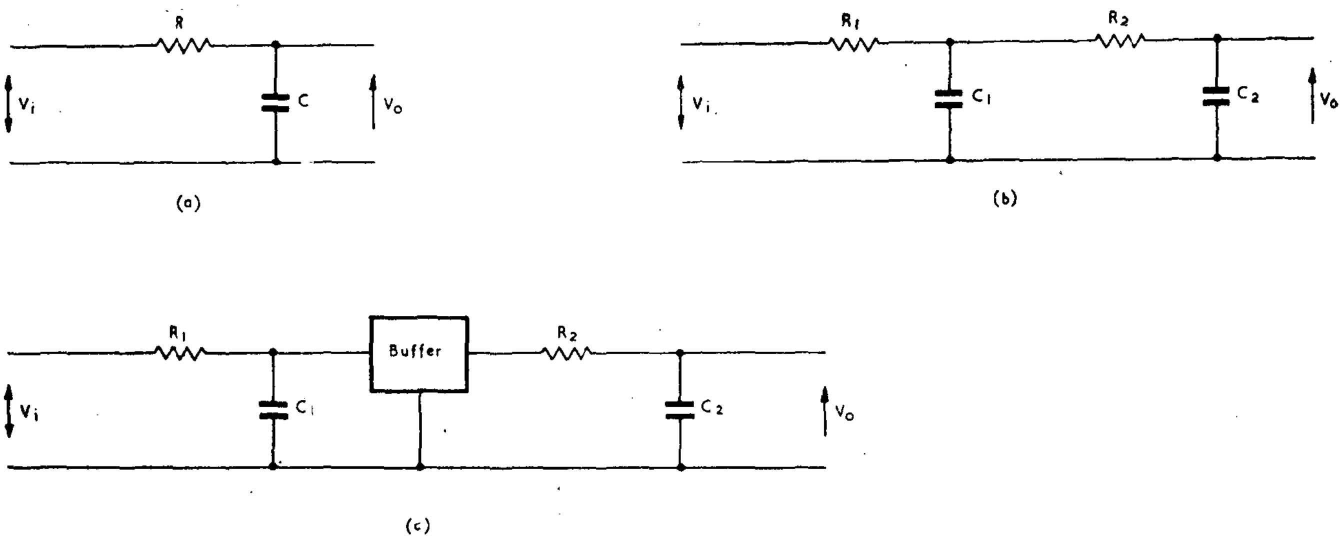

Fig. 13. First and second order networks

effect of R_2 and C_2. By applying more complex circuit laws (not important in this text) we can show that

$$\frac{V(s)}{V_1(s)} = \frac{1 + sT_2}{1 + s\,(T_1 + T_2 + R_1C_2) + s^2\,T_1T_2}$$

$$\therefore \frac{V_o(s)}{V_1(s)} = \frac{V_o(s)}{V(s)} \cdot \frac{V(s)}{V_1(s)} = \frac{1}{1 + sT_2} \cdot \frac{1 + sT_2}{1 + s\,(T_1 + T_2 + R_1C_2) + s^2T_1T_2}$$

$$= \frac{1}{1 + s\,(T_1 + T_2 + R_1C_2) + s^2T_1T_2} \qquad (29)$$

For Fig. 13(c) a buffer stage is included. A buffer can be considered to transfer the voltage but will have no input current, thereby *not* loading the first network. Thus

$$\frac{V_o(s)}{V_1(s)} = \frac{1}{1 + sT_1} \cdot \frac{1}{1 + sT_2} \qquad (30)$$

The effect of the interaction of terms is to introduce the R_1C_2 term into equation (29).

Now consider the effect of two time constants inside a closed loop. Take the closed loop temperature control of section 9.2 only include the amplifier time constant T_2. The block diagram is shown in Fig. 14.

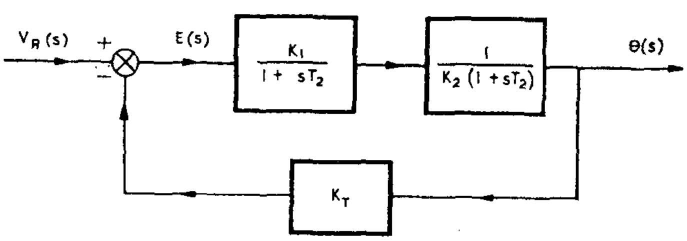

Fig. 14. Block diagram of a closed loop temperature control system

Here $$G(s) = \frac{K_1}{K_2} \cdot \frac{1}{(1 + sT_1)\,(1 + sT_2)}$$

and $H(s) = K_T$ (we have assumed no time constant for simplicity). The open loop gain is $K_1/K_2 \,.\, K_T$ so that if this is much bigger than unity

$$\frac{\theta(s)}{V_R(s)} \simeq \frac{1}{H(s)} \simeq \frac{1}{K_T}$$

More accurately

$$\frac{\theta(s)}{V_R(s)} = \frac{G(s)}{1 + G(s)\,H(s)}$$

$$\frac{\theta(s)}{V_R(s)} = \frac{\dfrac{K_1}{K_2} \cdot \dfrac{1}{(1 + sT_1)\,(1 + sT_2)}}{1 + \dfrac{K_1\,K_T}{K_2} \cdot \dfrac{1}{(1 + sT_1)\,(1 + sT_2)}}$$

$$= \frac{K_1}{K_2\,(1 + sT_1)\,(1 + sT_2) + K_1\,K_T}$$

$$= \frac{K_1}{K_2 + K_1K_T} \cdot \frac{1}{1 + \left(\dfrac{(T_1 + T_2)K_2}{K_2 + K_1K_T}\right)s + \left(\dfrac{T_1\,T_2\,K_2}{K_2 + K_1K_T}\right)s^2} \qquad (31)$$

We can try to split the denominator of this function into two equivalent time constants as in equation (24) by factorizing the equation.

The two roots m_1 and m_2 are given by

$$m = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{where } c = 1\ ;\ b = \frac{(T_1 + T_2)\,K_2}{K_2 + K_1K_T} \qquad (32)$$

$$\text{and } a = \frac{T_1T_2K_2}{K_2 + K_1K_T}$$

This gives $$\frac{\theta(s)}{V_R(s)} = \frac{K_1}{K_2 + K_1K_T} \cdot \frac{1}{\left(1 - \dfrac{s}{m_1}\right)\left(1 - \dfrac{s}{m_2}\right)} \qquad (33)$$

However, if we inspect these roots we see that if $4ac > b^2$ the roots are complex so that two equivalent time constants cannot be conceived. It is only in this example that b^2

can be less than 4ac due to the closed loop nature of the system.

The second order system must thus be investigated further. To do this we will consider a general equation :

$$a\ddot{c} + b\dot{c} + c = r \qquad (34)$$

Put r=K, a step input applied at t=o. The solution for c is in two parts a transient form (complementary function) and a steady state term (particular integral).

The complementary function is found by first calculating the roots of an auxiliary equation

$$a m^2 + b m + 1 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4a}}{2a} \qquad (35)$$

The complementary function has three possible values:

(i) Overdamped ($b^2 > 4a$)

The roots are real,

$$m_1 = -\frac{b}{2a} + \frac{1}{2a}\sqrt{b^2 - 4a}$$

and $$m_2 = -\frac{b}{2a} - \frac{1}{2a}\sqrt{b^2 - 4a}$$

and the complementary function (C.F.) is

$$Ae^{m_1 t} + Be^{m_2 t} \qquad (36)$$

(ii) Critically damped $b^2=4a$

$$m_1 = m_2 = -\frac{b}{2a}$$

and the complementary function (C.F.) is

$$(A + Bt)\, e^{-\frac{b}{2a}t} \qquad (37)$$

(iii) Underdamped (oscillatory) $b^2 < 4a$

The roots are a complex conjugate pair,

$$m_1 = -\frac{b}{2a} + j\frac{1}{2a}\sqrt{4a - b^2} = \sigma + j\omega$$

$$m_2 = -\frac{b}{2a} - j\frac{1}{2a}\sqrt{4a - b^2} = \sigma - j\omega$$

Where $\sigma = -\frac{b}{2a}$ and $\omega = \frac{1}{2a}\sqrt{4a - b^2}$

and the complementary function (C.F.) is

$$e^{\sigma t}(A \sin \omega t + B \cos \omega t) \qquad (38)$$

The particular integral (P.I.) is simply c=K since in the steady state all derivatives are zero.

The complete solution is c=C.F.+P.I.

The coefficients A and B are arbitrary constants dependent upon the initial conditions, i.e. the values of c and $\dot{c}$ at t=o.

Consider now the special case of an *undamped* system with b=o. The complementary function is now A Sin ωt+ B cos ωt since σ is zero. This term is an oscillatory constant amplitude function in response to a step change in input as shown in Fig. 15.

The undamped frequency of oscillation, called the NATURAL FREQUENCY,

$$\omega_n = \frac{1}{2a}\sqrt{4a} = \frac{1}{\sqrt{a}}$$

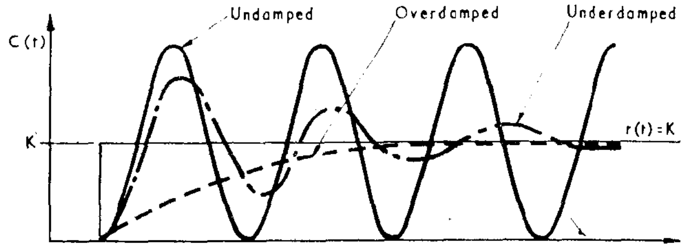

Fig. 15. Response of 3 second order systems to a step input signal

∴ we can replace a in equation (34) by $\frac{1}{\omega_n^2}$

However, the coefficient of $\dot{c}$, i.e. b, determines the actual degree of damping. For critical damping $b_c^2=4a$ or $b_c=2\sqrt{a}$.

Now define the damping ratio (Zeta) as

$$\xi = \frac{\text{actual damping coefficient}}{\text{coefficient for critical damping}}$$

$$= \frac{b}{b_c} = \frac{b}{2\sqrt{a}} = \frac{b\,\omega_n}{2}$$

$$\therefore b = \frac{2\xi}{\omega_n}$$

Thus the general second order equation can be written

$$\frac{1}{\omega_n^2}\ddot{c} + \frac{2\xi}{\omega_n}\dot{c} + c = r \qquad (39)$$

The transfer function corresponding to this is obtained from

$$\left(\frac{1}{\omega_n^2}s^2 + \frac{2\xi}{\omega_n}s + 1\right) C(s) = R(s)$$

$$\therefore \frac{C(s)}{R(s)} = \frac{1}{1 + \frac{2\xi}{\omega_n}s + \frac{1}{\omega_n^2}s^2} \qquad (40)$$

Equations (29), (30) and (31) are such systems.

Note that the exponential DECAY RATE =

$$\frac{b}{2a} = \xi\omega_n \qquad (41)$$

It is termed a *decay* rate since for the second order system a and b must be positive so that σ must be negative, i.e. the decay rate = −σ.

The TRANSIENT FREQUENCY OF OSCILLATION is given by

$$\omega = \frac{1}{2a}\sqrt{4a - b^2} = \frac{\omega_n^2}{2}\sqrt{\frac{4}{\omega_n^4} - \frac{4\xi^2}{\omega_n^2}} = \omega_n\sqrt{1 - \xi^2} \qquad (42)$$

This is applicable, of course, only for ξ < 1, otherwise the two roots are real, i.e. the second order term can be factorised as in equations (29) and (30) for which inspection should show that ξ must be bigger than 1.

9.7 Higher Order Systems

Systems of higher order than second can be represented by the general transfer function with a m^{th} order numerator and n^{th} order denominator with n > m for practical systems.