



Optoelectronics

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Optoelectronics is a practical and self-contained graduate-level textbook and reference, which will be of great value to both students and practising engineers in the field. Sophisticated concepts are introduced by the authors in a clear and coherent way, including such topics as quantum mechanics of electron-photon interaction, quantization of the electromagnetic field, semiconductor properties, quantum theory of heterostructures, and non-linear optics. The book builds on these concepts to describe the physics, properties, and performances of the main optoelectronic devices: light emitting diodes, quantum well lasers, photodetectors, optical parametric oscillators, and waveguides. Emphasis is placed on the unifying theoretical analogies of optoelectronics, such as equivalence of quantization in heterostructure wells and waveguide modes, entanglement of blackbody radiation and semiconductor statistics. The book concludes by presenting the latest devices, including vertical surface emitting lasers, quantum well infrared photodetectors, quantum cascade lasers, and optical frequency converters.

For Nadia, Anne, Julien, and Clara, for their patience
with all my love.

For Nadia who understands so many other things.

Preface

The field of optoelectronics is currently in full expansion, drawing to its classrooms and laboratories numerous science and engineering students eager to master the discipline. From the lecturer's perspective, optoelectronics is a considerable challenge to teach as it emerges from a complex interplay of separate and often seemingly disjointed subjects such as quantum optics, semiconductor band structure, or the physics of carrier transport in electronic devices. As a result, the student (or lecturer) is left to navigate through a vast literature, often found to be confusing and incoherent.

The aim of this text is to teach optoelectronics as a science in itself. To do so, a tailored presentation of its various sub-disciplines is required, emphasizing within each of these, those concepts which are key to the study of optoelectronics. Also, we were determined to offer a partial description of quantum mechanics oriented towards its application in optoelectronics. We have therefore limited ourselves to a utilitarian treatment without elaborating on many fundamental concepts such as electron spin or spherical harmonic solutions to the hydrogen atom. On the other hand, we have placed emphasis on developing formalisms such as those involved in the quantization of the electromagnetic field (well suited to a discussion of spontaneous emission), or the density matrix formalism (of value in treating problems in non-linear optics).

Similarly, our treatment of semiconductor physics ignores any discussion of the effect of the crystallographic structure in these materials. Rather, a priori use is made of the semiconductor band structures which implicitly incorporate these effects on the electrical and optical properties of these materials. In carrying out our rather utilitarian-minded presentation of these disciplines, we have claimed as ours Erwin Schrödinger's maxim that it mattered little whether his theory be an exact description of reality insofar as it proved itself useful.

We have sought in this work to underline wherever possible the coherence of the concepts touched on in each of these different areas of physics, as it is from this vantage point that optoelectronics may be seen as a science in its own right. There exists, for instance, a profound parallel between the behaviour of an electron in a quantum well and that of an electromagnetic wave in an optical waveguide. As well, one finds between the photon statistics of black bodies, the mechanics of quantum transitions within semiconductor band structures and the statistics of

charge carriers in these materials, an entanglement of concepts comprising the basis for infrared detection. In the same spirit, this work does not pretend to present an exhaustive list of all known optoelectronic devices. Such an effort could only come at the cost of the overall coherence aimed at in this work, and add to the type of confusion we have claimed as our enemy. The goal is rather to present those optoelectronic concepts which will allow an overall understanding of principles necessary in solving problems of a general or device-specific nature. Thus, only the analysis of *generic classes* of optoelectronic components will be undertaken here without entering into the labyrinth offered by more particular applications.

Lastly, regarding the problem of notation (a problem inherent to any multidisciplinary study), we have chosen simply to follow the lead of standard physics notation in any given chapter. Thus, the symbol ' ϵ ' may be used indiscriminately to represent the permittivity, the quantum confinement energy, or the saturation coefficient of a semiconductor laser. We could have attempted the introduction of various notations for each of these different uses based on the Latin, Greek, and Hebrew character sets, but we realized that even these would have soon been exhausted. We have thus chosen merely to redefine in each chapter the correspondence between the symbols and their respective notions.

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Properties of common semiconductors

	Si	Ge	GaAs	AlAs	InAs	GaP	InP	GaSb	InSb
<i>Bandgap</i>									
E_g (eV)	indirect	indirect	direct	indirect	direct	indirect	direct	direct	direct
@ $T = 0$ K	1.170	0.744	1.519	2.229	0.418	2.350	1.424	0.236	0.236
@ $T = 300$ K	1.124	0.664	1.424	2.17	0.354	2.272	1.344	0.70	0.18
Lattice constant, a_0 Å	5.43095	5.64613	5.6533	5.6600	6.0583	5.4505	5.8688	6.096	6.4794
Relative permittivity, $\epsilon_{\text{st}}/\epsilon_0$	11.9	16.2	13.1	10.06	15.15	11.1	12.56	15.69	16.8
<i>Effective mass</i>									
Electron longitudinal, m_{el}/m_0	0.9163	1.59	0.067	0.15(Γ)	0.023	0.254	0.073	0.047	0.014
Electron traverse, m_{et}/m_0	0.1905	0.0823			4.8				
Heavy hole, m_{hh}/m_0	0.537	0.284	0.50	0.79	0.40	0.67	0.60	0.8	0.42
Light hole, m_{lh}/m_0	0.153	0.043	0.087	0.15	0.026	0.17	0.12	0.05	0.016
<i>Luttinger parameters</i>									
γ_1	4.25	13.4	7.0	3.45	20.4	4.05	5.04	13.3	40.1
γ_2	0.32	4.3	2.3	0.68	8.3	0.49	1.6	4.4	18.1
γ_3	1.45	5.7	2.9	1.3	9.1	1.25	2.4	6.2	19.2
Intrinsic density, n_i (cm $^{-3}$)	1.5×10^{10}	2.4×10^{13}	1.8×10^6		1.3×10^{15}	3.0×10^6	1.2×10^8	4.3×10^{12}	2.0×10^{16}
<i>Mobility</i>									
Electron, μ_e (cm 2 Vs $^{-1}$)	1450	3900	8000	400	30000	200	5000	5000	80000
Hole, μ_h (cm 2 V $^{-1}$ s $^{-1}$)	370	1800	400	100	480	150	180	1500	1500

Further reading

General references useful in obtaining values for semiconductor properties:

- K. H. Hellwege, ed., *Landolt-Börnstein Numerical Data and Functional Relationships in Science and Technology*, Springer, Berlin.
O. Madelung, ed., *Semiconductors, Group IV Elements and III-V Compounds, in Data in Science and Technology*, Springer, Berlin (1996).
Recent review works:
B. L. Weiss, ed., *EMIS Databooks Series*, INSPEC, London.

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1 Quantum mechanics of the electron

1.1 Introduction

This chapter reviews the fundamental principles and techniques of quantum mechanics that are necessary to understand the subject of optoelectronics. Often, concepts are not presented in depth: the aim, rather, is to provide the tools and notation required to work through this book. Thus, in spite of their immense importance in other areas of physics, and the severe scientific injustice resulting from their being placed aside, we shall pass almost entirely in silence over Heisenberg's uncertainty principle, spherical harmonics, electron spin, etc. The reader wishing to deepen his/her understanding of these concepts is greatly encouraged to read or reread the remarkable work by C. Cohen-Tannoudji et al. (1992).

1.2 The postulates of quantum mechanics

We consider an electron of charge q and mass m_e subjected to a generalized potential of the form $V(\mathbf{r}, t)$ varying in three-dimensional space \mathbf{r} , and time t . Quantum mechanics tells us that the notion of a classical electron trajectory loses its meaning when the distance over which this potential varies is of the order of the *de Broglie wavelength* (λ_{DB}). This length is given by:

$$\lambda_{\text{DB}} = \frac{2\pi\hbar}{\sqrt{2m_e E}} \approx \frac{1.23 \text{ (nm)}}{\sqrt{V \text{ (V)}}} \quad (1.1)$$

where \hbar is *Planck's constant* ($1.04 \times 10^{-34} \text{ J s}^{-1}$), V is the average potential experienced by the particle, and E is the energy of the particle. We will see that in a crystalline solid where electrons are subjected to spatially varying potentials of the order of 5 eV ($1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$), their *de Broglie wavelength* turns out to be of the order of 5 Å. As this length corresponds to the interatomic distance between atoms in a crystalline lattice, conduction electrons in this medium will be expected to display interference effects specific to the mechanics of wave-motion. These effects (studied in Chapter 5) are the origin of the semiconductor band gap, and cannot readily be discussed in terms of classical theories based upon the notion of a well-defined trajectory.

Quantum mechanics also teaches us that we must forgo the idea of a trajectory

in favour of a more subtle description in terms of *quantum states* and *wavefunctions*. The electron is then represented by a *state vector* evolving in time $|\psi(t)\rangle$. One of the strongest postulates of quantum mechanics is that all these state vectors span a *Hilbert space*. For instance, the existence of linear combinations of states (which leads to dramatic effects such as molecular stability, energy bandgaps, . . .) is a direct consequence of this postulate. This vector space possesses a *Hermitian scalar product*, whose physical significance will be given later. We will use *Dirac notation* to represent the scalar product between two vector states $|\psi_1\rangle$ and $|\psi_2\rangle$ as $\langle\psi_2|\psi_1\rangle$. Now, we recall the properties of a Hermitian scalar product:

$$\left. \begin{aligned} \langle\phi|\psi\rangle &= \langle\psi|\phi\rangle^* \\ \langle\phi|\alpha\psi_1 + \beta\psi_2\rangle &= \alpha\langle\phi|\psi_1\rangle + \beta\langle\phi|\psi_2\rangle \\ \langle\alpha\phi_1 + \beta\phi_2|\psi\rangle &= \alpha^*\langle\phi_1|\psi\rangle + \beta^*\langle\phi_2|\psi\rangle \\ \langle\psi|\psi\rangle &\text{ real, positive, and zero if and only if } |\psi\rangle = 0 \end{aligned} \right\} \quad (1.2)$$

where the asterisk indicates that the complex conjugate is taken. By definition a physical state possesses a norm of unity, which is to say that $|\psi(t)\rangle$ is a physical state if:

$$\langle\psi(t)|\psi(t)\rangle = 1 \quad (1.3)$$

A certain number of linear operators act within this Hilbert space. A second postulate of quantum mechanics is that classically measurable quantities such as position, energy, etc. are represented by Hermitian operators A (i.e. operators such that $A^\dagger = A$, where \dagger is the *adjoint* or *Hermitian conjugate*) called *observables*, and that the result of the measurement of such an observable can only be one of the eigenvalues associated with the observable. If the ensemble of eigenvalues of the observable A forms a discrete set, then the set of all possible *measurements* of a system are given by the a_n solutions of the eigenvalue equation:

$$A|\psi_n\rangle = a_n|\psi_n\rangle \quad (1.4)$$

As the observable operators are Hermitian, it follows that their eigenvalues are necessarily real (consistent with the familiar fact that the result of a physical measurement is a real number). We also define the commutator of two operators A and B as:

$$[A, B] = AB - BA \quad (1.5)$$

It can be shown that if two operators commute (i.e. if their commutator equals zero), then they share a complete set of simultaneous eigenvectors. A noteworthy consequence of this is that physical states exist in which the results of measurement of both of these observables (A and B) can be obtained simultaneously with certainty: these are their common eigenstates.