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FOREWORD

ASCE TRANSACTIONS, 1961, Part I, contains, nominally, all papers published in the Journals of the Engineering Mechanics, Hydraulics, and Soil Mechanics and Foundations Divisions, Proceedings of the American Society of Civil Engineers. The following papers were not included in this part because discussion was not complete when the volume was closed:

JOURNAL OF THE ENGINEERING MECHANICS DIVISION

January 1960

Electrical Analog Computer for Limit Design of Structures
by M. Zaid and F. L. Ryder (Proc. Paper 2331)[a]

Comparative Study of a Segmental Arch Ring
by O. C. Zienkiewicz (Proc. Paper 2332)[a]

Relaxation Theory of Creep of Metals
by Francis H. Ree, Taikyue Ree, and Henry Erying (Proc. Paper 2333)[b]

Deflection Stability of Frames Under Repeated Loads
by E. P. Popov and R. E. McCarthy (Proc. Paper 2334)[a]

Travelling Loads on Rigid-Plastic Beams
by P. S. Symonds and B. G. Neal (Proc. Paper 2337)[a]

Design of Circular Plates Based on Plastic Limit Load
by L. W. Hu (Proc. Paper 2338)[a]

Commentary on Plastic Design in Steel: Compression Members
Progress Report No. 5 of the Joint WRC-ASCE Committee on Plasticity Related to Design (Proc. Paper 2342)[c]

April 1960

White Noise Representation of Earthquakes
by G. N. Bycroft (Proc. Paper 2434)[a]

Strength and Efficiency Aspects of Plate Structures
by George V. Gerard (Proc. Paper 2439)[a]

Dynamic Analysis of Elasto-Plastic Structures
by Glen Berg and Donald A. DaDeppo (Proc. Paper 2440)[a]

Behavior of Buckled Rectangular Plates
by Manuel Stein (Proc. Paper 2445)[a]

Commentary on Plastic Design in Steel: Connections
Progress Report No. 6 of the Joint WRC-ASCE Committee on Plasticity Related to Design (Proc. Paper 2453)[c]

Commentary on Plastic Design in Steel: Deflections
Progress Report No. 7 of the Joint WRC-ASCE Committee on Plasticity Related to Design (Proc. Paper 2454)[c]

February 1960

Hydrologic Studies by Electronic Computers in TVA
by Willard M. Snyder (Proc. Paper 2362)[a]

Scales of Viscous Analogy Models for Ground Water Studies
by Jacob Bear (Proc. Paper 2364)[a]

Boundary Layer Stimulation in Rectangular Conduits
by R. G. Cox and F. L. Bauer (Proc. Paper 2366)[a]

Scour at Bridge Crossings
by Emmett M. Laursen (Proc. Paper 2369)

Sediment Transport and Delta Formation
by E. Kuiper (Proc. Paper 2371)[a]

Trap Efficiency of Reservoirs, Debris Basins, and Debris Dams
by Charlie M. Moore, Walter J. Wood, and Graham W. Renfro (Proc. Paper 2374)[a]

March 1960

Development of Flow in Tank Draining
by David Burgreen (Proc. Paper 2415)[a]

April 1960

Conservancy Districts as Flood Control Organizations
by Cloyde C. Chambers (Proc. Paper 2429)

A Comparison of Stream Velocity Meters
by F. Wayne Townsend and F. A. Blust (Proc. Paper 2438)

Friction Losses on Lines with Service Connections
by David L. Muss (Proc. Paper 2449)

Sediment Problems of the Lower Colorado River
by Whitney M. Borland and Carl R. Miller (Proc. Paper 2452)

May 1960

Tolkmitt's Backwater and Dropdown Curve Tables
by R. D. Goodrich (Proc. Paper 2469)

Hood Inlet for Closed Conduit Spillways
by Fred W. Blaisdell (Proc. Paper 2478)

Uniform Water Conveyance Channels in Alluvial Material
by Daryl B. Simons and Maurice L. Albertson (Proc. Paper 2484)

Resistance to Flow in Alluvial Channels
by Daryl B. Simons and E. V. Richardson (Proc. Paper 2485)

June 1960

Translations of Foreign Literature on Hydraulics
Second Progress Report of the Task Force on List of Translations of the Committee on Hydromechanics of the Hydraulics Division (Proc. Paper 2514)[c]

Sedimentation Aspects in Diversion at Old River
by Fred B. Toffaleti (Proc. Paper 2525)

Generalized Solutions for Laterally Loaded Piles
by Hudson Matlock and Lymon C. Reese (Proc. Paper 2626)

December 1960

Tuttle Creek Dam of Rolled Shale and Dredged Sand
by K. S. Lane and R. G. Fehrman (Proc. Paper 2681)

In the foregoing list, the symbol [a] denotes a paper cleared for publication in the Journals prior to December 1, 1959, that will not be published in Transactions. The symbol [b] is used to signify papers that were part of Symposia that were not completed in time to publish the entire group in Transactions. The symbol [c] represents a committee report that will not be included in Transactions.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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TRANSACTIONS

Paper No. 3087

COLUMNS UNDER COMBINED BENDING AND THRUST

By Theodore V. Galambos,[1] M. ASCE, and Robert L. Ketter,[2] M. ASCE

With Discussion By Messrs. George Winter; and Theodore V. Galambos and Robert L. Ketter

SYNOPSIS

Interaction curves relating the axial thrust, applied end bending moment, and slenderness ratio are developed for the ultimate carrying capacity of pin-ended, wide-flange beam-columns. It is assumed that failure is due to excessive bending in the plane of the applied moments that is further considered to be the plane of the web. The two conditions of loading that are investigated are (1) equal end moments applied such that the resulting deformation is one of single curvature, and (2) end moment applied only at one extremity of the member. The influence of an assumed symmetrical residual stress pattern is considered in the computations, and curves are presented for slenderness ratios up to and including $L/r = 120$. For ease of design computations, the interaction curves are fitted into approximate equations. Comparisons are made with various column test results.

INTRODUCTION

When designing (or analyzing) a structure by the simple plastic theory, it is assumed that the member in question will deliver the full plastic moment value, M_p, noted in the computations. This, however, will not necessarily be

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the case if the member is subjected to an axial thrust in addition to bending moments (1).[3] To attain the desired moment value, it is necessary to supply a member having a greater fully plastic moment value than the one needed for pure bending that is, one that will develop the required end moment in the presence of the imposed axial thrust.

The problem that will be considered in this paper is the determination of the maximum amount of end bending moment that a member can sustain when it is also subjected to a given axial thrust. The material presented herein constitutes an extension of certain of the ideas advanced in an earlier paper (2). Two loading cases will be investigated:

1. Axial thrust plus equal end moments applied at both ends of the member such that it deforms in single curvature;
2. Axial thrust plus moment applied only at one end of the member.

These conditions are shown diagrammatically as loading conditions "c" and "d" in Fig. 1. In both cases it is assumed that the plane of the applied moments is that of the web of the section and that failure is the result of excessive bending in this same plane.

The stress-strain properties of the material are presupposed to be ideally elastic-plastic. That is, stress, σ, is proportional to strain, ϵ, until the yield stress, σ_y, is reached; thereafter the stress remains constant at $\sigma = \sigma_y$ as the strain increases indefinitely. This type of behavior is typical of mild structural (ASTM A7) steel if strain-hardening is neglected. There is, however, assumed to be a symmetrical residual stress pattern present in the member prior to the application of any external loads. The presumed pattern (Fig. 2) is consistent with measured residual stresses in wide-flange column type sections resulting from cooling of the section during and after rolling (2), (3).

Ketter, E. L. Kaminsky, A. M. ASCE, and L. S. Beedle, F. ASCE, have shown (2) that if the material is homogeneous and isotropic and if bending strains are assumed to be proportional to the distance from the neutral axis, then the thrust-moment-curvature relationship for the 8WF31 section will be that given in Fig. 3. In this figure two conditions are illustrated. The solid lines are for the cases in which residual stresses are neglected. The solutions that include the influence of the residual stress pattern shown in Fig. 2 are given by the dashed lines in Fig. 3.

Because the basic approach that will be used in solving the problem considered in this paper is one of numerical integration, and because this integration will proceed from a knowledge of the curvature values of Fig. 3, that, as was stated above, were computed for the 8WF31 section, the resulting interaction curves will in the strictest sense apply only to the 8WF31 section. It should be noted, however, that this section has one of the more severe thrust-moment-curvature relationships of the column sections rolled because of its low shape-factor (1.11 as compared to 1.14 for most sections). Using the interaction curves for other shapes should therefore result in a somewhat conservative or, at least, equal prediction of strength for the member in question.

For ease of presentation and generalization, load and section property parameters have been non-dimensionalized wherever possible. It was necessary, however, to consider a fixed value of Young's Modulus of E = 30,000,000 psi. Because specifications require a minimum yield stress of σ_y = 33,000 psi for A7 steels, this value was used in the computations as the base yield stress.

[3] Numerals in parentheses, thus (1), refer to corresponding items in the Bibliography.

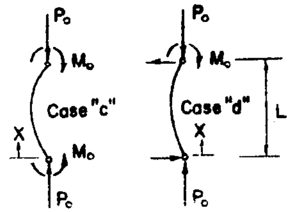

FIG. 1.—CONDITIONS OF LOADING

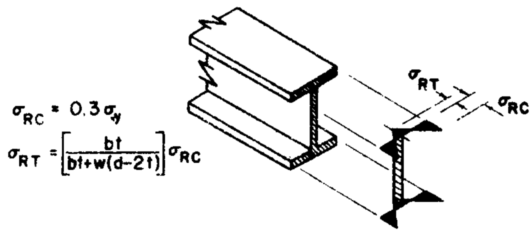

FIG. 2.—ASSUMED COOLING RESIDUAL STRESS PATTERN

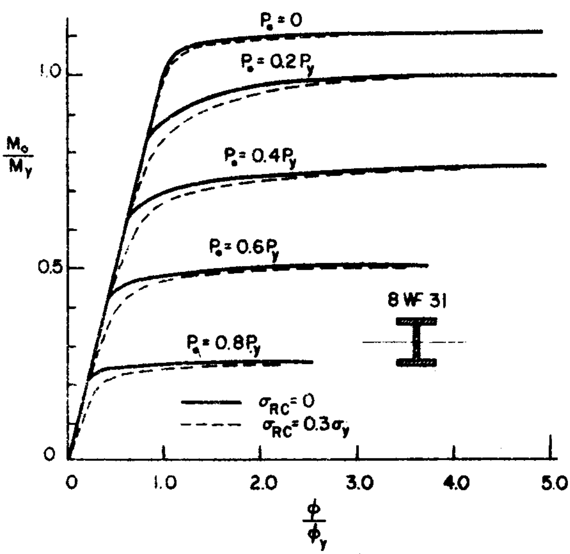

FIG. 3.—MOMENT-THRUST-CURVATURE RELATIONSHIP

The results of this report then apply without modification to rolled wide-flange columns of A-7 structural steel. Similar procedures could be used for other materials and loading conditions. However, to afford a means of comparing test results with predicted strengths and to facilitate the solution of problems where the material has a yield point other than 33,000 psi, a method of modifying the solution will be presented subsequently.

Notation.—The letter symbols adopted for use in this paper are defined where they first appear and are arranged alphabetically, for convenience of reference, in Appendix II.

DETERMINATION OF INTERACTION CURVES

The approach that will be used in the solution of the problem in question will be one of numerical integration (4). The computation will proceed from an assumed deflection configuration and will take into account the non-linearity between moment and curvature as strains exceed the initial yield strain.

Since deflections must be assumed, it is desirable to know the equation of the column centerline at initiation of yielding for each of the conditions of loading. These can be determined from a consideration of the equations presented by Timoshenko (5). In terms of the parameters used in this report, the equations are as follows:

a) Moments applied at both ends of the member,

$$y = \frac{S}{A}\left[\frac{M_o/M_y}{P_o/P_y}\right]\left[\frac{\sin kx}{\sin kL} + \cos kx - \cot kL \sin kx - 1\right] \qquad (1)$$

b) Moment applied only at one end of the member

$$y = \frac{S}{A}\left[\frac{M_o/M_y}{P_o/P_y}\right]\left[\frac{\sin kx}{\sin kL} - \frac{x}{L}\right] \qquad (2)$$

In these equations S is the section modulus, A refers to the cross-sectional area, M_y is the yield moment ($M_y = S\sigma_y$); P_y is the axial load causing full yielding ($P_y = A\sigma_y$), x equals the distance along member as shown in Fig. 1, y is the lateral deflection of the column centerline in the plane of bending, and $k = \sqrt{P_o/EI}$.

For the assumed values of $E = 30{,}000{,}000$ psi and $\sigma_y = 33{,}000$ psi

$$\left.\begin{aligned} kL &= 0.03317\left(\frac{L}{r}\right)\sqrt{\frac{P}{P_y}} \\ &\text{and} \\ kx &= 0.03317\left(\frac{x}{L}\right)\left(\frac{L}{r}\right)\sqrt{\frac{P}{P_y}} \end{aligned}\right\} \qquad (3)$$

From Eqs. 1 and 2, it can be seen that for the conditions of constant axial thrust and elastic behavior there is a linear relationship between the applied end moment, M_o, and the resulting deformation. The maximum value of M_o for which this situation holds is referred to as the initial yield value, and the