ADVANCES IN

FRACTURE RESEARCH

Editors

K. SALAMA, K. RAVI-CHANDAR,
D. M. R. TAPLIN, P. RAMA RAO

Volume 3

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in
Fracture Research

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Outline Contents

Volume 3

Computational Fracture Mechanics	1885
Damage Mechanics	2135
Mixed-Mode Fracture	2241

Volume 1

Brittle Fracture	1
Ductile Fracture	123
Dynamic Fracture	583

Volume 2

Fatigue	867
Creep and Environmental Fracture	1479

Volume 4

Fracture of Metallic Materials	2411
Fracture of Nonmetallic Materials	2659
Composites and Failure of Interfaces	2937

Volume 5

NDE and Experimental Techniques	3097
Fractography	3363
Applications	3493

Volume 6

Additional Papers	3583
Subject Index	4063

CONTENTS OF VOLUME 3

VI. COMPUTATIONAL FRACTURE MECHANICS **1885**

Computational Fracture Mechanics*
H. Liebowitz 1887

On Some Recent Advances in Computational Methods in the Mechanics of Fracture*
S.N. Atluri and T. Nishioka 1923

Numerical Calculations for Problems of Ductile Fracture*
R.M. McMeeking 1971

Progress in the Assessment of Complex Components*
E. Sommer 1999

Plasticity and Thickness Effect
L. Anquez 2021

Crack Tip Parameters for a Central Crack in an Elastic Plastic Plate Under Mixed Mode and Repetitive Loading
W.S. Blackburn 2029

Analyses of Crack Growth in Ductile Solids
A. Needleman and V. Tvergaard 2041

Numerical Evaluation of Creep Experiments
W. Schmitt, R. Kienzler and T. Hollstein 2049

Analysis of J-Integral and HRR Singularity Using Experimental and Computational Hybrid Method with Image Processing
G. Yagawa, N. Soneda and S. Yoshimura 2059

An Economic Finite Element Strategy for LEFM Problems
K. Badari Narayana, T.S. Ramamurthy, B. Dattaguru and K. Vijayakumar 2067

Advanced Analytical Methods for Interpreting Crack Run-Arrest Events in Reactor Pressure Vessel Steels
C.R. Bass, C.E. Pugh, D.J. Naus and J. Keeney-Walker 2075

A Singular Element for a Variable Crack Length
B. Fedelich and A. Ehrlacher 2083

Mesh Generation by Conformal Mapping in Two-Dimensional Fracture Problems
O. Haber, H. Grebner and A. Hofler 2091

Computer Modelling of the Evolution of Hydraulic Fracturing Phenomena in Hydrocarbon Reservoirs
R. Romagnoli and R. Varvelli 2099

Finite Element Method of Thin Shell J Integral Evaluation
A. Sedmak, M. Berkovic, J. Jaric and S. Sedmak 2111

Energy Criteria for Stable Crack Growth Simulation Under Biaxial Loading Using Finite Element Method
R.N. Singh and C.V. Ramakrishnan 2119

An Integral Equation Method based on Resultant Forces on a Piece-Wise Smooth Crack in a Finite Plate
W.L. Zang and P. Gudmundson 2127

VII. DAMAGE MECHANICS **2135**

Unified Damage Approach to Crack Initiation*
J. Lemaitre 2137

Stable Path of Interacting Crack Systems and Micromechanics of Damage
Z.P. Bazant, M.R. Tabbara and M.T. Kazemi 2141

Energy Analysis of Crack-Damage Interaction
A. Chudnovsky and S. Wu 2153

Fracture of Fibrous Metal Matrix Composites
G.J. Dvorak and Y.A. Bahei-El-Din 2161

Damage Equations for Physically-Based Creep Life
B.F. Dyson and F.A. Leckie 2169

A Method for Mechanical State Characterization of Inelastic Composite Laminates with Damage
R.A. Schapery 2177

Continuum Modeling of the Development of Intralaminar Cracking in Composite Laminates
R. Talreja 2191

A General Damage Criterion for Solids
C.C. Hsiao and Y.S. Cheng 2201

Void Formation in Short-Fiber Composites
A. Needleman and S.R. Nutt 2211

Damage Analysis of Materials with Microvoids
Du Shanyi and Liu Yingjie 2219

Plastic Damage Evolution in Low-Carbon Steels
Wei Hua Tai 2225

The Damage Criterion for Ductile Fracture and its Applications
Wei Hua Tai 2231

VIII. MIXED MODE FRACTURE **2241**

Mixed Mode Cohesive Crack Propagation
A. Carpinteri, S. Valente and P. Bocca 2243

Mixed Mode Fracture in Concrete
A. Hillerborg 2259

On Mixed-Mode Plane Stress Ductile Fracture
Y.W. Mai and B. Cotterell 2269

Various Crack Concepts for Curved Fracture in Concrete
J.G. Rots 2279

About the Penny-Shaped Dugdale Crack Under Shear and Triaxial Loading
W. Becker and D. Gross 2289

Fracture Analysis of Non-Coplanar Crack Extension under Mixed-Mode Loading by the Crack Closure Integral Method
F.-G. Bucholz and H.A. Richard 2301

Shear Failure of Al-Cu Single Crystals
C.Q. Chen, H.X. Li and W. Yao 2309

Use of Two Singular Point Finite Elements in the Analysis of Kinked Cracks
B.K. Dutta, A. Kakodkar and S.K. Maiti 2315

Some Aspects of Fatigue Crack Growth under Mixed Mode Loading
K. Henn and H.A. Richard 2323

High-Speed Frictional Heating Effect on the Stress Intensity Factors of a Near-Surface Line Crack
F.D. Ju and T.Y. Chen 2331

The Stress Intensity Factors at the Tip of a Kinked and Curved Crack
J.B. Leblond and M. Amestoy 2339

Mode Transformation of Growing Stage I Cracks in Polycrystals
Chingshen Li and T. Bretheau 2347

Mixed Modes of the Crack Propagation under Biaxial Cyclic Load
V.N. Shlyannikov and V.A. Dolgorukov 2355

Torsional Vibration of a Penny-Shaped Crack in a Transversely Isotropic Material
Y.M. Tsai 2363

Tribo Fracture - Basic Concepts and Phenomena
O. Vingsbo 2369

Fracture Curve of a Thin-Walled Tube in Torsion Test
Jie-Qing 2377

The Line Spring Model for Surface Cracked Shell Subjected to Antisymmetric Loadings
Lu Yinchu and Tang Guojin 2387

Experimental Study on Mixed Mode Crack Propagation
Zhao Yishu 2395

The Line-Spring Model Analysis of Surface Crack at Weld Joint of Two Tubes
Lu Yinchu and Liu Xi 2403

VI. COMPUTATIONAL FRACTURE MECHANICS

Computational Fracture Mechanics

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ABSTRACT

The field of Computational Fracture Mechanics is reviewed. The paper focuses on the impact of computational methodology on furthering the understanding of fundamental fracture phenomena. The current numerical approaches to the solution of fracture mechanics problems, e.g. finite element methods, finite difference methods and boundary element methods are reviewed. The application of these techniques to the problems of linear elastic fracture problems is discussed. Particular emphasis is placed on three dimensional problems and the issues involved with surface crack geometries and stress intensity factor calculations.

Numerical solutions of two dimensional ductile fracture problems are surveyed. A special focus is placed on the effect of stable crack growth on the field quantities and the implications of numerical solutions for fracture prediction. Creep fracture problems are discussed. The similarities and differences between creep and ductile fracture problems are highlighted. The importance of large strain phenomena and accurate modeling of nonlinear effects are highlighted.

The current state of knowledge of continuum fields for elastostatic cracks, elastodynamic cracks, ductile cracks and viscoplastic cracks is summarized. The range of applicability of asymptotic solutions (especially in the nonlinear regimes) is highlighted.

Major research needs in computational fracture mechanics are detailed. Emphasis is placed on coupled theoretical and numerical approaches. Prospects for future research trends are proffered. Application of fracture mechanics and computational fracture approaches are explored.

KEYWORDS

Crack propagation; failure prediction; finite element; fracture mechanics; nonlinear methods; numerical methods; research needs.

INTRODUCTION

The field of fracture mechanics has progressed significantly over the past thirty years. Fracture mechanics now provides a firm theoretical basis for the prediction of fracture and the fracture-proof design of new structures for many applications (most notably for applications with solely elastic response). For other problems (where ductility or environmental effects are present), fracture mechanics has progressed toward an understanding and theoretical framework for the future. While much additional research is required before fracture mechanics can be considered a mature discipline, it is recognized that significant advancement has been made. Fracture mechanics is based on the assumption of a continuum material behavior of the structural component under analysis. The effect of atomic spacing and material microstructure, therefore, is assumed to be totally represented by the constitutive equations employed in the continuum model. Hence, this assumption is the major limiting factor in the development of a quantitative, cohesive theory of fracture. The ultimate theory of fracture should attempt to couple the microscopic and macroscopic fracture characteristics in a coherent manner. This task is a major requirement of future research in fracture mechanics.

The advent of the digital computer made it possible to solve engineering and scientific problems by using numerical techniques. Many problems which could not be addressed analytically could (at least in theory) be solved numerically. As computers have become faster, cheaper, more powerful and more widely available, the number of problems which are addressed numerically has grown exponentially. The field of fracture mechanics has benefited dramatically from the use of the digital computer. Routine use of Linear Elastic Fracture Mechanics (LEFM) in fracture-proof design can be largely attributed to the ability to solve fracture problems routinely using digital computers. Critical technology problems involving material and geometric nonlinearities have been addressed successfully using numerical solutions. Indeed, many application areas would have been significantly hindered (if not stopped) without the numerical solution of fracture problems. In addition, much fundamental understanding of the behavior of materials containing cracks has been gained through numerical simulation of fracture problems.

The purpose of this paper is to provide a critical examination of the impact of numerical methods on the field of fracture mechanics. For the purposes of this discussion, fracture mechanics problems will be subdivided into three major classes: Linear Elastic Fracture Mechanics (LEFM) problems (both static and dynamic), problems involving composite materials, and ductile fracture problems (including rate dependent problems). These broad topics represent the major areas of challenge and application of the field of fracture mechanics.

The paper starts with a discussion of the major numerical approaches available for the numerical solution of boundary value problems. Emphasis is placed on the Boundary Integral Equation Method (BIEM) and the Finite Element Method (FEM). These approaches are the major methods employed for the solution of fracture mechanics problems. Historical note is made concerning integral equation methods and finite difference methods. Emphasis in this section is on the strengths, weaknesses and successes of the methods to date.

The problem areas of LEFM and ductile fracture problems are then considered in turn. The emphasis in each section is placed on highlighting the impact of numerical solutions on the understanding of each problem area and the application of the methodology to design considerations. Also considered is the role of asymptotic and analytic ideal problem solutions in the numerical solution of real engineering problems. An important issue is the value of numerical solutions and the delineation of their limitations.

After surveying the major problem areas and their state of the art, the discussion turns to the major needs of fracture mechanics and the role that numerical methods can play in fulfilling these needs. The majority of this centers on the role of computer simulation, visualization and the interpretation of results. Emphasis is on coupling accurate numerical solutions to physical insight and understanding. A very important concern is the consideration of the numerical solution needs in the formulation stage.

The paper concludes with a discussion of the major obstacles and challenges that face researchers in the numerical solution of fracture mechanics problems. Coupling of numerical and theoretical advances and approaches is emphasized. An attempt is made to focus on those issues which can shed important light on the open questions in the field of fracture mechanics.

NUMERICAL METHODS FOR SOLUTION OF FRACTURE PROBLEMS

The problems of fracture mechanics reduce to the solution of boundary value problems (which may be static or dynamic) which have mixed boundary conditions. These mixed boundary conditions can give rise to singularities in the stress and strain fields. The problems may involve both material and geometric nonlinearities which complicate the formulation and render prediction of convergence extremely difficult. Because little can be done with these problems analytically, numerical methodologies are required. The advent of large scale computers coupled with the rapid growth in the field of algorithmic methods render many of the problems of fracture mechanics tractable today.

The finite difference method is the oldest technique for the solution of boundary value problems and was widely employed in the 1960s. The method directly involves the solution of the governing differential system in an approximate manner by subdividing the domain of interest into a connected series of discrete points called nodes. These nodes are the sampling points for the solution and are linked using the finite difference operators to the governing equations. For example, the second order finite difference operator for the second partial derivative of a two dimensional field variable is given by

$$\left.\frac{\partial^2 \psi}{\partial x^2}\right|_{x_i, y_j} = \frac{\psi(x_{i+1}, y_j) - 2\psi(x_i, y_i) + \psi(x_{i-1}, y_j)}{(\Delta x)^2} \quad (1)$$

where ψ is the field variable and x and y are the independent spatial variables. This is a second order difference operator and the error is proportional to the square of the mesh spacing in x ($O(\Delta x^2)$). Employment of the finite difference operators results in a system of algebraic equations for the discrete nodal values of the field variable.

Gradients can be evaluated by employing finite difference operators to the discrete solution.

Finite difference methods can be used to discretize both space and time. In addition, they provide easy error estimation techniques. Unfortunately, finite difference methods are difficult to use for irregularly shaped domains. Often absurd discretization is required for accurate solution. In addition, it is difficult to implement meshes without equal grid spacing. Convergence is difficult to gauge with this characteristic. An excellent discussion of the finite difference approach to the solution of partial differential equations can be found in Lapidus and Pinder (1982).

Finite difference methods have not performed very well for problems involving singularities. One major reason for this is that the fine meshing required near a singularity cannot easily be reduced for the rest of the domain. Special finite difference techniques which directly handle singularities can be developed; however, they have not been very successful for practical applications. Computational requirements for convergence are larger than for finite element and boundary element solutions. The finite difference method is not seriously employed for fracture problems today.

In addition to finite difference methods, integral equation methods are a historic approach to the solution of fracture problems and are still used by some researchers today. The basic approach employed involves an analytic formulation of the elasticity problem to the point of a singular integral equation. The singularity is then extracted and the result is a nonsingular integral equation which can be solved quite accurately with any number of techniques. This approach yields excellent solutions, however, it requires an extensive analytic formulation which is different for each new problem. The method is quite useful, nonetheless, for establishing benchmark solutions to compare with other methods as the degree of accuracy can be guaranteed. The method is only applicable to elasticity problems (no nonlinearities). For three dimensional problems, it is almost impossible to derive the integral equations in a finite period of time. An excellent discussion of the method can be found in Muskhelishvili (1953).

Two major numerical approaches are available for the solution of fracture mechanics problems today: the Boundary Integral Equation Method (BIEM) and the Finite Element Method (FEM). These techniques have been widely researched and developed. For two dimensional Linear Elastic Fracture Mechanics (LEFM) problems, either can be employed with much confidence and accuracy. Both BIEM and FEM are actually a class of approaches with many variants which allow a flexible approach for modeling many areas of application. The discussion of each given below will focus on the methods as they commonly are applied to fracture mechanics problems and the variants employed by some authors for better solution characteristics.

The BIEM method is a numerical approach to the solution of linear boundary value problems with known Green's function solutions. The boundary of the domain of interest is discretized using "elements" which are interconnected at discrete points called nodes. For a three dimensional problem, the mesh is two dimensional; for two dimensional problems, the mesh is one dimensional. The boundary value problem is formulated as an equivalent surface or line integral using the Green's function solution and the governing differential system. For linear elasticity in two dimensions, the formulation is based on Betti's theorem and the resulting system of equations is given by

$$C_{lk}u_k + \int_\Gamma u_k T_{lk}\, d\Gamma = \int_\Gamma t_k\, U_{lk}\, d\Gamma \qquad (l,k = 1,2) \tag{2}$$

where u_k and t_k are the surface displacement and traction vectors, is the domain boundary, and U_{lk} and T_{lk} are related to the Green's function solutions for displacement and tractions. At each boundary point, either u or t is specified and the other variable is unknown. These relate to the physical field variables in question. A complete discussion of the approach can be found in Banerjee and Butterfield (1981).

The BIEM method is a quickly convergent, highly robust method for the solution of linear boundary value problems. It is relatively easy to employ and general purpose commercial software can be developed around the method (the BEASY code is a widely available example; see BEASY in References). Because the surface of the domain need only be discretized, it is easier to use the BIEM than the FEM (to be discussed subsequently). For static problems, the BIEM method reduces to the solution of a system of dense linear equations which may be nonsymmetric (although methods of symmeterizing the systems recently have been very successful). If surface data is the only quantity required (as is the case in many fracture problems where the only interesting results are the stress intensity factors and the compliance), the BIEM is often computationally superior to the FEM for two dimensional problems. If interior data is required, the method is computationally costly. For three dimensional problems, BIEM solutions are often very expensive as the resulting linear system is dense, unbanded and often nonsymmetric. Ongoing research, however, is addressing this problem rapidly. BIEM solutions often yield excellent results for field quantities and their gradients (e.g. displacements and strains). Primary unknown predictions on par with FEM solutions usually predict better gradients within the BIEM concept.

For applications in fracture mechanics, the BIEM has received a good bit of attention recently. For two dimensional problems, the BIEM can be employed for the solution of fracture problems with much success. Mesh generation is quite simple and users can master the techniques rapidly (much more so than for the FEM). Accurate solutions can be obtained and reasonable error estimates can be predicted. It is certainly competitive with the FEM if not better for these problems. The numerical techniques employed for fracture mechanics problems are summarized in Table 1.

Three dimensional LEFM problems have been solved using the BIEM without great success. These solutions are quite costly and often do not produce good solutions. As an example, consider the problem of an edge cracked rectangular bar subjected to uniaxial uniform tensile stress as shown in Fig. 1. The resulting stress intensity factor distribution is shown in Fig. 2 and is compared with well established finite element results. It can be seen that near the midplane the results agree well. Far from the midplane, however, resolution degrades. Because it is well known that FEM solutions of surface crack problems overestimate the boundary layer effect near the free surface, the BIEM results are in error (Rooke et al., 1987). Interior crack problems have been solved successfully; however, this is not a sufficient test of the method. Ongoing research hopefully will address this problem, although the BIEM is not a current competitor for three dimensional problems.

Table 1. Numerical methods for the solution of fracture problems

Method	Strengths	Weaknesses
Finite Difference	Easy to employ Error estimates available	Slow convergence Uniform mesh requirements Cannot model singularities
Finite Elements	Good convergence Singularities can be modeled	Modeling is difficult Few exiting error estimators
Boundary Elements	Modeling is easier Error estimation is easier	Computationally more expensive for most problems Converge slowly for singular problems
Hybrid Approaches	Good for specific problems Generally very accurate	Usually developed for restricted problem class Often difficult to implement

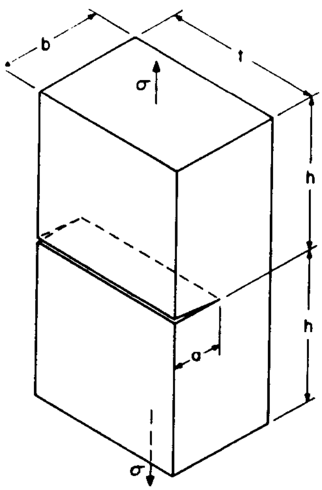

Fig. 1 Edge cracked rectangular bar subjected to uniaxial uniform tensile stress

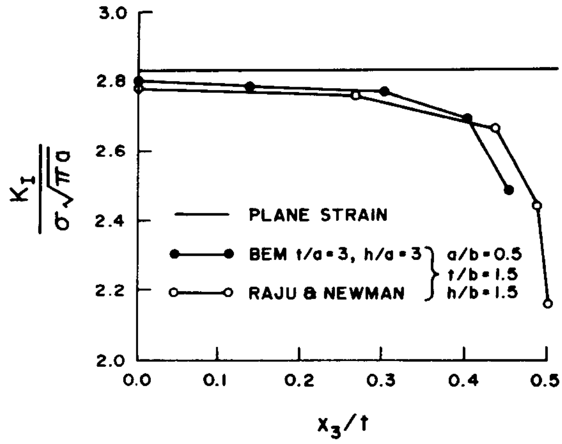

Fig. 2. Variation of the stress intensity factor along the crack front

Much effort has been focused recently on the extension of BIEM to nonlinear problems where known Green's functions do not exist. This work is in its infancy and it is fair to say that the approach has yet to impact the field of fracture mechanics. Indeed, available solutions to problems with extensive nonlinear material behavior are disappointing (e.g. Wilson et al., 1985). Ongoing research may establish BIEM approaches to nonlinear problems which produce reasonable answers. For nonlinear problems, analytical Green's functions are not available. A variational approach with assumed trial and weight functions must be employed. The formulation is similar to that employed by the finite element method. The BIEM, therefore, will have the same approximate formulation as the FEM.

The FEM is the most widely employed numerical method for the solution of fracture mechanics problems. The formulation of the FEM is based on a variational statement of the governing physics. For the problems of linear elasticity, the principle of Virtual Work, given by

$$\int_V \sigma_{ij}\ \delta\varepsilon_{ij}\ dV = \int_S \sigma_{ij}\ n_j\ \delta u_i\ dS \quad , \tag{3}$$

is employed where σ_{ij} is the stress tensor, $\delta\varepsilon_{ij}$ is the virtual strain tensor due to virtual displacements δu_i and n_j is the normal vector to the surface of applied tractions. The domain is discretized into subdomains (elements) which are interconnected through common discrete points (nodes). The primary unknown field variables are nodal values. The formulation reduces the problem to the solution of a system of algebraic equations in terms of the nodal variables (for dynamic problems, the result is a system of ordinary differential equations). Finite element systems tend to be relatively banded and symmetric for most problems. The resulting systems can be solved using a number of techniques. For nonlinear problems, algorithms are also available, however, accuracy and convergence are much larger problems.