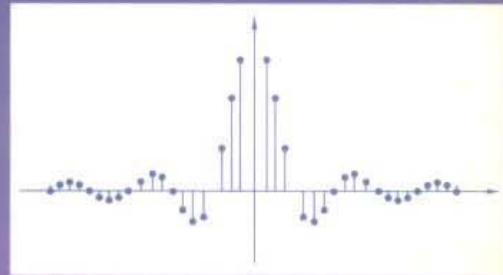


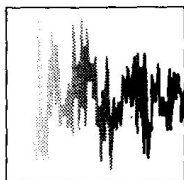
*Third Edition*

# DIGITAL SIGNAL PROCESSING

*Principles, Algorithms, and Applications*



John G. Proakis  
Dimitris G. Manolakis



# Digital Signal Processing

**Principles, Algorithms, and Applications**

**Third Edition**

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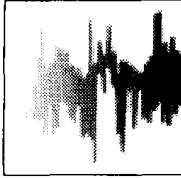
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## Preface

This book was developed based on our teaching of undergraduate and graduate level courses in digital signal processing over the past several years. In this book we present the fundamentals of discrete-time signals, systems, and modern digital processing algorithms and applications for students in electrical engineering, computer engineering, and computer science. The book is suitable for either a one-semester or a two-semester undergraduate level course in discrete systems and digital signal processing. It is also intended for use in a one-semester first-year graduate-level course in digital signal processing.

It is assumed that the student in electrical and computer engineering has had undergraduate courses in advanced calculus (including ordinary differential equations), and linear systems for continuous-time signals, including an introduction to the Laplace transform. Although the Fourier series and Fourier transforms of periodic and aperiodic signals are described in Chapter 4, we expect that many students may have had this material in a prior course.

A balanced coverage is provided of both theory and practical applications. A large number of well designed problems are provided to help the student in mastering the subject matter. A solutions manual is available for the benefit of the instructor and can be obtained from the publisher.

The third edition of the book covers basically the same material as the second edition, but is organized differently. The major difference is in the order in which the DFT and FFT algorithms are covered. Based on suggestions made by several reviewers, we now introduce the DFT and describe its efficient computation immediately following our treatment of Fourier analysis. This reorganization has also allowed us to eliminate repetition of some topics concerning the DFT and its applications.

In Chapter 1 we describe the operations involved in the analog-to-digital conversion of analog signals. The process of sampling a sinusoid is described in some detail and the problem of aliasing is explained. Signal quantization and digital-to-analog conversion are also described in general terms, but the analysis is presented in subsequent chapters.

Chapter 2 is devoted entirely to the characterization and analysis of linear time-invariant (shift-invariant) discrete-time systems and discrete-time signals in the time domain. The convolution sum is derived and systems are categorized according to the duration of their impulse response as a finite-duration impulse

response (FIR) and as an infinite-duration impulse response (IIR). Linear time-invariant systems characterized by difference equations are presented and the solution of difference equations with initial conditions is obtained. The chapter concludes with a treatment of discrete-time correlation.

The  $z$ -transform is introduced in Chapter 3. Both the bilateral and the unilateral  $z$ -transforms are presented, and methods for determining the inverse  $z$ -transform are described. Use of the  $z$ -transform in the analysis of linear time-invariant systems is illustrated, and important properties of systems, such as causality and stability, are related to  $z$ -domain characteristics.

Chapter 4 treats the analysis of signals and systems in the frequency domain. Fourier series and the Fourier transform are presented for both continuous-time and discrete-time signals. Linear time-invariant (LTI) discrete systems are characterized in the frequency domain by their frequency response function and their response to periodic and aperiodic signals is determined. A number of important types of discrete-time systems are described, including resonators, notch filters, comb filters, all-pass filters, and oscillators. The design of a number of simple FIR and IIR filters is also considered. In addition, the student is introduced to the concepts of minimum-phase, mixed-phase, and maximum-phase systems and to the problem of deconvolution.

The DFT, its properties and its applications, are the topics covered in Chapter 5. Two methods are described for using the DFT to perform linear filtering. The use of the DFT to perform frequency analysis of signals is also described.

Chapter 6 covers the efficient computation of the DFT. Included in this chapter are descriptions of radix-2, radix-4, and split-radix fast Fourier transform (FFT) algorithms, and applications of the FFT algorithms to the computation of convolution and correlation. The Goertzel algorithm and the chirp- $z$  transform are introduced as two methods for computing the DFT using linear filtering.

Chapter 7 treats the realization of IIR and FIR systems. This treatment includes direct-form, cascade, parallel, lattice, and lattice-ladder realizations. The chapter includes a treatment of state-space analysis and structures for discrete-time systems, and examines quantization effects in a digital implementation of FIR and IIR systems.

Techniques for design of digital FIR and IIR filters are presented in Chapter 8. The design techniques include both direct design methods in discrete time and methods involving the conversion of analog filters into digital filters by various transformations. Also treated in this chapter is the design of FIR and IIR filters by least-squares methods.

Chapter 9 focuses on the sampling of continuous-time signals and the reconstruction of such signals from their samples. In this chapter, we derive the sampling theorem for bandpass continuous-time-signals and then cover the A/D and D/A conversion techniques, including oversampling A/D and D/A converters.

Chapter 10 provides an indepth treatment of sampling-rate conversion and its applications to multirate digital signal processing. In addition to describing decimation and interpolation by integer factors, we present a method of sampling-rate

conversion by an arbitrary factor. Several applications to multirate signal processing are presented, including the implementation of digital filters, subband coding of speech signals, transmultiplexing, and oversampling A/D and D/A converters.

Linear prediction and optimum linear (Wiener) filters are treated in Chapter 11. Also included in this chapter are descriptions of the Levinson–Durbin algorithm and Schür algorithm for solving the normal equations, as well as the AR lattice and ARMA lattice-ladder filters.

Power spectrum estimation is the main topic of Chapter 12. Our coverage includes a description of nonparametric and model-based (parametric) methods. Also described are eigen-decomposition-based methods, including MUSIC and ESPRIT.

At Northeastern University, we have used the first six chapters of this book for a one-semester (junior level) course in discrete systems and digital signal processing.

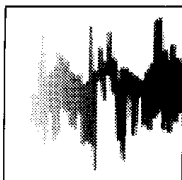
A one-semester senior level course for students who have had prior exposure to discrete systems can use the material in Chapters 1 through 4 for a quick review and then proceed to cover Chapter 5 through 8.

In a first-year graduate level course in digital signal processing, the first five chapters provide the student with a good review of discrete-time systems. The instructor can move quickly through most of this material and then cover Chapters 6 through 9, followed by either Chapters 10 and 11 or by Chapters 11 and 12.

We have included many examples throughout the book and approximately 500 homework problems. Many of the homework problems can be solved numerically on a computer, using a software package such as MATLAB®. These problems are identified by an asterisk. Appendix D contains a list of MATLAB functions that the student can use in solving these problems. The instructor may also wish to consider the use of a supplementary book that contains computer based exercises, such as the books *Digital Signal Processing Using MATLAB* (P.W.S. Kent, 1996) by V. K. Ingle and J. G. Proakis and *Computer-Based Exercises for Signal Processing Using MATLAB* (Prentice Hall, 1994) by C. S. Burrus et al.

The authors are indebted to their many faculty colleagues who have provided valuable suggestions through reviews of the first and second editions of this book. These include Drs. W. E. Alexander, Y. Bresler, J. Deller, V. Ingle, C. Keller, H. Lev-Ari, L. Merakos, W. Mikhael, P. Monticciolo, C. Nikias, M. Schetzen, H. Trussell, S. Wilson, and M. Zoltowski. We are also indebted to Dr. R. Price for recommending the inclusion of split-radix FFT algorithms and related suggestions. Finally, we wish to acknowledge the suggestions and comments of many former graduate students, and especially those by A. L. Kok, J. Lin and S. Srinidhi who assisted in the preparation of several illustrations and the solutions manual.

John G. Proakis  
Dimitris G. Manolakis



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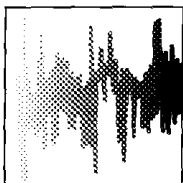
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# 1

## Introduction

Digital signal processing is an area of science and engineering that has developed rapidly over the past 30 years. This rapid development is a result of the significant advances in digital computer technology and integrated-circuit fabrication. The digital computers and associated digital hardware of three decades ago were relatively large and expensive and, as a consequence, their use was limited to general-purpose non-real-time (off-line) scientific computations and business applications. The rapid developments in integrated-circuit technology, starting with medium-scale integration (MSI) and progressing to large-scale integration (LSI), and now, very-large-scale integration (VLSI) of electronic circuits has spurred the development of powerful, smaller, faster, and cheaper digital computers and special-purpose digital hardware. These inexpensive and relatively fast digital circuits have made it possible to construct highly sophisticated digital systems capable of performing complex digital signal processing functions and tasks, which are usually too difficult and/or too expensive to be performed by analog circuitry or analog signal processing systems. Hence many of the signal processing tasks that were conventionally performed by analog means are realized today by less expensive and often more reliable digital hardware.

We do not wish to imply that digital signal processing is the proper solution for all signal processing problems. Indeed, for many signals with extremely wide bandwidths, real-time processing is a requirement. For such signals, analog or, perhaps, optical signal processing is the only possible solution. However, where digital circuits are available and have sufficient speed to perform the signal processing, they are usually preferable.

Not only do digital circuits yield cheaper and more reliable systems for signal processing, they have other advantages as well. In particular, digital processing hardware allows programmable operations. Through software, one can more easily modify the signal processing functions to be performed by the hardware. Thus digital hardware and associated software provide a greater degree of flexibility in system design. Also, there is often a higher order of precision achievable with digital hardware and software compared with analog circuits and analog signal processing systems. For all these reasons, there has been an explosive growth in digital signal processing theory and applications over the past three decades.